EECS 16A   Lecture 8

Logistics

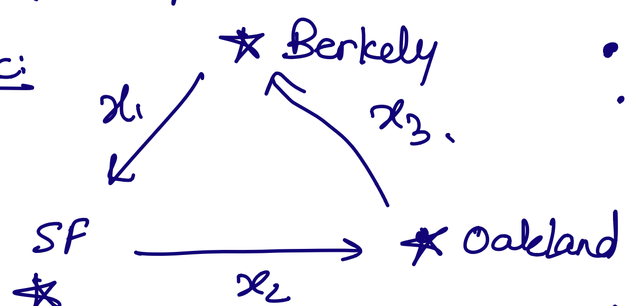
- OH : Discord option
- Roundtable : Th. 4:30-5:30
- How are you doing?
- Progress tracker ←

Today:

- Vector Spaces
- Subspaces
- Basis / Dimension
- Nullspaces / Columnspaces
- Determinants

Last time: Nullspace : All $\vec{x}$ such that $A\vec{x} = \vec{0}$

e.g. Traffic:



- Cal PATH
- Traffic

$$A\vec{x} = \vec{0}$$

No cars stop in cities

$$x_1 - x_2 = 0$$
$$x_2 - x_3 = 0$$
$$x_3 - x_1 = 0$$

$$\Rightarrow \boxed{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

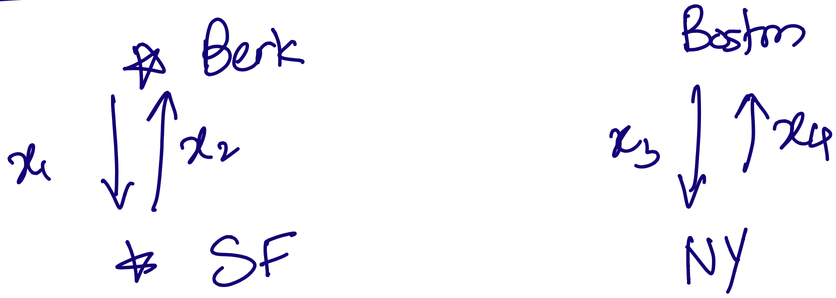
$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Basic   free

$\boxed{0x = 1}$

\# of free vars: dim of null space

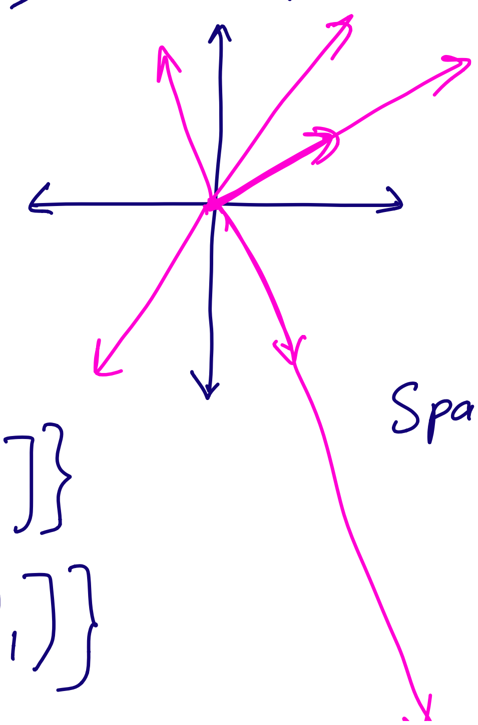
\# of basic vars: dim of the colspace



→ See that you have 2 free variables.

