EECS 16A Lecture 7

• Revive your study groups.

Last time:

Capacitors: Buckets for charge.
Store energy.

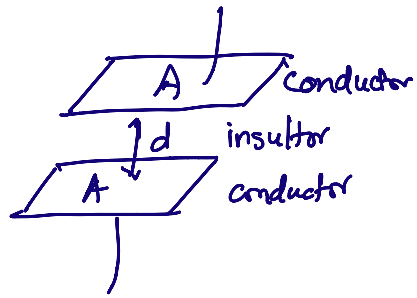
Current:
$$I = C\frac{dV}{dt}$$

Charge:
$$Q = CV$$

Energy
$$E = \frac{1}{2}CV^2$$

$$C = \frac{\varepsilon \cdot A}{d}$$

Capacitance

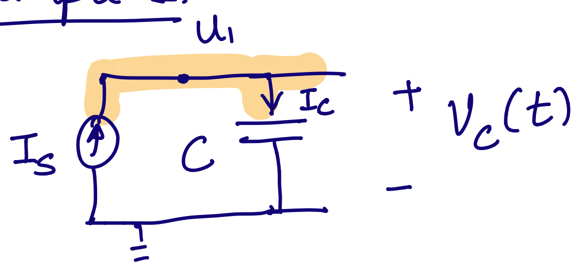


Today: (1) Circuits with capacitors.

(2) Capacitor equivalence

(3) Capacitive touchscreen. (Charge sharing)

Example 1:



Integration:

$$\int_a^b f(x)\,dx = F(a) - F(b)$$

$$\int f(x) = F(x)$$

KCL: $I_s = I_c$

$$I_c = C \cdot \frac{dV_c}{dt} \quad \text{(Element equation)}$$

$$I_s = C \cdot \frac{dV_c}{dt}$$

$V_c(0)$: initial voltage across capacitor at time 0.
$V_c(t)$: volt. @ time t

Integrate

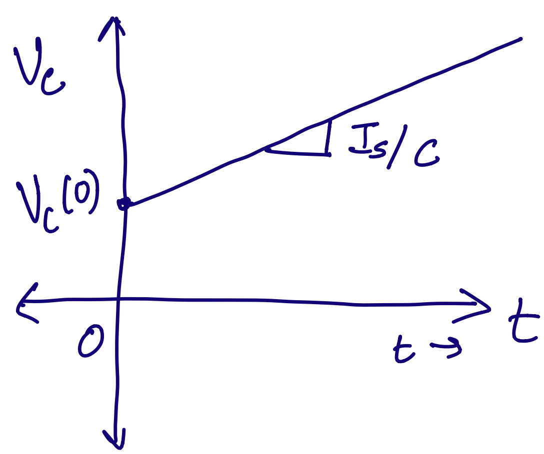
$$\int_0^t I_s \, dt = \int_0^t C \cdot \frac{dV_c}{dt} \cdot dt$$

C: constant

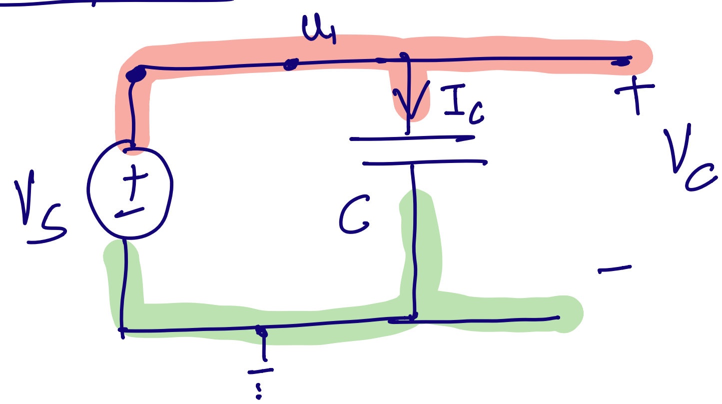
$$\boxed{I_s \cdot t = C\left[V_c(t) - V_c(0)\right]}$$

$$\int_0^t \frac{dV_c(t)}{dt} dt = V_c(t)\Big|_0^t = V_c(t) - V_c(0)$$

$$\boxed{V_c(t) = \frac{I_s \cdot t}{C} + V_c(0)}$$



Example 2:



