EECS 16A

Module 3, Lecture 3.

Logistics

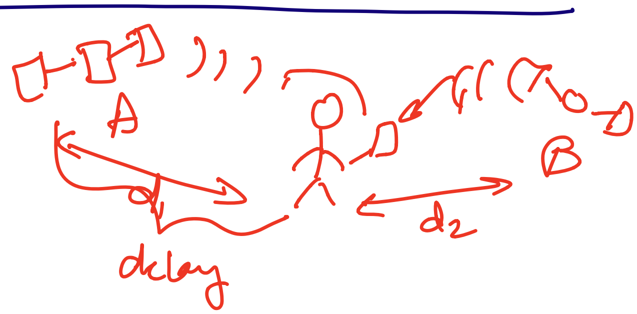
- MT2 grades released.
- OH today

Today.

- Trilateration
- Projections
- The Least Squares algorithm.

essentially the same thing.

GPS:



~~(1)~~ Which satellite is transmitting

→ Inner products

~~(2)~~ Distance to the satellite

→ Propagation delays.

→ Cross-correlation. (Moving inner product)

(3) Distances ⟷ location

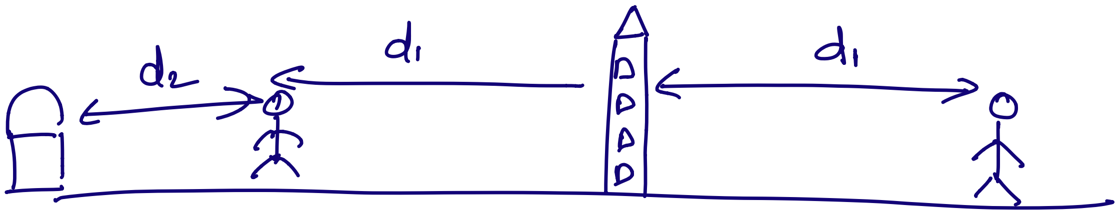
Trilateration

(4) Dealing with noise.

Projection.

Least Squares algorithm.

Distance $\longleftrightarrow$ Position (1D World)

