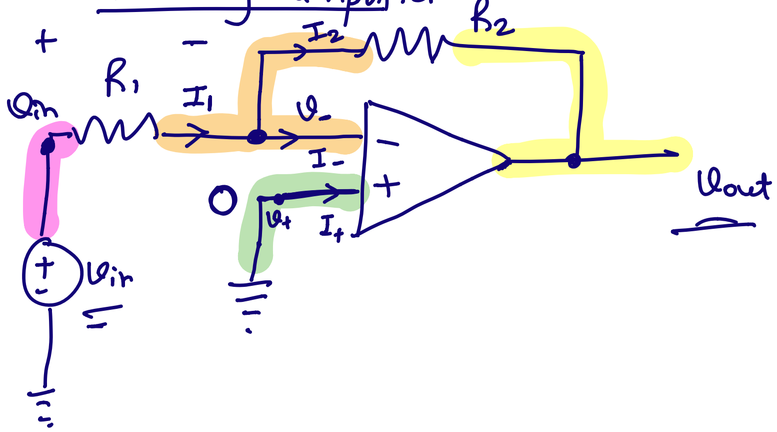
EECS 16A   Nov 10, 2020

→ logistics
- Optimal lab.
- Redo.

Today:

- Inverting amplifer
- Module 3
  - ↳ Classification.
  - Inner Product
  - Design for GPS system.

Inverting amplifier.



V_{out} ↑ ↓

v_- ↑ ↓

$(v_+ - v_-)$ ↓ ↑

$V_{out} = A(v_+ - v_-)$

V_{out} = ↓ ↑

Checking negtive feedback.

(1) Check that a perturbation in the output gets "supressed"

If V_{out} ↑ → the feedback path should bring it back down.

If $v_{out} \downarrow \rightarrow$ ____________ $\downarrow$ ____________

____________ fb. path $\uparrow$

② Ignore any independent sources.

Negative feedback $\Rightarrow$ Golden Rules.

$v_+ = v_-$ $\equiv$ $v_+ = v_- = 0$

$I_+ = I_- = 0$

KCL: $I_1 = I_2 + \underbrace{I_-}$

$\Rightarrow I_1 = I_2 + 0$

$I_1 = I_2$

$$I_1 = \frac{v_{in} - v_-}{R_1}$$

$$I_2 = \frac{v_- - v_{out}}{R_2}$$

$$I_1 = I_2$$

$$\frac{v_{in} - v_-}{R_1} = \frac{v_- - v_{out}}{R_2}$$

$$\frac{v_{in}}{R_1} = -\frac{v_{out}}{R_2}.$$

$$\Rightarrow \boxed{v_{out} = -\frac{R_2}{R_1} \cdot v_{in}}$$

Module 1: Systems + Modeling.

Module 2: One specific model

Module 3 :

- Classification.
- Estimation.
- Prediction.
- Clustering.

Techniques :

- Modeling
- "Optimization"

GPS System : 24 Satellites

- Distance from each satellite
- Position of satellites.
- Speed of the satellite
- How to use distance $\longrightarrow$ location.
- Which satellite am I talking to ?
- Identifiers.

Inner Product : $\vec{v}, \vec{w}$ be two vectors $\in \mathbb{R}^n$.

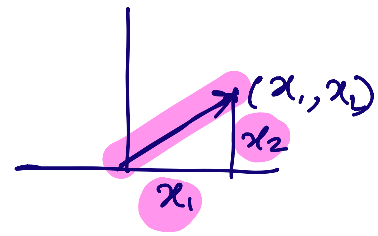
$$\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}.$$

Define: Inner product $\langle \vec{v}, \vec{w} \rangle = \vec{v}^T \vec{w}$

$$= \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ \vdots \\ w_n \end{bmatrix}$$

$1 \times n$ $\qquad$ $n \times 1$

$$= v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$$



"Dot Product"

"Correlation"

Properties:

① $\langle \vec{v}, \vec{w} \rangle = \langle \vec{w}, \vec{v} \rangle$

② $\langle \vec{v}, \vec{v} \rangle = v_1^2 + v_2^2 + \cdots + v_n^2$

$= \|\vec{v}\|^2$ (Norm of the vector squared)

"Magnitude squared"

norm = magnitude = length.

$$= \left(v_1^2 + v_2^2 + \cdots + v_n^2 \right)^{1/2}$$

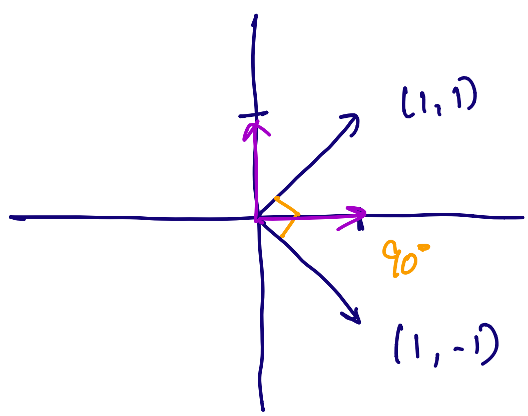
$\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. $\qquad \vec{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\langle \vec{v}, \vec{w} \rangle = 1\cdot 1 + 1\cdot 2 = 3$$

$$\vec{v} = \begin{bmatrix}1\\1\end{bmatrix}, \quad \vec{w} = \begin{bmatrix}1\\-1\end{bmatrix}$$

$$\langle \vec{v}, \vec{w} \rangle = 1\cdot 1 + 1\cdot(-1) = 0$$

If inner product $= 0 \Rightarrow$ "orthogonal"



