EECS 16A

Logistics

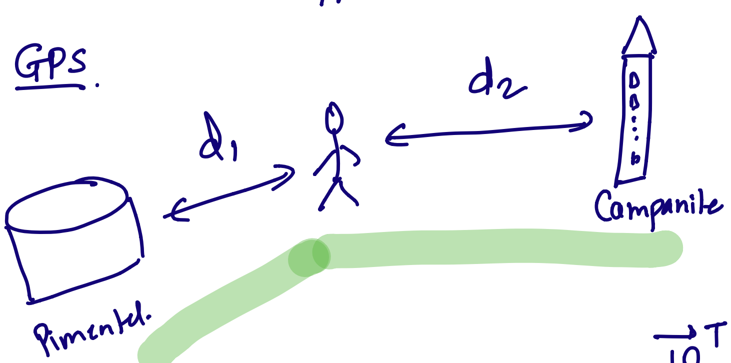
- Redo due on Friday.
- Still grading, scores not out before.

Today.

- ML Problem 1: Classify Sattelite
- ML Problem 2: Estimation of propagation delays.
- Max correlation approach.

GPS.

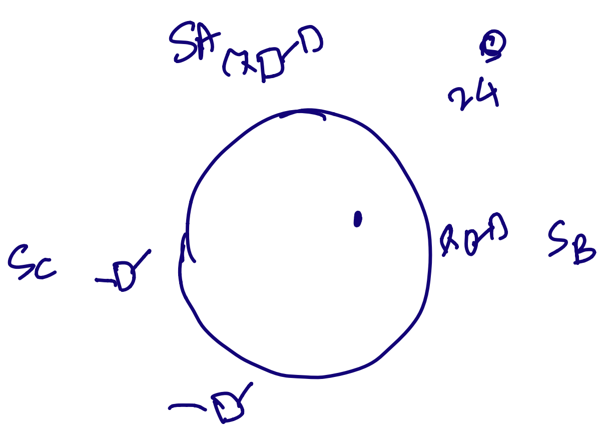
Use landmarks to identify location.



- **Recall:** Inner Product: $\langle \vec{v}, \vec{w} \rangle = \sum_{i=1}^{n} v_i w_i$

$\vec{v}^T \vec{w}$

$\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ $\vec{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$



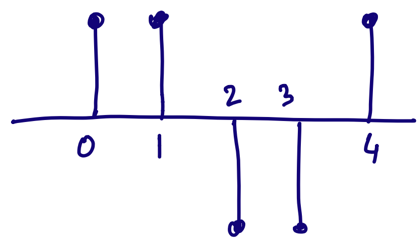
Last time:

$$\vec{s}_A = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \qquad \|\vec{s}_A\|^2 = 5$$

$$\vec{s}_B = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}. \qquad \|\vec{s}_B\|^2 = 5$$

Length 5 vectors.

1024 Gold Code

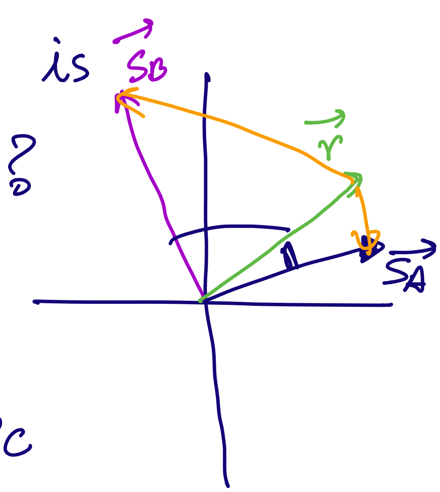


"Signal representation" $\vec{r} = \vec{s} + \vec{n}$

$$\vec{r} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \qquad \boxed{\vec{r} \doteq \begin{bmatrix} 0.9 \\ 1.1 \\ -1.2 \\ 0 \\ 1 \end{bmatrix}}$$

Classify with noisy received signal.

Which signature $\vec{s}_A$ or $\vec{s}_B$ is "closest" to $\vec{r}$?



