

EECS 16A

Module 2, Lecture 6

Logistics

- MT score released.
- ON today: 1-1 conversations
- Advising today 4pm.

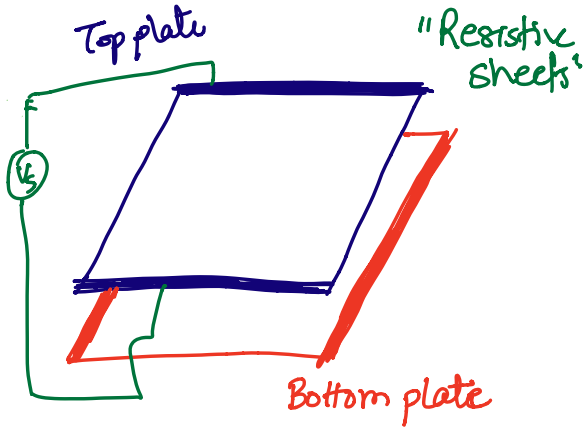
Today:

- What does it mean to "make a model"?
- 2D Touchscreen recap.
- Equivalence: Resistance in series and parallel
- Superposition: Power of Module 1 in Module 2.
Linearity of Circuits.

• Page Rank

• Power of modeling

Physical Reality:
Two resistive sheets.



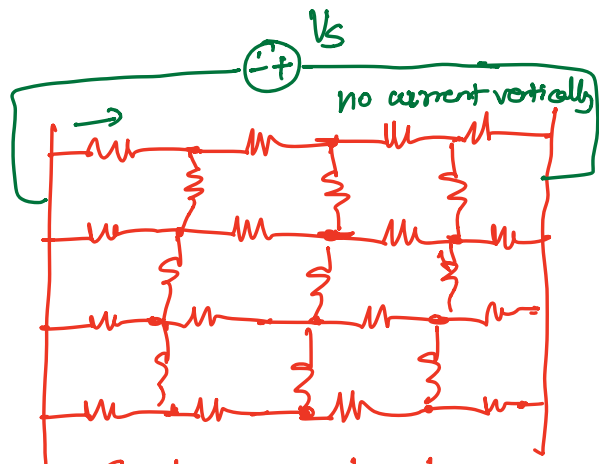
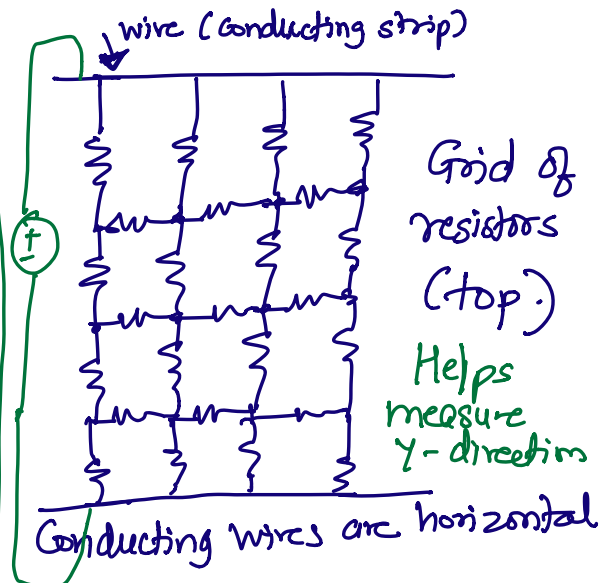
Dark shaded areas

