EECS 16A   Lecture 2   Sept 1, 2020.

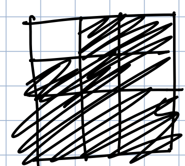
Logistics: ① Discussions : Try different sections Mond AND Wed
Find a TA that works for you.

Less crowded:

Shashank	8-9 am
Tsegereda	2-3 pm
Mosea	1-2 pm
Miyuki	12-1 pm.

② Homework 1 due on Friday 9/4.
↳ Study group survey.

③ Office hours - Piazza post is coming.



$x_2 + x_9 = b$

linear combinations of x_2, x_9.

Linear equations

System of linear equations

$2x + 3y = 8$ (E1)

$3x - y = 1$ (E2)

How to solve systems with million of eq^n + variables?

x , y

$2x + 3y = 8 \quad (E1)$

$3x - y = 1 \quad (E2)$

$$\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 3 & -1 & 1 \end{array}\right]$$

Augmented matrix form.

① Normalize E1 so that the coefficient of x is 1.

$E1 \leftarrow (E1)/2$

Mult by 3

$x + \frac{3}{2}y = 4 \quad (E1*)$

$3x - y = 1 \quad (E2)$

$$\begin{array}{c} \left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 3 & -1 & 1 \end{array}\right] \end{array} \begin{array}{l} R1 \\ R2 \end{array}$$

② Use $(E1*)$ to eliminate 'x' from $(E2)$

Take $(E2) - 3(E1*)$ $\qquad (R2) - 3\cdot(R1)$

$(3\cancel{x} - y) - \cancel{3}(x + \frac{3}{2}y) = 1 - 3\times 4$

$-y - \frac{9y}{2} = -11$

$-\frac{11}{2}y = -11 \quad (E2*)$

$$\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 0 & -\frac{11}{2} & -11 \end{array}\right] \begin{array}{l} R1 \\ R2 \end{array}$$

$-\frac{11}{2}y = -11$

③ Solve for y, by normalizing $(E2*)$

$y = 2$

$$\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 0 & 1 & 2 \end{array}\right]$$

Upper triangular matrix.

④ Back substitute $\rightarrow (E1*)$

$x + \frac{3}{2}y = 4$

$x + 3 = 4$

$x = 1$

$(R1) - (\frac{3}{2})\cdot R2$

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

identity matrix.

Gaussian Elimination!

linalg.solve $\longrightarrow$ Ancient China

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}}_{\text{Matrix}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{vector}} = \underbrace{\begin{bmatrix} 8 \\ 1 \end{bmatrix}}_{\text{vector.}}$$

Just one more representation

Vector: Ordered list of elements.

e.g. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{\text{3 dim vector.}}$

2D vector

$\vec{x} \in \mathbb{R}^{\cancel{2}}$

arrow on to   in   2 dim of real numbers

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\vec{y} \in \mathbb{R}^3$ $\quad$ $\vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

Matrix: Grid of numbers

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2×3 Matrix

$$A \in \mathbb{R}^{2\times 3}$$

Gaussian Elimination.

$$2x + 3y = 8 \quad (E1)$$
$$2x + 3y = 6 \quad (E2)$$

$$\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 2 & 3 & 6 \end{array}\right]$$

① Normalize (R1)

$$(R1) \leftarrow (R1)/2$$

$$\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 2 & 3 & 6 \end{array}\right]$$

want 0 here

② Eliminate x from second row.

$$(\text{Row } 2) - 2\times(\text{Row } 1)$$

$$\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 0 & 0 & -2 \end{array}\right]$$

Equation

$$0x + 0y = -2$$

→ Conclude: No solution to this system.

In an upper triangular matrix, the diagonal entries are called pivots.

If you have a 0 on the diagonal, this means you have to be careful.

Example 3

$$x + 4y = 6$$
$$2x + 8y = 12$$

$$\left[\begin{array}{cc|c} 1 & 4 & 6 \\ 2 & 8 & 12 \end{array}\right]$$

create 0

① Normalize.
↳ Not needed
Already done

② Subtract 2(R1) from R2.

$$\left[\begin{array}{cc|c} 1 & 4 & 6 \\ 0 & 0 & 0 \end{array}\right]$$

$0x + 0y = 0$ →

Infinite number of solutions!