Vector Spaces → see notes

e.g. $\mathbb{R}^2$

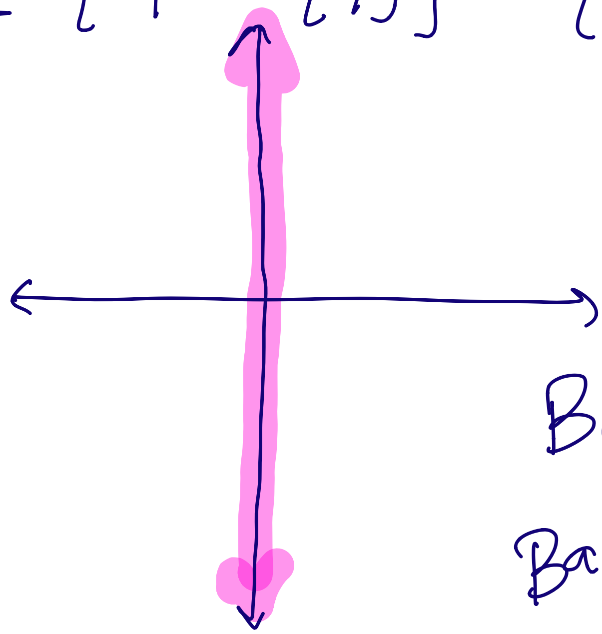


$\text{Span}\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}\right\}$

$\text{Span}\left\{\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right\}$

$\text{Span}\left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\right\}$

e.g $\left\{ \text{Span} \begin{bmatrix}0\\1\end{bmatrix} \right\} = \left\{ \vec{v} \;\middle|\; \vec{v} = \alpha \begin{bmatrix}0\\1\end{bmatrix}, \alpha \in \mathbb{R} \right\}$



Vectorspace

Basis: $\left\{ \begin{bmatrix}0\\1\end{bmatrix} \right\}$

Basis: $\left\{ \begin{bmatrix}0\\2\end{bmatrix} \right\}$

e.g. $\{ \vec{0} \}$ → Vector space :)

e.g. $\left\{ \begin{bmatrix}0\\1\end{bmatrix} \right\}$

$\begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\2\end{bmatrix}$

→ Not in the set!

$\vec{0} \notin$ set

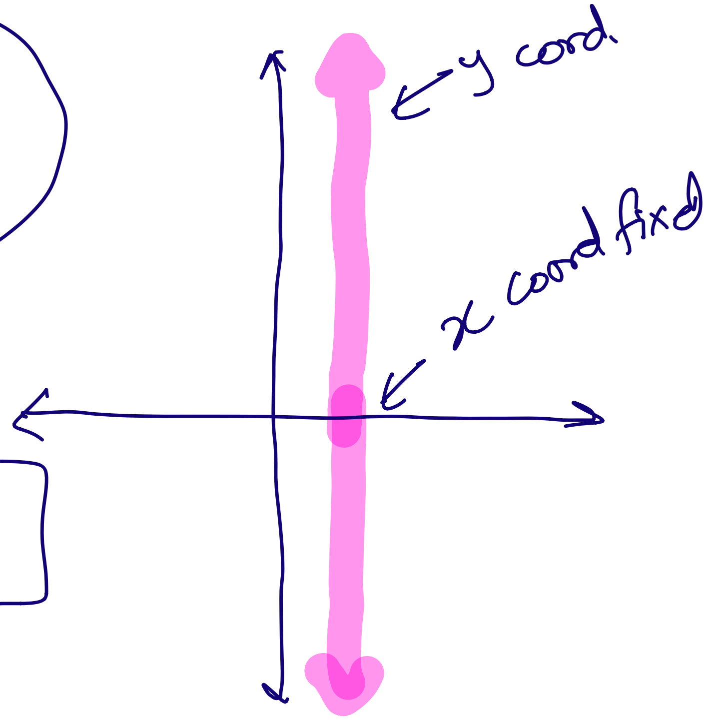
$\Rightarrow$ NOT a VS.

e.g. $\left\{ \vec{v} = \begin{bmatrix}1\\0\end{bmatrix} + \alpha \begin{bmatrix}0\\1\end{bmatrix} \;\middle|\; \alpha \in \mathbb{R} \right\}$.

$$\begin{bmatrix} 1 \\ \alpha \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \in$ Set

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$



Basis $\{\vec{v_1}, \vec{v_2} \cdots \vec{v_n}\}$ is called a Basis for a Vector Space V, if

1. $\vec{v_1}, \vec{v_2} \cdots \vec{v_n}$ are all linearly independent!

2. All $\vec{v} \in V$ are in the span $\{\vec{v_1} \cdots \vec{v_n}\}$

$\rightarrow \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ is a **Basis** for the vectors space $\mathbb{R}^2$

$\rightarrow \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right\} \rightarrow$ NOT a Basis

$\rightarrow \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

$\rightarrow$ NOT a basis.

Dimension of a VS.

The number of vectors in the basis of a vector space.

Subspace: V is a vector space.

W is a subset of vectors in V

If W is also a vector space,
then W is a subspace of V.