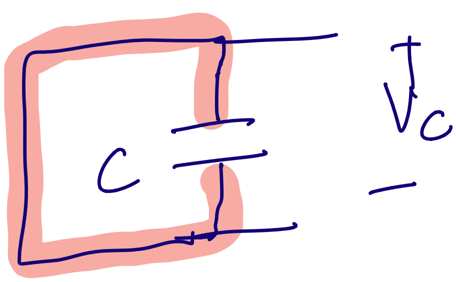
$V_c = u_1$

$$I_c = C\frac{dV_c}{dt}$$

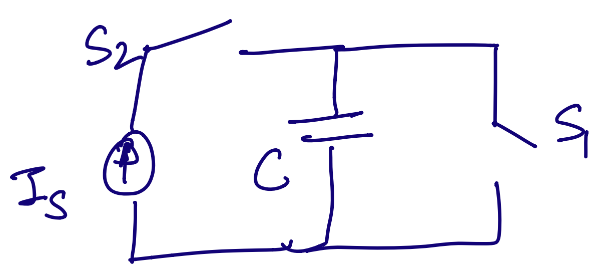
$$\frac{dV_c}{dt} = \frac{dV_s}{dt} = 0$$

$$I_c = C \cdot 0 = 0$$

Example 2a:

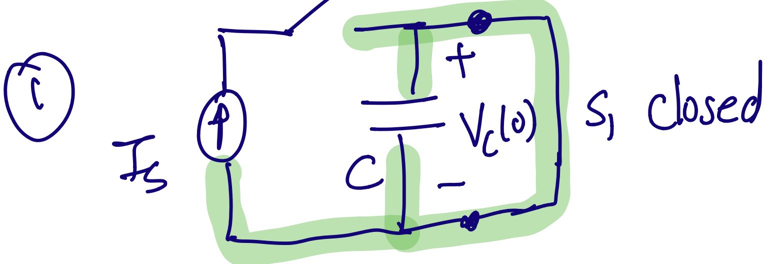


$V_c = 0$



Time: Closed S_1.

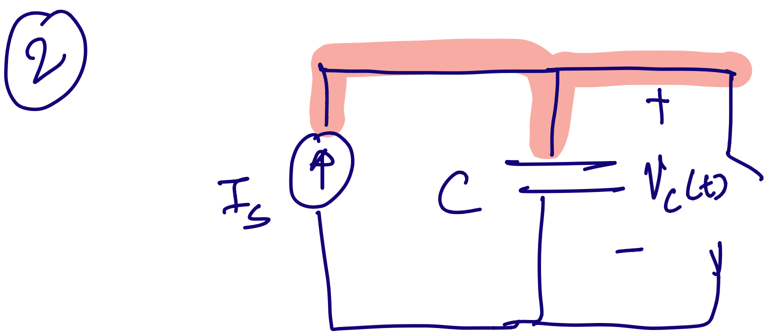
Open S_2, close S_1



Set inital condition of capacitor

$V_c(0) = 0$ Set the inital condition

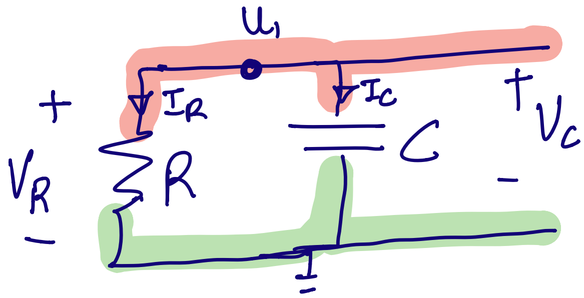
Then: Open S_1 close S_2.



$$V_c(t) = \frac{I_s \cdot t}{C} + V_c(0)$$

$$= \frac{I_s \cdot t}{C} + 0$$

Example 3 :



$\boxed{V_c(0) = 0}$

① $I_R = \dfrac{V_R}{R}$

KCL: $I_R + I_C = 0$ ☆

② $I_C = C \cdot \dfrac{dV_c}{dt}$

$V_R = V_c$

What does steady state mean?

⇒ The state where nothing is changing.

$V_c(t) = V_c(t + \Delta t)$

$\Rightarrow \dfrac{dV_c}{dt} = 0$

"change over time"

