Office hours:

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}.$$

$\lambda_1 = 5,\quad \lambda_2 = -1.$

$\vec{v_1} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}\quad \vec{v_2} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\underline{A\vec{v_1} = 5\,\vec{v_1}} \longrightarrow \text{span}\{\vec{v_1}\} \in$ Eigenspace corresponding to e-val. 5

$A\vec{v_2} = -\vec{v_2} \longrightarrow \text{span}\{\vec{v_2}\}$

$\vec{v_3} = \cancel{\beta_1}\,\alpha\,\vec{v_1}$

$$\underline{A\cdot\vec{v_3}} = A\cdot\alpha\cdot\vec{v_1} = \alpha\, A\cdot\vec{v_1} = \alpha\cdot 5\cdot\vec{v_1}$$
$$= 5\alpha\cdot\vec{v_1}$$
$$= \underline{5\,\vec{v_3}}$$

$\vec{v_4} = \beta\cdot\vec{v_2}$

$$A\cdot\vec{v_4} = A\cdot\beta\cdot\vec{v_2} = \beta\, A\cdot\vec{v_2} = \beta\cdot(-\vec{v_2})$$
$$= -\vec{v_4}$$

$\vec{v_5} = \alpha\vec{v_1} + \beta\vec{v_2}$

$$A \cdot \vec{v_5} = A \cdot \alpha \cdot \vec{v_1} + A \beta \vec{v_2}$$

$$= \alpha \cdot A \vec{v_1} + \beta A \vec{v_2}$$

$$= 5\alpha \cdot \vec{v_1} + (-1)\beta \cdot \vec{v_2}$$

Gaussian Elimination

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \xrightarrow{\cancel{\ }} \xrightarrow{1}$$

$ad - bc = 0$

$\Rightarrow ad = bc$

$\Rightarrow \frac{a}{c} = \frac{b}{d}$

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & b/a & \frac{1}{a} & 0 \\ c & d & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - c \cdot R_1} \left[\begin{array}{cc|cc} 1 & b/a & \frac{1}{a} & 0 \\ 0 & d - \frac{cb}{a} & 0 - \frac{c}{a} & 1 - 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cc|cc} 1 & b/a & \frac{1}{a} & 0 \\ 0 & \frac{ad-bc}{a} \cdot \frac{a}{ad-bc} & -\frac{c}{a}\left(\frac{a}{ad-bc}\right) & \cdot \frac{a}{ad-bc} \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|cc} 1 & b/a & 1/a & 0 \\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$$R_1 - \frac{b}{a} \cdot R_2$$

$$\longrightarrow \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{a} - \frac{b}{a}\left(\frac{-c}{ad-bc}\right) & 0 - \frac{b}{a}\left(\frac{a}{ad-bc}\right) \\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|cc} 1 & 0 & \cancel{\frac{1}{a}\left(1 + \frac{bc}{ad-bc}\right)} \; \cancel{\frac{ad-bc+bc}{ad-b}} \; \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ 0 & 1 & \frac{-c}{(\quad)} & \frac{a}{(\quad)} \end{array}\right]$$

$$\text{inv}: \quad \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\text{if } ad-bc \neq 0$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{bmatrix}$$

CHECK

$\downarrow$

CH poly.

$$a\lambda^2 + b\lambda + c = 0$$