- Nullspace (A) ✓

- Columnspace (A) = Span of the columns of matrix A.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \rightarrow \mathrm{Col}(A) = \mathrm{Span}\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \rightarrow \mathrm{Col}(A) = \mathrm{Span}\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$$ lin. dependant

$$= \mathrm{span}\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

$\left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\}$ is a basis for col(A)

Rank: "Rank" of a matrix is the dimension of the columnspace (i.e. the maximum number of linearly independant columns in the matrix)

# of lin indep columns in a matrix

= # of — " — row in the mat.

Proof in 16B

Systems of eq^n.

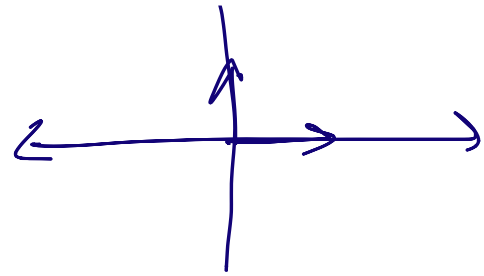
$\underbrace{A}_{\text{Square}}$ is invertible $\iff$ A has linearly independent columns "full rank"

$\iff$ $A\vec{x} = \vec{b}$ has a unique solution

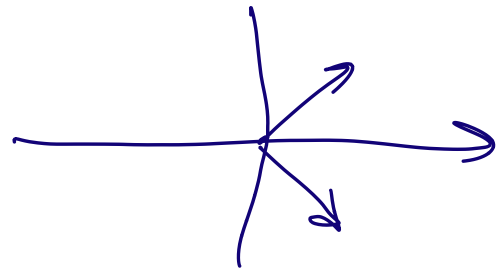
$\iff$ Null (A) is trivial (containly only the $\{\vec{0}\}$)

$\iff$ Determinant of A is $\neq 0$.

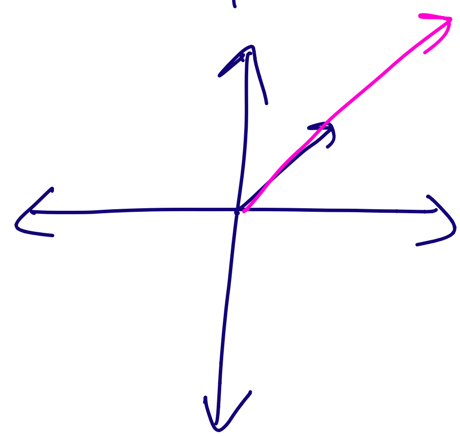
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow$$



$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \longrightarrow$$

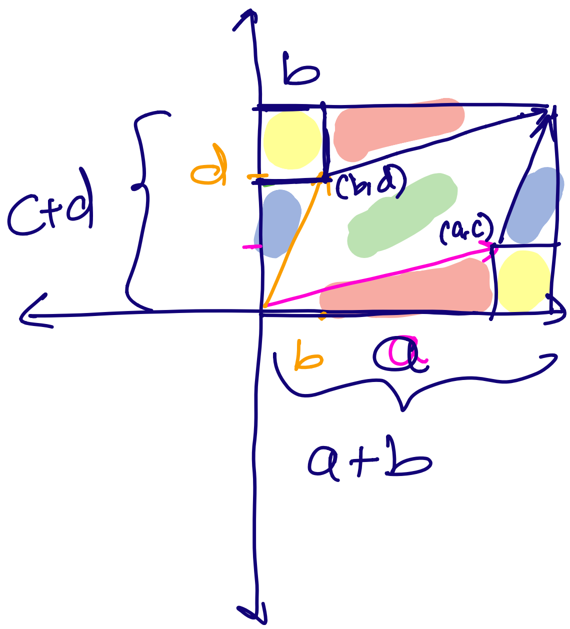


$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \longrightarrow$$



Determinants

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



Area of rectangle: $(a+b)(c+d)$

Area of red Δ : $\left(\frac{1}{2}ac\right) \times 2$

Area of the blue Δ : $\left(\frac{1}{2}bd\right) \times 2$

Area of yellow $\square$ = $bc \times 2$

$(a+b)(c+d) - ac - bd - 2bc$

$= ad - bc$ = area of parallelogram

$$\mathrm{Det}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$$

$$= ad - bc \, .$$