$$\frac{V_R}{R} + C\frac{dV_c}{dt} = 0$$

$$\frac{V_c}{R} + C\frac{dV_c}{dt} = 0$$

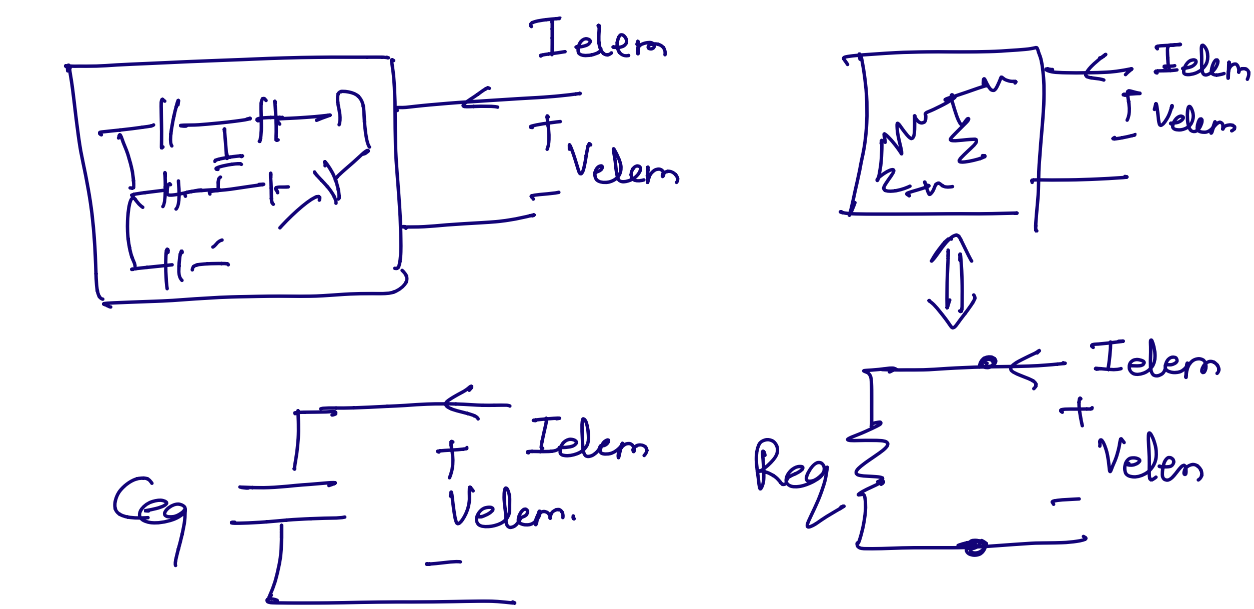
$$\Rightarrow \quad \frac{dV_c}{dt} = -\frac{V_c}{RC} \quad (*)$$

"Steady State" $\Longrightarrow$ $\dfrac{dV_c}{dt} = 0$. By def^n of steady state

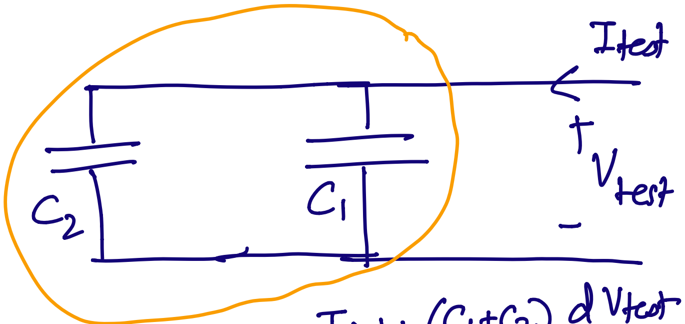
$\Longrightarrow^{(*)}$ $0 = \dfrac{-V_c}{RC}$ at steady state

$\Longrightarrow$ $V_c = 0$ at steady state

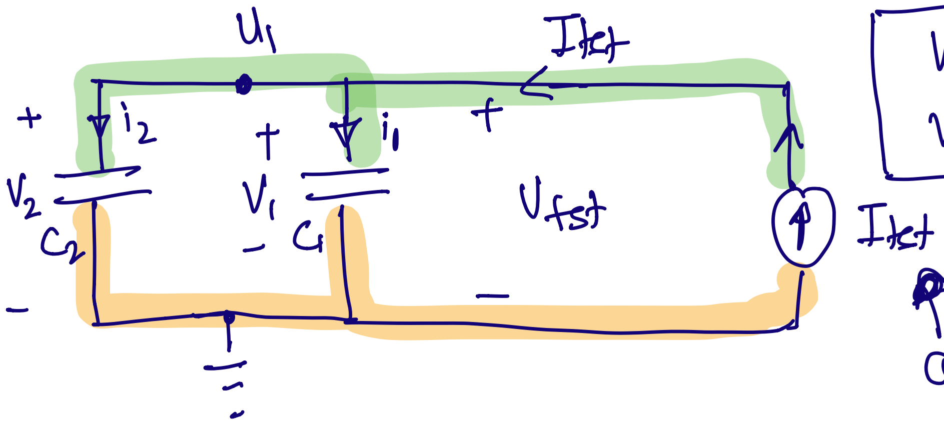
Equivalence of Capacitors



Example: Capacitors in Parallel.



$$I_{test} = (C_1+C_2)\frac{dV_{test}}{dt} \qquad I_{test} = C_{eq}\cdot\frac{dV_{test}}{dt}$$



$$\boxed{\begin{aligned} V_1(0) &= 0 \\ V_2(0) &= 0 \end{aligned}}$$

adding for analysis

KCL: $\quad I_{test} = i_1 + i_2 \qquad V_1 = V_2 = V_{test}$

$$i_1 = C_1\cdot\frac{dV_1}{dt} \qquad i_2 = C_2\cdot\frac{dV_2}{dt} = C_2\cdot\frac{dV_1}{dt}$$

