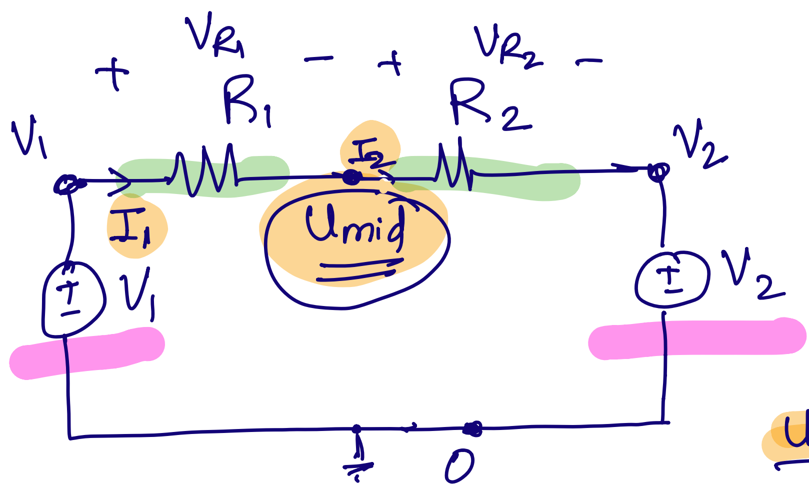
EECS 16A Lecture ~~8~~ 6

- Superposition: Linear Algebra tricks for circuits.
- Capacitors

Logistics

- Extended circuits review.

Superposition : Power of the linearity of circuits



NVA

Unknown: U_{mid}, I_1

Known: V_1, V_2, R_1, R_2

KCL: $\boxed{I_1 = I_2}$

Element eq^{n}s: $V_{R_1} = I_1 R_1$, $V_{R_2} = I_2 R_2$

Substitute:

$$V_{R_1} = V_1 - U_{mid} = I_1 R_1 \quad (1)$$

$$V_{R_2} = U_{mid} - V_2 = I_2 R_2 = I_1 R_2 \quad (2)$$

$$\boxed{\begin{aligned} (1) \Rightarrow \quad & U_{mid} + I_1 R_1 = V_1 \\ (2) \quad & U_{mid} - I_1 R_2 = V_2 \end{aligned}}$$

$$I_1 = \frac{V_1 - V_2}{R_1 + R_2}$$

$$U_{mid} = \frac{R_1 V_2 + R_2 V_1}{R_1 + R_2}$$

$$\vec{x} = \begin{bmatrix} U_{mid} \\ I_1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & R_1 \\ 1 & -R_2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} U_{mid} \\ I_1 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{\vec{b}}$$

$$\vec{x} = A^{-1} \vec{b}$$

$$A\vec{x} = \vec{b}$$

Check: Is A invertible?

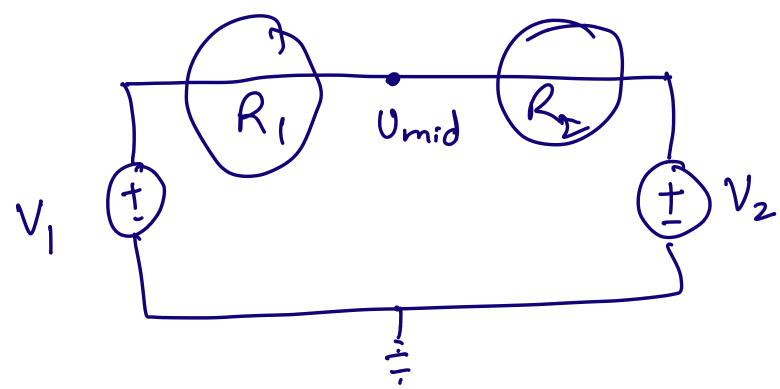
$$A = \begin{bmatrix} 1 & R_1 \\ 1 & -R_2 \end{bmatrix}$$

$$\begin{bmatrix} R_1 \\ -R_2 \end{bmatrix} \overset{?}{=} \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\alpha \in \mathbb{R}$

Is this possible?