or

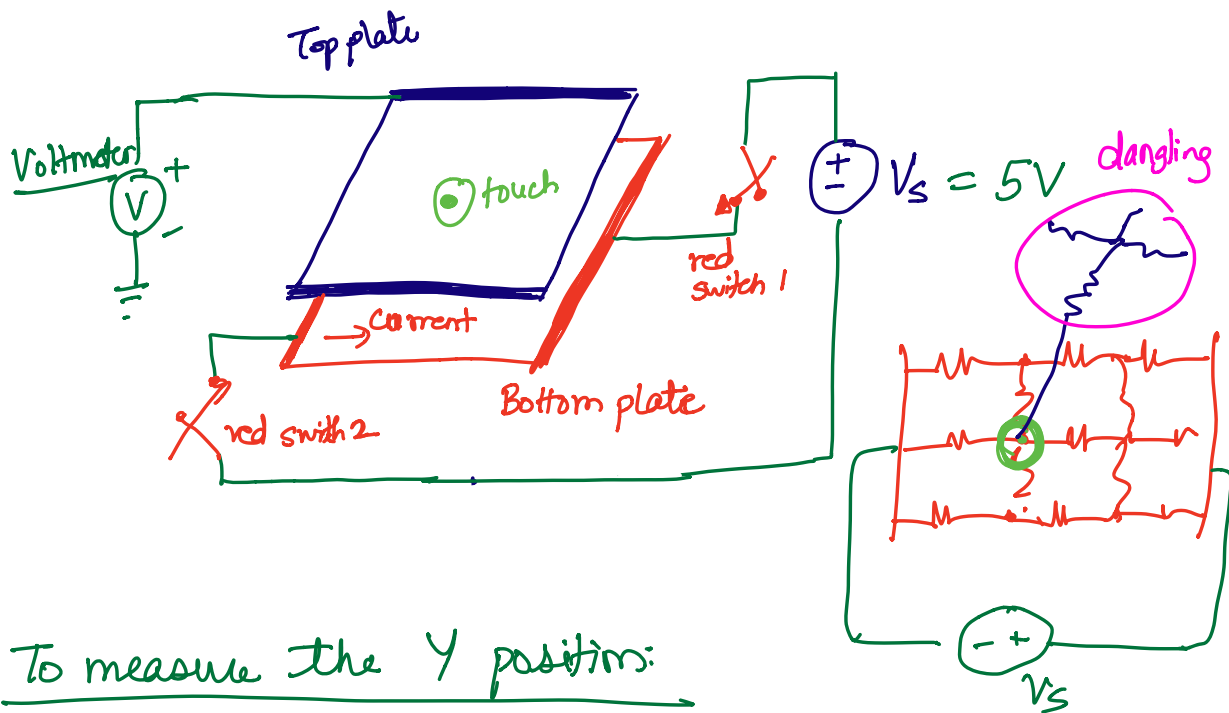
are conducting strips
like wires.

The grid of resistors is a
MODEL for the physical
reality of the continuous
plate.

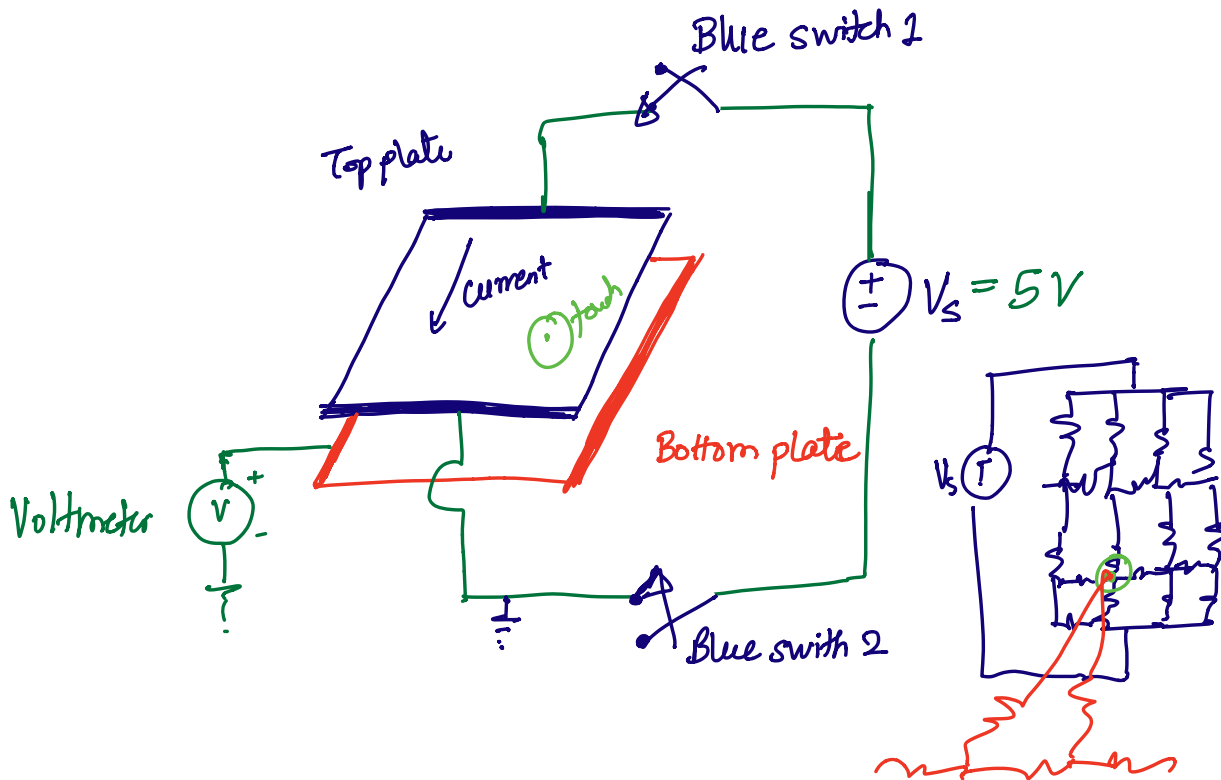
Model for reality.