"Design choice"

Choose an error metric

$\vec{e}_A = \vec{r} - \vec{s}_A$ $\qquad$ $\vec{e}_B = \vec{r} - \vec{s}_B$

Find: Satellite such that thee error $\vec{e}$ is minimized.

Look at the norm: $\|\vec{e}\|^2$

Find which of $\vec{s}_A, \vec{s}_B$ such that $\|\vec{e}\|^2$ is minimized.

$\boxed{\text{minimize } \|\vec{e}\|^2 \text{ over all satellites}}$

Optimization problem.

$\vec{e} = \vec{r} - \vec{s}$

$$\|\vec{e}\|^2 = \langle \vec{e}, \vec{e} \rangle = \vec{e}^T \vec{e}$$

$$= (\vec{r} - \vec{s})^T (\vec{r} - \vec{s})$$

$$= (\vec{r}^T - \vec{s}^T)(\vec{r} - \vec{s})$$

$$= \vec{r}^T \vec{r} + \vec{s}^T \vec{s} - \vec{s}^T \vec{r} - \vec{r}^T \vec{s}$$

$$= \|\vec{r}\|^2 + \|\vec{s}\|^2 - \langle \vec{s}, \vec{r} \rangle - \langle \vec{r}, \vec{s} \rangle$$

$$= \|\vec{r}\|^2 + \|\vec{s}\|^2 - 2\langle \vec{s}, \vec{r} \rangle$$

fixed   fixed

$$\boxed{\text{minimize } \|\vec{e}\|^2 \text{ over all } \vec{s}}$$

$\Longleftrightarrow$ minimize $-2\langle \vec{s}, \vec{r} \rangle$ over all $\vec{s}$

$$\Longleftrightarrow \boxed{\text{maximize } \langle \vec{s}, \vec{r} \rangle \text{ over all } \vec{s}}$$

Algorithm: Which satellite is transmitting?

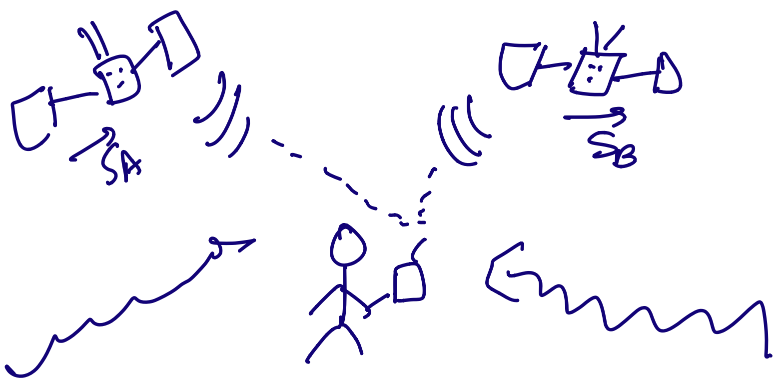
For all satellites $\vec{s_i}$

Compute $\langle \vec{r}, \vec{s_i} \rangle$

If $\langle \vec{r}, \vec{s_i} \rangle$ is LARGE.

Say satellite i is transmitting.

What about multiple satellites transmitting?



Electromagnetic waves.
"Speed of light"

Superposition

$$\vec{r} = \vec{s}_A + \vec{s}_B + \vec{n}$$

$$\boxed{\langle \vec{r}, \vec{s}_A \rangle} = \langle \vec{s}_A + \vec{s}_B + \vec{n}, \vec{s}_A \rangle$$

$$= (\vec{s}_A + \vec{s}_B + \vec{n})^T \cdot \vec{s}_A$$

$$= \vec{s}_A^{\,T} \vec{s}_A + \vec{s}_B^{\,T} \cdot \vec{s}_A + \vec{n}^T \cdot \vec{s}_A$$

$$= \langle \vec{s}_A, \vec{s}_A \rangle + \langle \vec{s}_B, \vec{s}_A \rangle + \langle \vec{n}, \vec{s}_A \rangle$$

approximates 1000

small approximates 20

small approx 10.