→ A is always invertible, except when $R_1 = R_2 = 0$



① If math breaks ⇒ physical world is also probably unhappy :(

$$\underbrace{\begin{bmatrix} 1 & R_1 \\ 1 & -R_2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_{mid} \\ I_1 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{\vec{b}}$$

$$\vec{x} = A^{-1}\vec{b}$$

$$\vec{b} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ V_2 \end{bmatrix}$$

$$= \vec{b_1} + \vec{b_2}$$

$$\vec{x} = A^{-1}\vec{b}$$

$$= A^{-1}(\vec{b_1} + \vec{b_2})$$

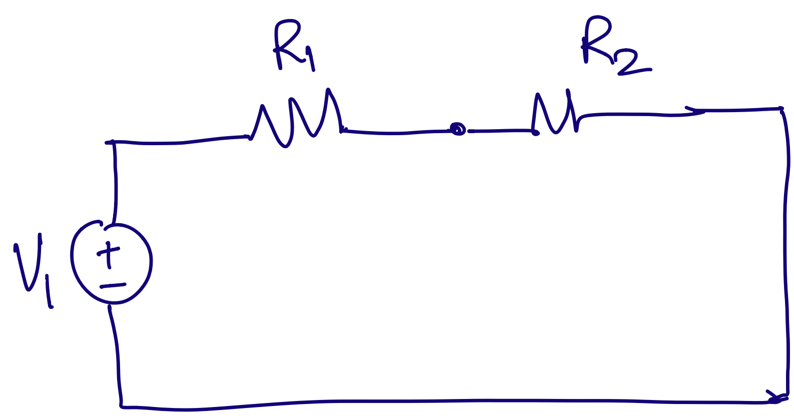
$$= A^{-1}\vec{b_1} + A^{-1}\vec{b_2}$$

$$= A^{-1}\begin{bmatrix} V_1 \\ 0 \end{bmatrix} + A^{-1}\begin{bmatrix} 0 \\ V_2 \end{bmatrix}.$$

$A^{-1}\begin{bmatrix} V_1 \\ 0 \end{bmatrix}$: what does this represent?

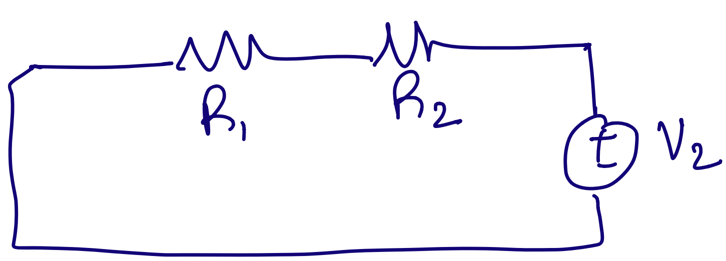
↳ Solution to

$$A\vec{x}_1 = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$ ie. our original circuit but $V_2 = 0$



↳ $A\vec{x} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

<u>Now:</u> $A\vec{x}_1 = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$



What does $A^{-1}\begin{bmatrix} 0 \\ V_2 \end{bmatrix}$?

I can solve the circuit by breaking it up into two parts:

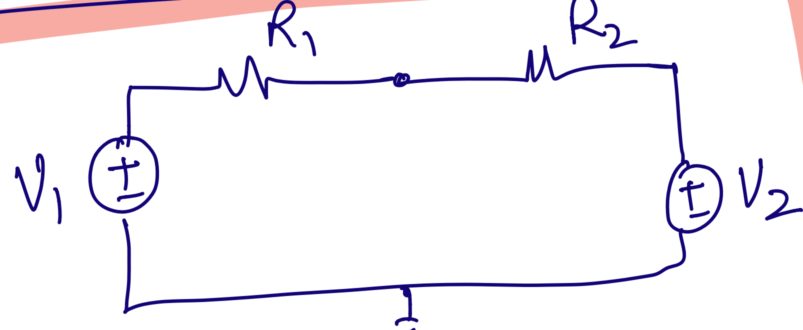
① with only V_1, $V_2 = 0$

② with only V_2 but $V_1 = 0$.