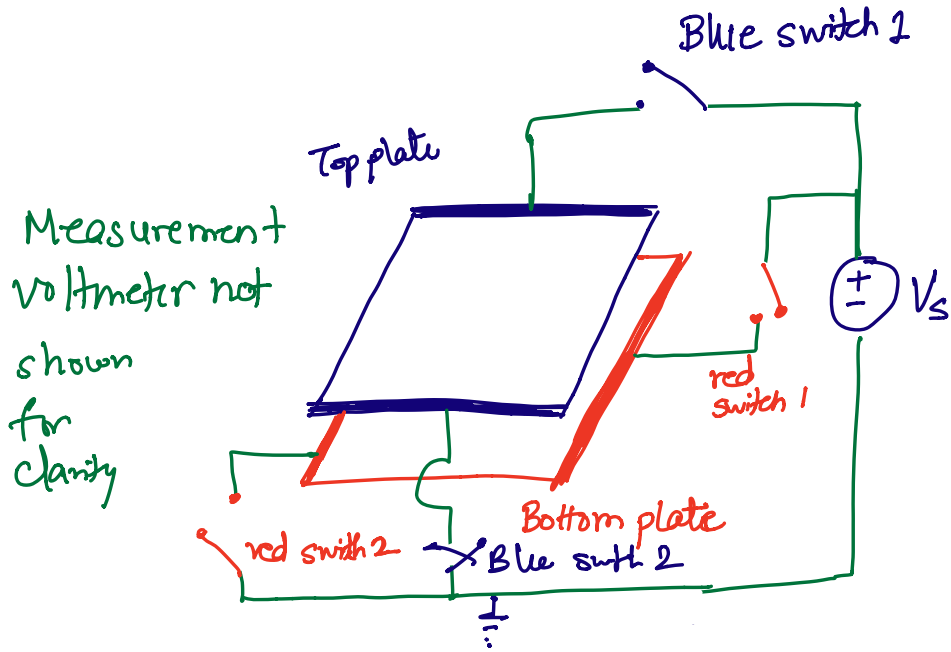
To measure the X position (horizontal position)



To measure the Y position:



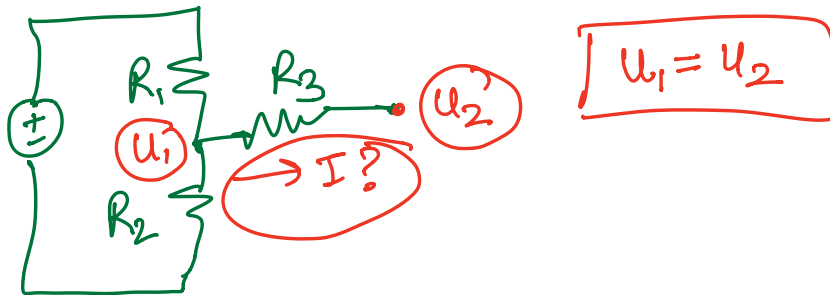
Putting it together:



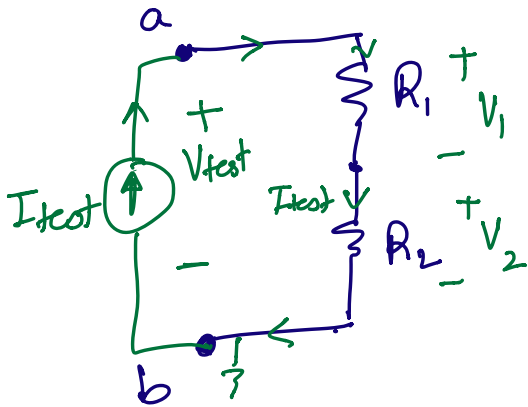
Dark shaded area: conducting strip


Turn both blue switches on, red OFF

red ON blue OFF



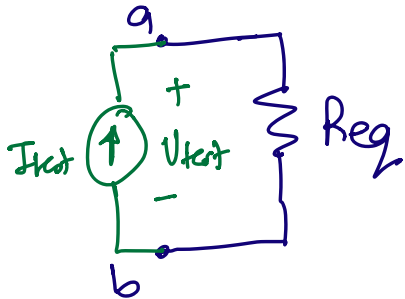
Equivalence : $I - V$ at nodes a, b
 Current and voltage to be the same for both circuits.



$$V_1 = I_{test} \cdot R_1$$

$$V_2 = I_{test} \cdot R_2$$

$$\begin{aligned} V_{test} &= V_1 + V_2 \quad (\text{KVL}) \\ &= I_{test} \cdot R_1 + I_{test} \cdot R_2 \\ &= I_{test} \cdot (R_1 + R_2) \end{aligned}$$



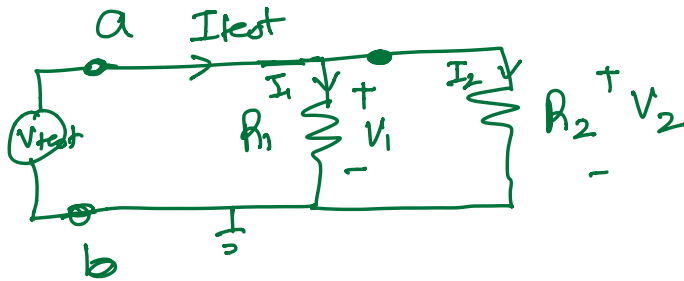
$$V_{test} = I_{test} (R_{eq})$$

$$\Rightarrow R_{eq} = R_1 + R_2$$

"Equivalence of circuits"

Resistors in series.

Equivalence 2: Resistance in parallel.



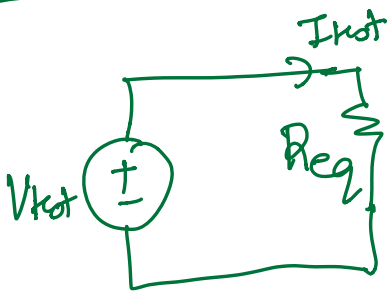
$$I_{\text{test}} = I_1 + I_2.$$

$$V_1 = I_1 R_1, \quad , \quad V_2 = I_2 R_2$$

$$V_1 = V_2 = V_{\text{test}}$$

$$V_{\text{test}} = I_1 R_1, \quad , \quad V_{\text{test}} = I_2 R_2$$

$$I_{\text{test}} \because I_1 + I_2 = \frac{V_{\text{test}}}{R_1} + \frac{V_{\text{test}}}{R_2} = V_{\text{test}} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$