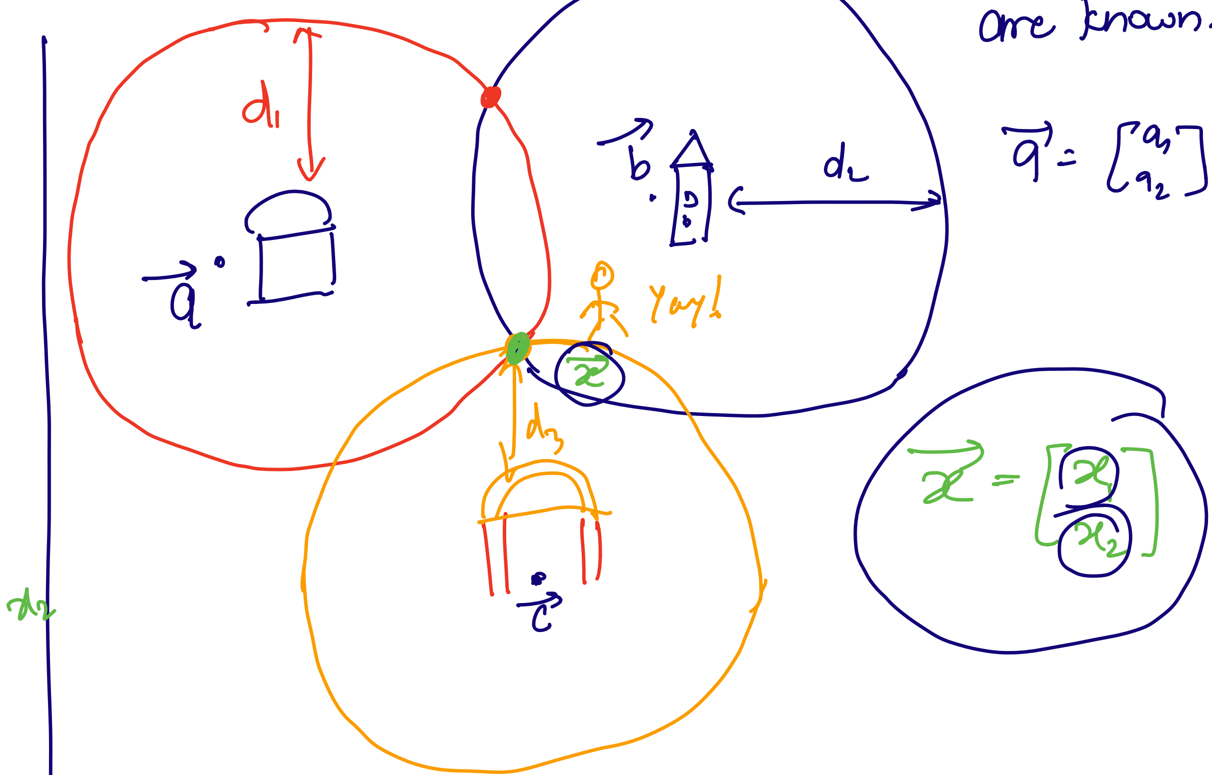


1D World $\rightarrow$ 2 Landmarks to identify my position.

2D world ?

$\vec{a}, \vec{b}, \vec{c}$ are known.

$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$



$\rightarrow$ Need 3 landmarks.

x_1

Find coordinates of green dot in terms of

$\vec{a}, \vec{b}, \vec{c}, d_1, d_2, d_3$

Unknown $\vec{x}$

$$① \quad \|\vec{x} - \vec{a}\|^2 = d_1^2$$

$$\|\vec{x}\| = \sqrt{\vec{x}^T \vec{x}}$$

$$② \quad \|\vec{x} - \vec{b}\|^2 = d_2^2$$

$$③ \quad \|\vec{x} - \vec{c}\|^2 = d_3^2$$

$$① \Rightarrow (\vec{x} - \vec{a})^T (\vec{x} - \vec{a}) = d_1^2$$

$$\vec{x}^T\vec{x} + \vec{a}^T\vec{a} - \vec{x}^T\vec{a} - \vec{a}^T\vec{x} = d_1^2$$

$$\|\vec{x}\|^2 + \underbrace{\|\vec{a}\|^2}_{\text{Known}} - 2\underbrace{\langle \vec{x}, \vec{a} \rangle}_{\text{Unknown}} = \underbrace{d_1^2}_{\text{Known}} \quad ④$$

$$② \Rightarrow \|\vec{x}\|^2 + \|\vec{b}\|^2 - 2\langle \vec{x}, \vec{b} \rangle = d_2^2 \quad ⑤$$

④ − ⑤

$$\|\vec{a}\|^2 - \|\vec{b}\|^2 - 2\langle \vec{x}, \vec{a} \rangle + 2\langle \vec{x}, \vec{b} \rangle = d_1^2 - d_2^2 \quad ⑥$$

⑥ is Linear!!!

(3) $\Rightarrow$ $\|\vec{x}\|^2 + \|\vec{c}\|^2 - 2\langle \vec{x}, \vec{c} \rangle = d_3^2$ (7)

(4) $-$ (7)

$$\Rightarrow \boxed{\|\vec{a}\|^2 - \|\vec{c}\|^2 - 2\langle \vec{x}, \vec{a} \rangle + 2\langle \vec{x}, \vec{c} \rangle = d_1^2 - d_3^2}$$

$\rightarrow$ Gaussian Elimination!!!

Trillateration algorithm

Projection and Least Squares

$\boxed{A\vec{x} = \vec{b}}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

4 satellites.

A square, invertible $\Rightarrow$ use $A^{-1}\vec{b} = \vec{x}$

A not square?

- More equations than unknowns.
- Our equations might be noisy.