$$\vec{x} = \begin{bmatrix} u_{mid} \\ I_1 \end{bmatrix} = A^{-1}\vec{b}$$

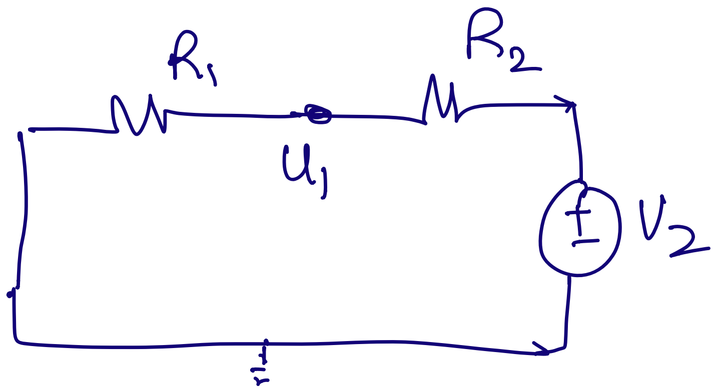
$$= A^{-1}\begin{bmatrix} V_1 \\ 0 \end{bmatrix} + A^{-1}\begin{bmatrix} 0 \\ V_2 \end{bmatrix}$$

$\boxed{\text{Superposition}}$



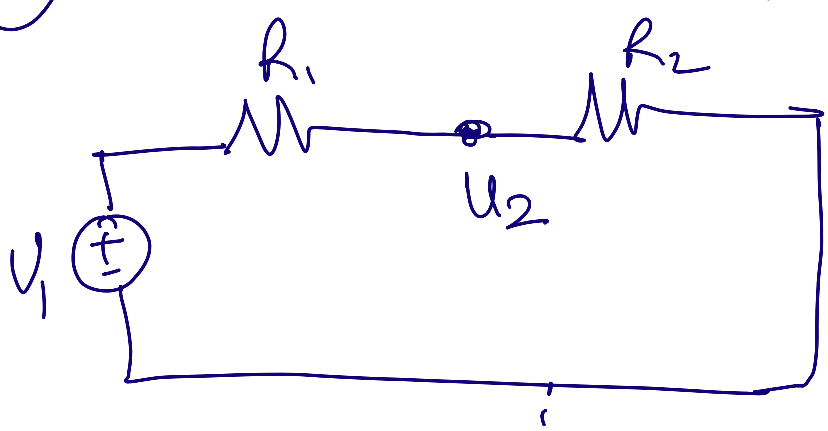
Voltage summer if $R_1 = R_2$

① Zero out the first voltage.



$$u_1 = \frac{R_1}{R_1+R_2} \cdot V_2$$

② Zero out the second voltage



$$u_2 = \frac{R_2}{R_1+R_2} \cdot V_1$$

③ Combining:

$$U_{mid} = U_1 + U_2$$

$$= \frac{R_1}{R_1+R_2} V_2 + \frac{R_2}{R_1+R_2} \cdot V_1$$

Two things:

① If $R_1 = R_2 = R$

$$U_{mid} = \frac{R\ (V_1 + V_2)}{2R}$$

$$= \frac{1}{2} (V_1 + V_2)$$

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② "Physical Meaning" of Superposition

Setup 1

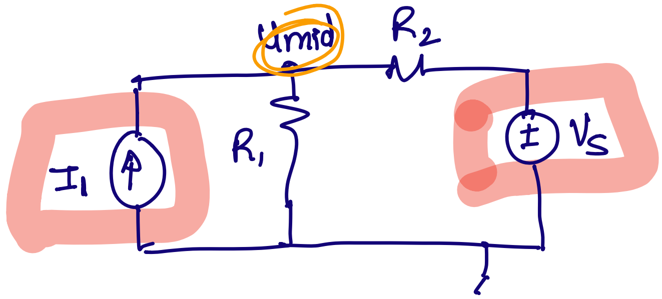
$$\begin{bmatrix} 1 & R_1 \\ 1 & -R_2 \end{bmatrix} \underbrace{\begin{bmatrix} u_{mid} \\ I_1 \end{bmatrix}}_{\vec{x}} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Instead of this if I had:

Setup 2

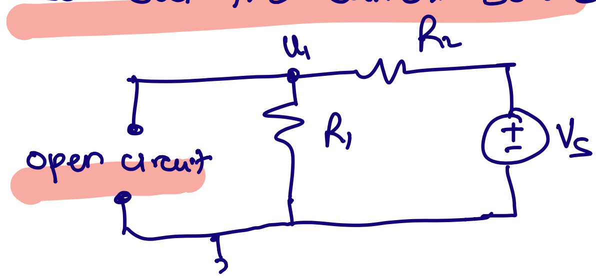
$$\begin{bmatrix} 1 & R_1 \\ 1 & -R_2 \end{bmatrix} \underbrace{\begin{bmatrix} u_{mid} \\ I_1 \end{bmatrix}}_{2\vec{x}} = \begin{bmatrix} 2V_1 \\ 2V_2 \end{bmatrix}$$

Example 2



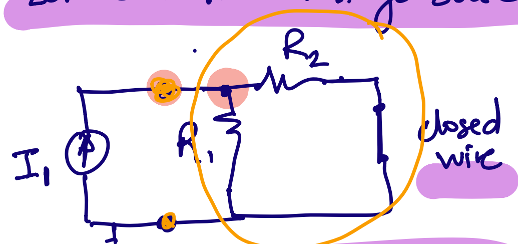
What does it mean to zero out a current source?

① Zero out the current source

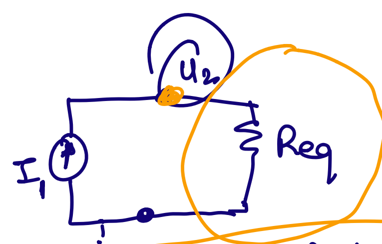


$$u_1 = \frac{R_1}{R_1+R_2} \cdot V_s$$

② Zero out the voltage source.



$\Leftrightarrow$



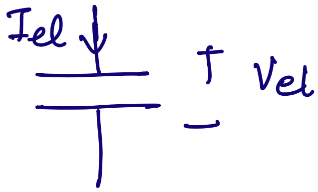
$$R_{eq} = R_1 \,//\, R_2$$

$$u_2 = I_1 R_{eq}$$
$$= I_1 \cdot \frac{R_1 R_2}{R_1+R_2}$$

③ Combine:

$$u_{mid} = u_1 + u_2$$
$$= \frac{R_1 V_s}{R_1+R_2} + \frac{I_1 R_1 R_2}{R_1+R_2}$$

Capacitors What are they?



"I-V" relationship for a capacitor:

$$Q = C \cdot V$$

Charge = Capacitance · Voltage

[C] [F] [V]