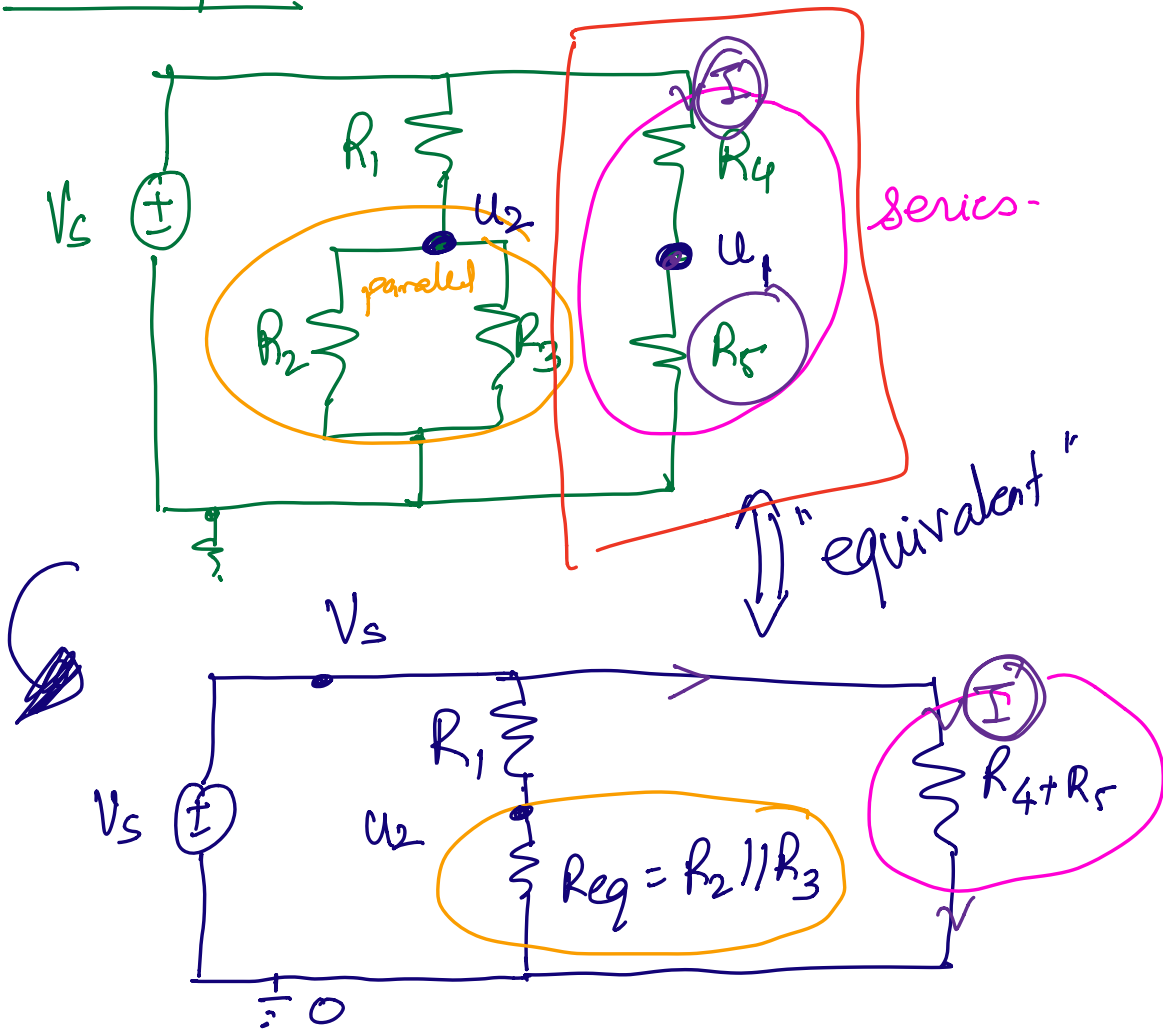
$$\bullet V_{\text{test}} = I_{\text{test}} \cdot R_{\text{eq}}$$

$$= I_{\text{test}} = \frac{V_{\text{test}}}{R_{\text{eq}}}$$

$$\Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{Resistance in parallel}$$