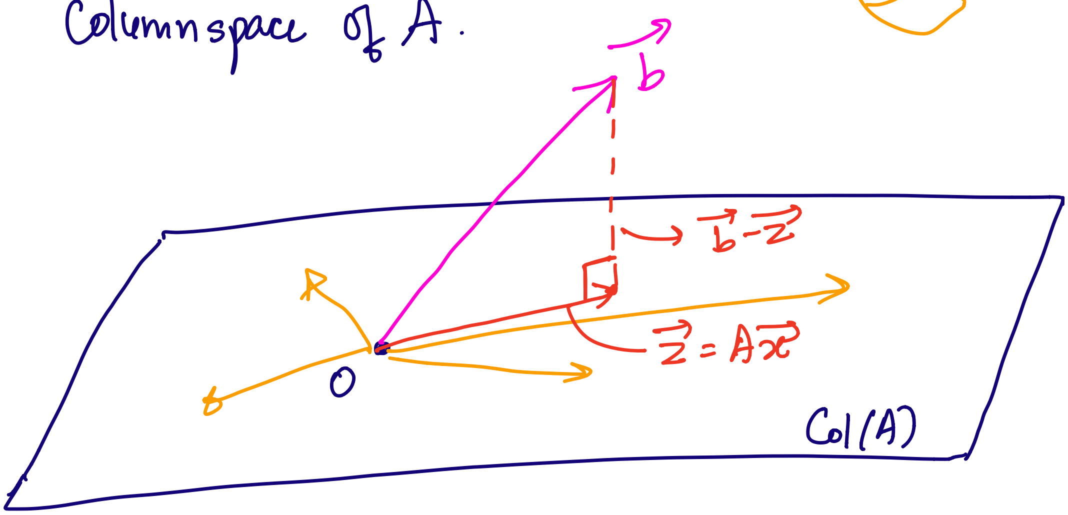
$$A\vec{x} = \vec{b} + \vec{e}$$

Want to find $\vec{x}$ such that $A\vec{x}$ is as close to $\vec{b}$ as possible!

$A\vec{x}$ $\quad A = \begin{bmatrix} \vec{a}_1, & \vec{a}_2 & \cdots & \vec{a}_n \end{bmatrix}$

$A\vec{x}$ $\quad A\vec{x} = \begin{bmatrix} \vec{a}_1 & \cdots & \vec{a}_n \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}$

Columnspace of A.



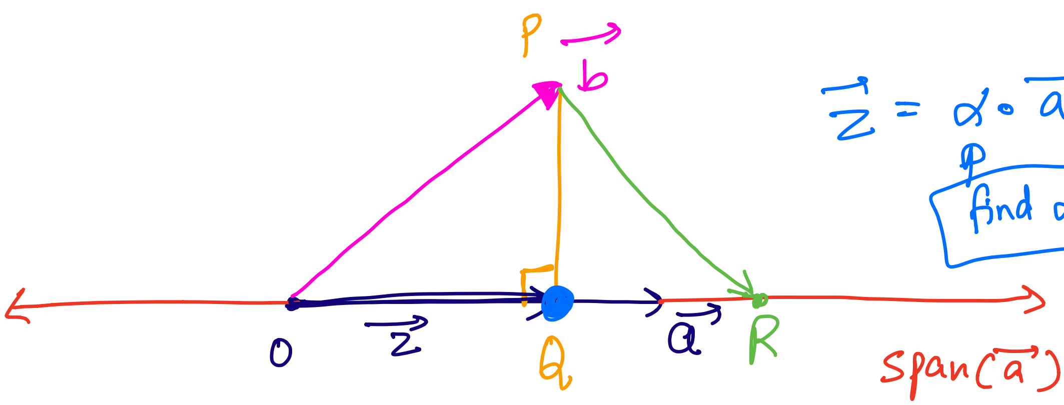
To find $A\vec{x} = \vec{z}$ that is closest to $\vec{b}$

we want to find the point "directly below" $\vec{b}$ in $Col(A)$.

"Orthogonal projection" of $\vec{b}$ onto $Col(A)$.

Simple Case: 1D Projection.

$$A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \vec{a} \qquad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$



$\vec{z} = \alpha \cdot \vec{a}$

$\boxed{\text{find } \alpha}$????

Intuition: PR should be > PQ.

Can we prove this?

Thm: Shortest distance between a point and a line is given by the perpendicular from that point to the line.

Proof: PQR always forms a right angle Δ, with right angle at Q.

$$PQ^2 + QR^2 = PR^2$$

$PR > PQ$ PR is always larger than PQ

PR Hypotenuse.

$\Rightarrow$ $\vec{z}$ is the closest point.

$\vec{z}$ is called the projection of $\vec{b}$ onto $\vec{a}$.

Office hours.