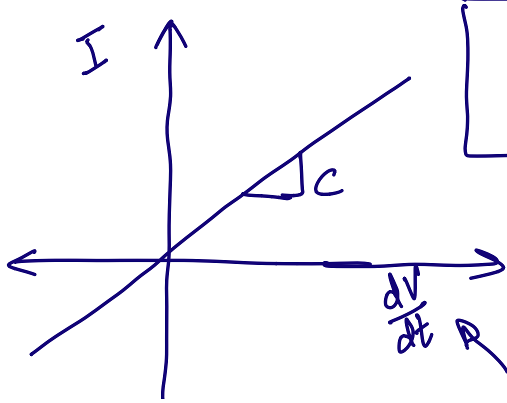
Coulombs Farads Voltage

I current: $= \dfrac{dQ}{dt}$

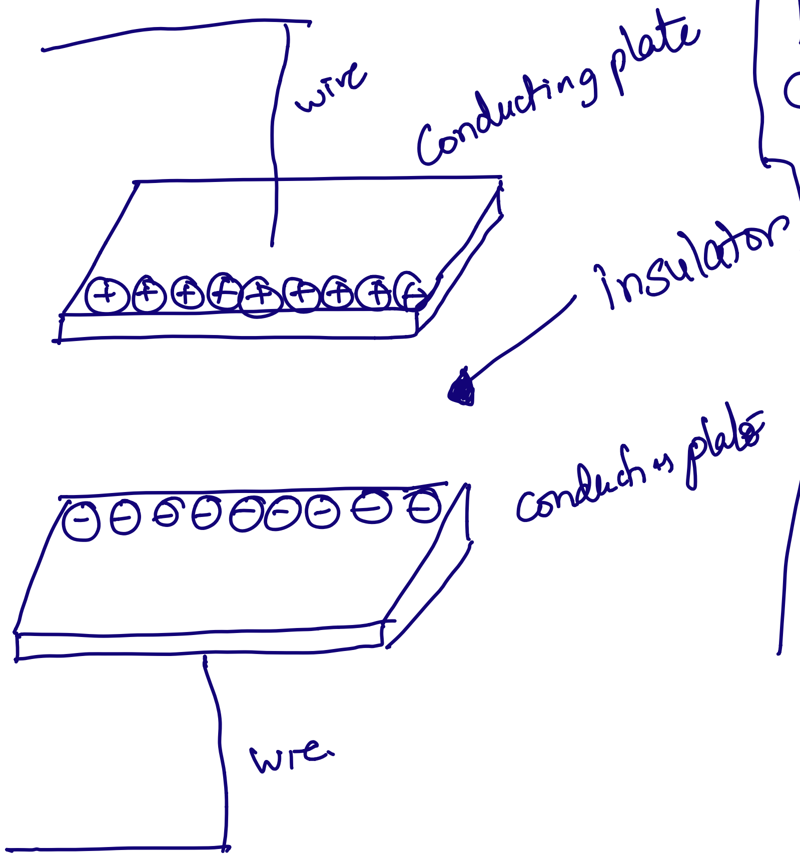
Taking derivatives: $I = \dfrac{dQ}{dt} = \dfrac{d(CV)}{dt}$

$$\boxed{I = C \cdot \frac{dV}{dt}}$$

C is a constant.



Not V



Insulator cannot carry any current

Charge accumulates on both sides

"Bucket" for charge

$$Q_{el} = C \cdot V_{el} \Longrightarrow V_{el} = \frac{Q_{el}}{C}$$

Voltage $\Longleftrightarrow$ energy?

$$V_{el} = \frac{dE_{el}}{dQ_{el}}$$

$$\Longrightarrow \frac{dE_{el}}{dQ_{el}} = \frac{Q_{el}}{C}$$

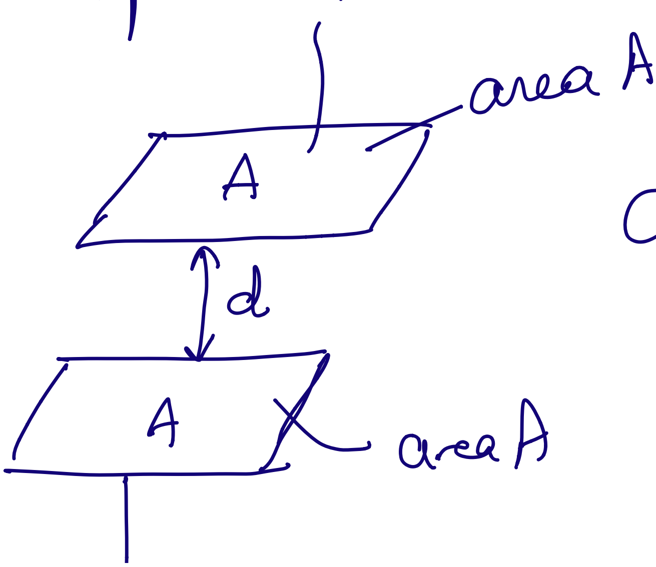
$$\Rightarrow \quad E_{el} = \int_0^Q \frac{1}{C} \cdot Q_{el} \, dQ_{el}$$

$$= \frac{1}{2} Q^2/C$$

$$= \frac{1}{2} \frac{C^2 V^2}{C} = \frac{1}{2} C V^2$$

Energy stored on a capacitor: $\frac{1}{2} C \cdot V^2$

How do the physical properties of a capacitor matter?



Two plates of a capacitor.

Capacitance 'C' $= \varepsilon \cdot \frac{A}{d}$

ε = permittivity. $\left[\frac{F}{m}\right]$

$\varepsilon_0 = 8.85 \; pF/m$ " Permittivity of free space "

$= 8.85 \times 10^{-12} \; F/m$