$$\underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 0 \end{bmatrix}}$$

$$\text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

1. lin indep ✓

2. ✓ span = col(A)

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$ Basis for col(A)

2 vectors

$\Rightarrow$ Dim Col(A) = 2.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

GE

dim 2   4

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

x_3

$x_2 + x_3 = 0$

$x_2 = -x_3$

$x_1 + x_2 = 0 \implies x_1 - x_3 = 0 \implies x_1 = x_3$

$\therefore$ All vectors $\begin{bmatrix} x_3 \\ -x_3 \\ x_3 \end{bmatrix} \in Null(A)$

$$\text{Span}\left\{\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right\} = Null(A)$$

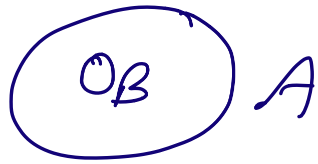
Proove: $\underbrace{\text{span}\{\vec{v_1}, \vec{v_2} \cdots \vec{v_n}\}}_{A} = \underbrace{\text{span}\{\vec{v_1} + \vec{v_2}, \cdots, \vec{v_2} \cdot \vec{v_n}\}}_{B}$.

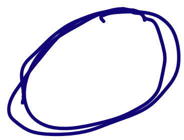
① If every $\vec{v} \in A$ is also in B.

the

$\vec{v}A$ B

~~If~~ $\vec{v} \in B$ is also in A





$\vec{v} \in A$ Known

$\vec{v} = \alpha_1 \vec{v_1} + \alpha_2 \vec{v_2} + \cdots + \alpha_n \vec{v_n}$

Want to show:

$\vec{v} \in B$. $\vec{v}$ can be written as

$\beta_1(\vec{v_1} + \vec{v_2}) + \beta_2 \vec{v_2} + \cdots + \beta_n \vec{v_n}$

for some $\beta_1, \beta_2 \cdots \beta_n$.

Choose $\underbrace{\beta_1 = \alpha_1}$ $\Longrightarrow$ $\beta_1 + \beta_2 = \alpha_2$

$\beta_2 = \alpha_2 - \alpha_1$

$\beta_2 = \alpha_2 - \alpha_1$

$\beta_3 = \alpha_3$

$\vdots$

$\beta_n = \alpha_n$

How do we know $N(A)$ is a vectorspace?

$\vec{x}_1 \in N(A)$

$\vec{x}_2 \in N(A)$.

$\vec{x}_1 + \vec{x}_2 \in$

$$A(\vec{x}_1 + \vec{x}_2) = A\vec{x}_1 + A\vec{x}_2$$

$$= \vec{0} + \vec{0} = \vec{0}$$

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \longrightarrow \text{Col}(A)$$

$A \to$ Gaussian Elim

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad A\vec{x} = \vec{0}$$

x_3 is free.

$x_2 + x_3 = 0$. $\qquad x_1 + x_3 = 0$

$x_2 = -x_3 \qquad x_1 = -x_3$.

all sol. $\begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix}$

$\text{span}\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}.$

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

x_3 free
x_2 free
$x_1 + x_2 + x_3 = 0$
$x_1 = -x_3 - x_2$

$$\text{sol's} = \begin{bmatrix} -x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} -x_2 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} -x_3 \\ 0 \\ x_3 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \text{Span}\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\} \leftarrow A\vec{x} = 0$$