Office Hours

Statement: $\{\vec{v_1}, \vec{v_2} \cdots \vec{v_n}\} = S_1$

$\{\vec{v_1}+\vec{v_2}, \vec{v_2}, \cdots \vec{v_n}\} = S_2$.

Show that spans of S_1, S_2 are equal.

$\Longrightarrow$ If $\vec{z}$ is in span of (S_1), then $\vec{z}$ is in span (S_2)

And if $\vec{y}$ is in span(S_2), then it is also in span(S_1)

Know:

$\{\vec{v_1}, \vec{v_2} \cdots \vec{v_n}\} = S_1$

Say $\vec{z}$ is in span of S_1

We know:

$\vec{z} = \alpha_1 \vec{v_1} + \alpha_2 \vec{v_2} + \cdots + \alpha_n \vec{v_n}$.

Definition of span.

$$\vec{z} = \sum_{i=1}^{n} \alpha_i \vec{v_i} = \overbrace{\alpha_1 \vec{v_1} + \alpha_2 \vec{v_2} + \cdots + \alpha_n \vec{v_n}}^{\text{linear combination.}}$$

$\alpha_1, \alpha_2 \cdots \alpha_n \in \mathbb{R}$

any real numbers.

To show:

$\vec{z}$ is also in span (S_2).

Want to show: $\vec{z}$ is a linear combination of

$$\{\vec{u}_1+\vec{u}_2, \vec{u}_2, \vec{u}_3 \cdots \vec{u}_n\}$$

$$\vec{u}_1+\vec{u}_2 = \vec{m}$$

$$\{\vec{m}, \vec{u}_2, \vec{u}_3 \cdots \vec{u}_n\}$$

Want to show:

$$\vec{z} = \beta_1(\vec{u}_1+\vec{u}_2) + \beta_2 \cdot \vec{u}_2 + \cdots + \beta_n \vec{u}_n.$$

$$= \beta_1 \vec{u}_1 + \beta_1 \vec{u}_2 + \beta_2 \vec{u}_2 + \cdots + \beta_n \vec{u}_n$$

$$\vec{z} = \alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + \cdots + \alpha_n \vec{u}_n \quad (1)$$

$$= \alpha_1 (\vec{u}_1 + \vec{u}_2) + (\alpha_2 - \alpha_1)\vec{u}_2 + \alpha_3 \vec{u}_3 \quad (2)$$

$$\alpha_4 \vec{u}_4 + \cdots + \alpha_n \vec{u}_n$$

Choose:

$$\beta_1 = \alpha_1$$

$$\beta_2 = \alpha_2 - \alpha_1$$

$$\beta_3 = \alpha_3$$

$$\vdots$$

$$\beta_n = \alpha_n$$

$\vec{z}$ as a linear combination of $(\vec{u}_1+\vec{u}_2), \vec{u}_2 \cdots \vec{u}_n$?

$$\alpha_1 \vec{v_1} + \alpha_2 \vec{v_2} = \beta_1 (\vec{v_1} + \vec{v_2}) + \beta_2 \vec{v_2}$$

$$= \beta_1 \vec{v_1} + (\beta_1 + \beta_2) \vec{v_2}$$

$$\alpha_1 = \beta_1$$

$$\alpha_2 = \beta_1 + \beta_2$$

$$\Rightarrow \beta_1 = \alpha_1$$

$$\beta_2 = \alpha_2 - \beta_1 = \alpha_2 - \alpha_1$$