Notation: $R_{eq} = R_1 \parallel R_2$

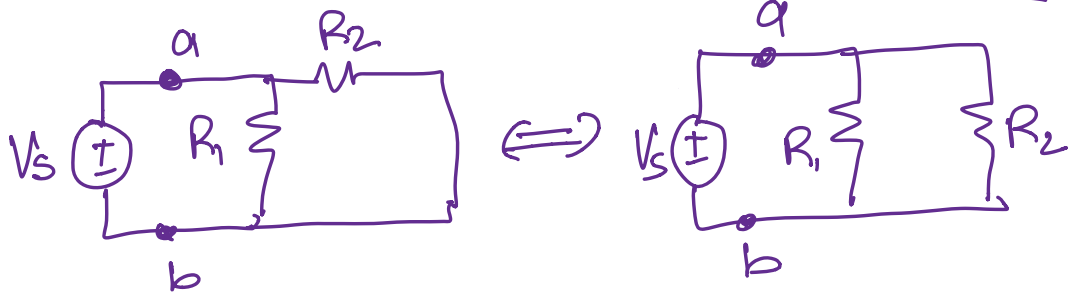
Example:



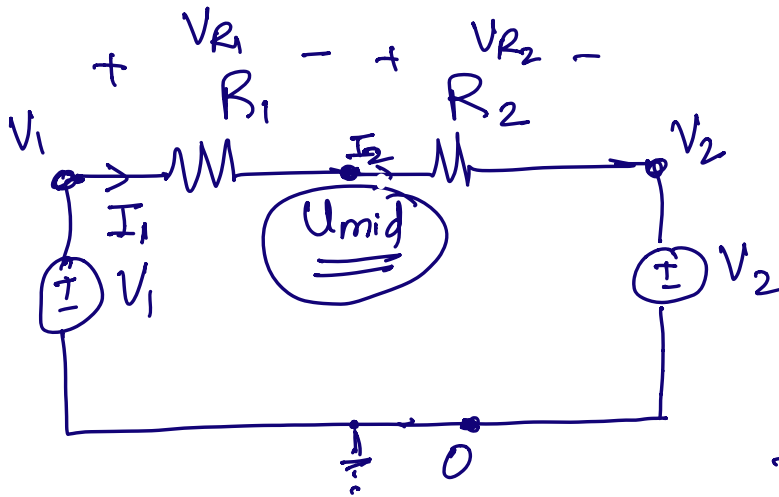
What happens to u_1 ?

$$u_2 = \frac{R_2 \parallel R_3}{R_1 + (R_2 \parallel R_3)} \cdot V_s$$

$$V_S = I(R_4 + R_5)$$



Superposition : Power of the linearity of circuits



NVA

Unknown:

U_{mid} , I_1

Known: V_1, V_2, R_1, R_2

KCL: $I_1 = I_2$

Element eqⁿs: $V_{R_1} = I_1 R_1, V_{R_2} = I_2 R_2$

Substitute:

$$V_{R_1} = V_1 - U_{\text{mid}} = I_1 R_1 \quad (1)$$

$$V_{R_2} = U_{\text{mid}} - V_2 = I_2 R_2 = I_1 R_2 \quad (2)$$

(1) \Rightarrow

$$U_{\text{mid}} + I_1 R_1 = V_1$$

(2)

$$U_{\text{mid}} - I_1 R_2 = V_2$$

$$I_1 = \frac{V_1 - V_2}{R_1 + R_2}$$

$$U_{\text{mid}} = \frac{R_1 V_2 + R_2 V_1}{R_1 + R_2}$$

$$\vec{x} = \begin{bmatrix} U_{\text{mid}} \\ I_1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & R_1 \\ 1 & -R_2 \end{bmatrix}}_A \begin{bmatrix} U_{\text{mid}} \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \vec{b}$$

$$\vec{x} = A^{-1} \vec{b}$$