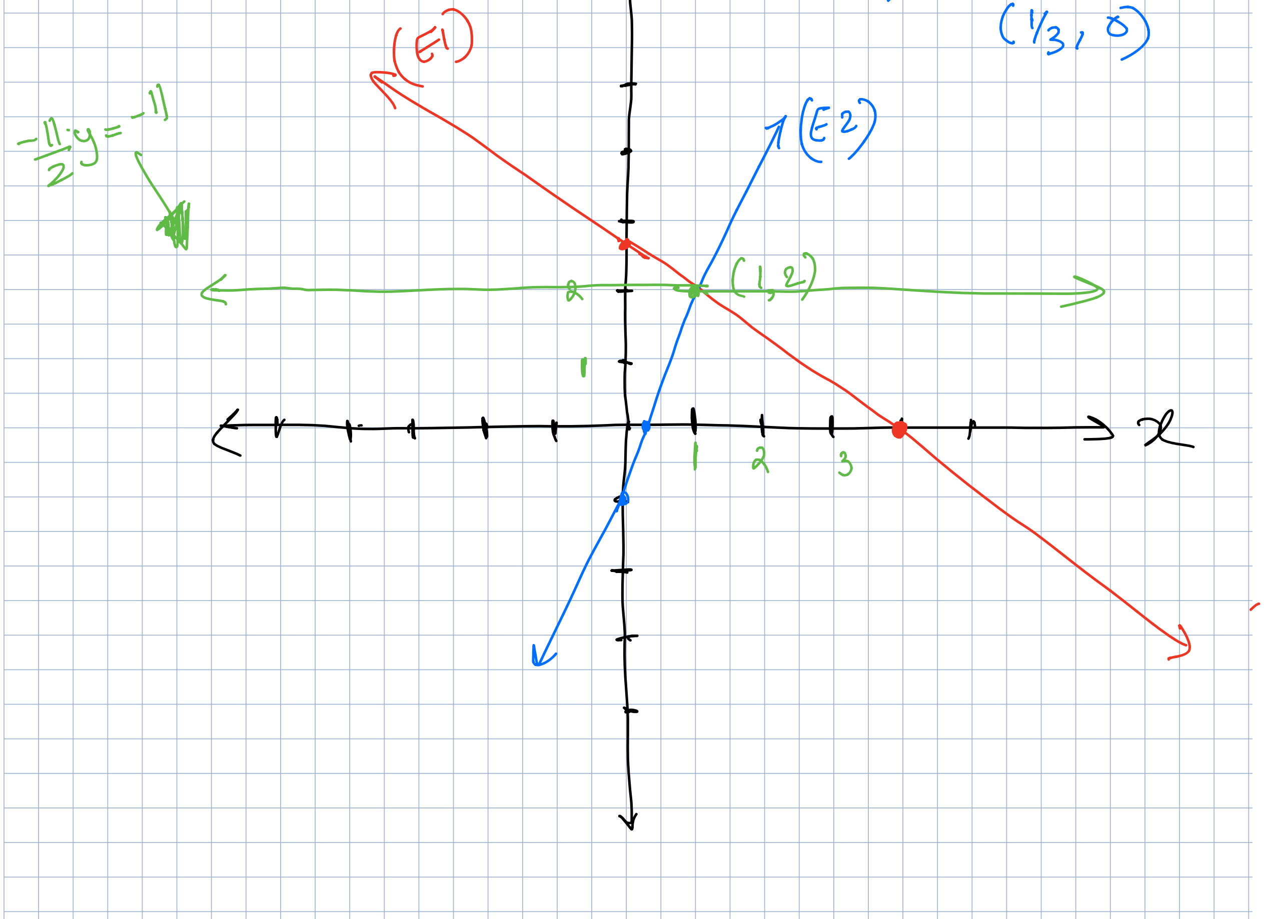
Geometric perspective:

$2x + 3y = 8 \longrightarrow$ Intercepts: $(0, 8/3)$ ≈ 2.6

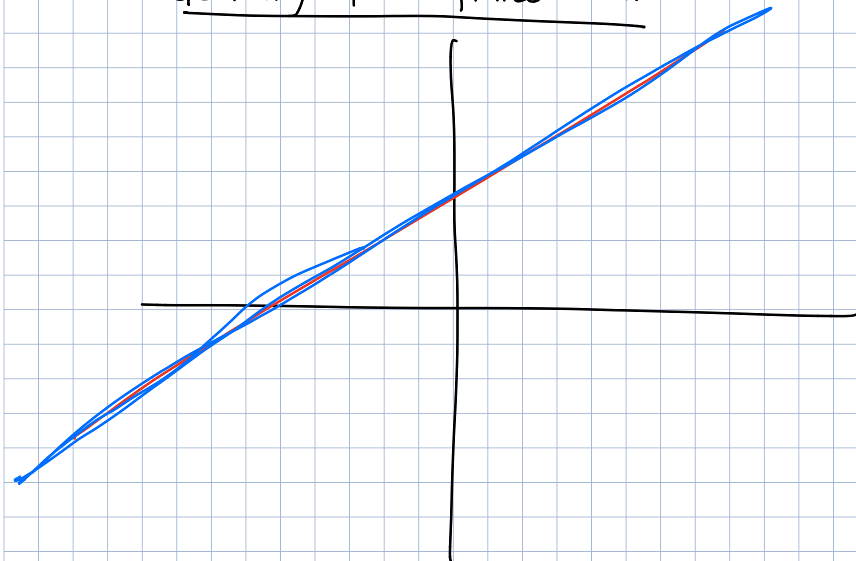
$(4, 0)$

(E2) $3x - y = 1$ Intercepts: $(0, -1)$

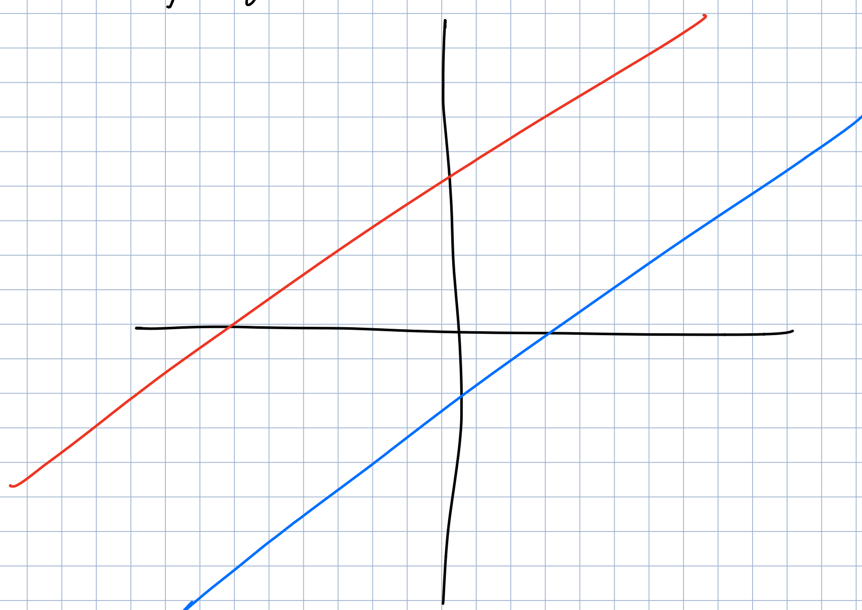
$(1/3, 0)$



Geometry for infinite solutions



Geometry of no solutions



Gaussian Elim.

n variables - m equations.

$$\left[\begin{array}{cccc|c} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} & \beta_1 \\ \alpha_{21} & \alpha_{22} & & \alpha_{2n} & \beta_2 \\ & & & & \\ \alpha_{m1} & \alpha_{m2} & & \alpha_{mn} & \beta_m \end{array}\right]$$

$\beta_1 \rightarrow \alpha_{11}x_1 + \alpha_{12}x_2 + \cdots + \alpha_{1n}x_n = \beta_1$

α_{ij}s $\rightarrow$ coefficients known.

$x_1, x_2 \ldots$ variables unknown.

$\beta_1, \beta_2 \ldots$ known measurements.

Every row represents a measurement

Every column — " ——— a variable.

---

① ~~Start~~ Start with row 1.
If α_{11} is 0, swap rows to have a non zero coefficient for x_1 in row 1.

② Normalize so that x_1 has a coefficient of 1 in the first row.

③ Use first row to eliminate x_1 from all other equations / rows.

④ Move on to 2nd variable: Repeat with row 2.

5. Get to an upper triangular matrix
6. Back-substitute.