Threshold:

If $\langle \vec{r}, \vec{s}_A \rangle \geq$ threshold

$\longrightarrow$ detect $\vec{s}_A$!

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received, noise

$$\langle \vec{r}, \vec{s}_C \rangle = \langle \vec{s}_A + \vec{s}_B + \vec{n}, \vec{s}_C \rangle$$

$$= \langle \vec{s}_A, \vec{s}_C \rangle + \langle \vec{s}_B, \vec{s}_C \rangle + \langle \vec{n}, \vec{s}_C \rangle$$

$\vec{r} = \vec{s}_A + \vec{s}_B + \vec{n}$

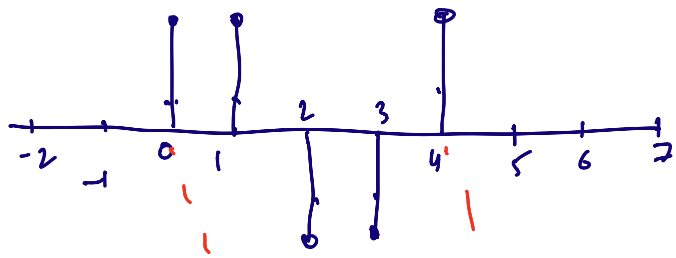
20 small

small 20

small 20

≈ 60

$$\vec{S_A} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$



$S_A[0] = 1$

$S_A[1] = 1$

$S_A[2] = -1$

$S_A[3] = -1$

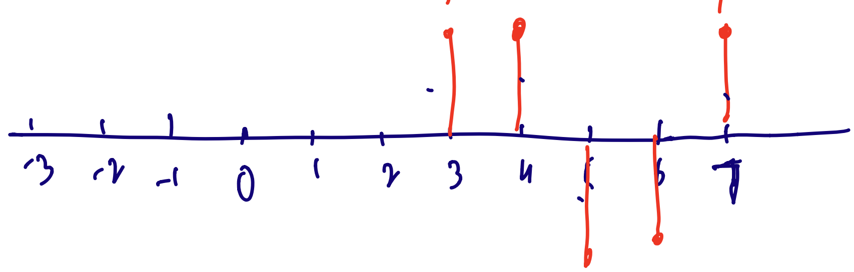
$S_A[4] = 1$

$S_A[5] = 0$

$S_A[-1] = 0$

Shift by 3.

$\vec{r}$



$\vec{r}$

$r[3] = 1$ $\qquad$ $r[0] = 0$

$r[4] = 1$

$r[5] = -1$

$r[6] = -1$

$r[7] = 1$

$$\underline{\underline{r[n]}} = \underline{\underline{S_A[n-3]}}$$

What is received at time n, is what was sent at time n-3.

"Cross correlation"

Define: $\text{Corr}_r (S_A)[k] = \sum_{i=-\infty}^{\infty} r[i]\, S_A[i-k]$

Inner prod: $\langle \vec{r}, \vec{s} \rangle = \sum_{i=1}^{n} r_i s_i$

$$\text{Corr}_r (S_A)[0] = r(3) S_A(3) + r(4) \cdot S_A(4) + r(5) \cdot S_A(5) + r(6) \cdot S_A(6) + r(7) \cdot S_A(7)$$

$$= 1(-1) + 1 \cdot 1 + (-1) \cdot 0 \ldots$$

$$= -1 + 1 + 0 \cdots 0$$

$= 0$

$$\mathrm{corr}_r(s_A)\cdot[3] = r(3)\,s_A(0) + r(4)\,s_A(1) + r(5)\,s_A(2) + r(6)\cdot s_A(3) + r(7)\,s_A(4) + 0\ldots$$

$$= 1\cdot(1) + 1(1) + (-1)(-1) + (-1)(-1) + 1(1)$$

$$= 1 + 1 + 1 + 1 + 1$$

$$= 5 \checkmark$$ LARGE

Office hours