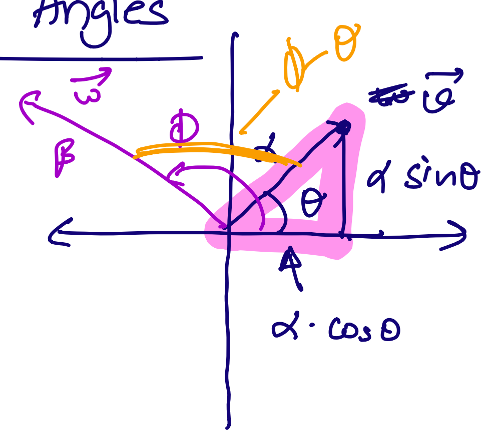
$$\vec{v} = \begin{bmatrix}1\\0\end{bmatrix}, \quad \vec{w} = \begin{bmatrix}0\\1\end{bmatrix}$$

$$\langle \vec{v}, \vec{w} \rangle = 0$$

Inner product $\longleftrightarrow$ Angles

$$\vec{v} = \alpha \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} = \begin{bmatrix}\alpha\cos\theta\\ \alpha\sin\theta\end{bmatrix}$$

$$\vec{w} = \beta \begin{bmatrix}\cos\phi\\ \sin\phi\end{bmatrix} = \begin{bmatrix}\beta\cos\phi\\ \beta\sin\phi\end{bmatrix}$$



$\alpha = \|\vec{v}\|$

$\beta = \|\vec{w}\|$

$\langle \vec{v}, \vec{w} \rangle$

$= (\alpha \cos\theta)(\beta \cos\phi) + (\alpha \sin\theta)(\beta \sin\phi)$

$= \alpha\beta \underbrace{(\cos\theta \cos\phi + \sin\theta \sin\phi)}$

$= \alpha\beta \, \underline{(\cos(\theta - \phi))}$ $\underline{\text{Trig identity}}$

$= \alpha\beta \cos(\phi - \theta)$

$= \|\vec{v}\| \, \|\vec{w}\| \cos(\phi - \theta)$ ≤ 1

If $(\theta - \phi) = 90°$, then $\langle \vec{v}, \vec{w} \rangle = 0$

"Orthogonal".

$$\langle \vec{v}, \vec{w} \rangle = \|\vec{v}\| \, \|\vec{w}\| \cos(\phi - \theta)$$

$$\leq \|\vec{v}\| \, \|\vec{w}\| \cdot 1$$

$$= \|\vec{v}\| \, \|\vec{w}\|$$

$$\Rightarrow \quad \boxed{\langle \vec{v}, \vec{w} \rangle \le \|\vec{v}\| \, \|\vec{w}\|}$$

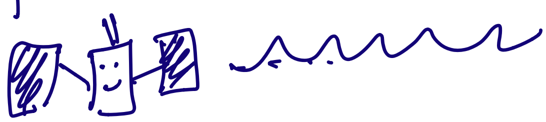
Cauchy - Schwartz inequality.

$$\langle \vec{v}, \vec{w} \rangle \ge \|\vec{v}\| \, \|\vec{w}\| \, (-1)$$

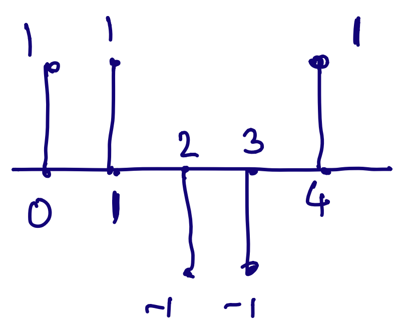
$$-\|\vec{v}\| \, \|\vec{w}\| \le \langle \vec{v}, \vec{w} \rangle \le \|\vec{v}\| \, \|\vec{w}\|$$

Satellite Classification

A



$$\vec{S}_A = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$



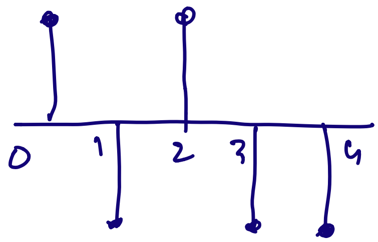
Signature pattern

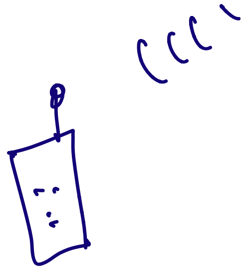
"Gold code"

$$\|\vec{SA}\| = \sqrt{5}$$

$$\|\vec{S_B}\| = \sqrt{5}$$

$$\vec{S_B} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$





$$\vec{r} = \begin{bmatrix} 0.9 \\ 1.1 \\ -1.2 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{r} = \vec{s} + \vec{n}$$

received vector　　signal　　noise

signal may be $\vec{s_A}$, $\vec{s_B}$

or a combination.

"Classify in the presence of noise"

Office Hours