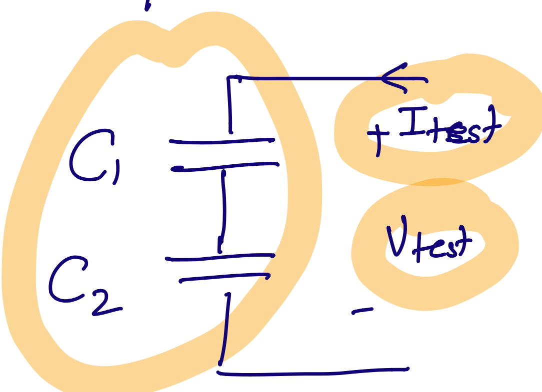
$$I_{test} = C_1\frac{dV_1}{dt} + C_2\frac{dV_2}{dt} = C_1\frac{dV_1}{dt} + C_2\frac{dV_1}{dt}$$

$$I_{test} = (C_1+C_2)\frac{dV_1}{dt} = (C_1+C_2)\frac{dV_{test}}{dt}$$

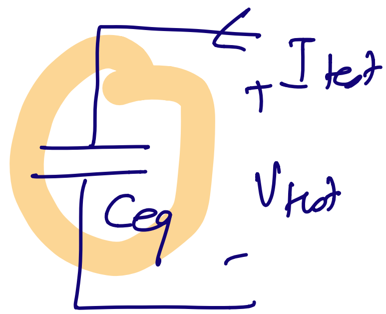
$$I_{test} = (C_1 + C_2)\frac{dV_{test}}{dt}$$

$$\Rightarrow \quad C_{eq} = C_1 + C_2$$

Capacitors in series

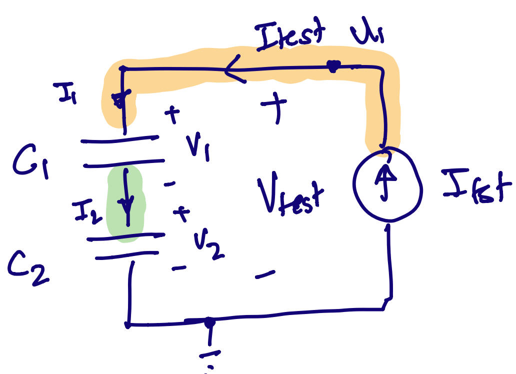


$\Rightarrow$



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Connect a current source I_{test}



- KCL : $I_1 = I_{test}$

  $I_1 = I_2 = I_{test}$

- KVL: $V_{test} = V_1 + V_2$

$$I_1 = C_1 \frac{dV_1}{dt}$$

$$I_2 = C_2 \cdot \frac{dV_2}{dt}$$

$$\frac{dV_{test}}{dt} = \frac{dV_1}{dt} + \frac{dV_2}{dt}$$

$$\Rightarrow \frac{dV_2}{dt} = \frac{dV_{test}}{dt} - \frac{dV_1}{dt}$$

Since $I_1 = I_2$,

$$C_1 \frac{dV_1}{dt} = C_2 \frac{dV_2}{dt}$$

$$= C_2 \left(\frac{dV_{test}}{dt} - \frac{dV_1}{dt} \right)$$

$$\Rightarrow (C_1 + C_2) \frac{dV_1}{dt} = C_2 \cdot \frac{dV_{test}}{dt}$$

$$\frac{dV_1}{dt} = \frac{I_1}{C_1} = \frac{I_{test}}{C_1}$$

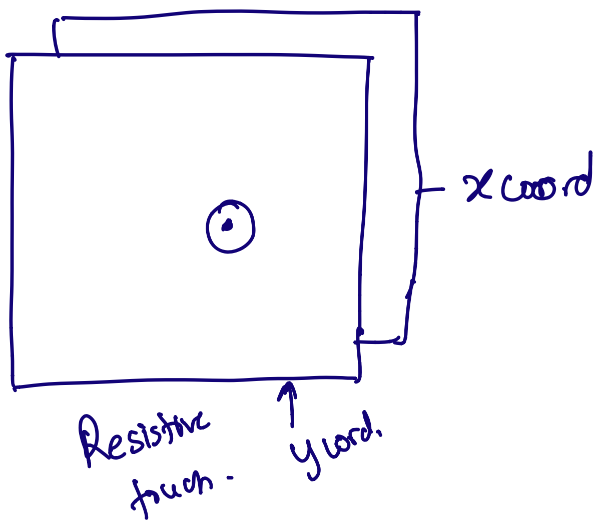
$$\Rightarrow (C_1 + C_2) \cdot \frac{I_{test}}{C_1} = C_2 \cdot \frac{dV_{test}}{dt}$$

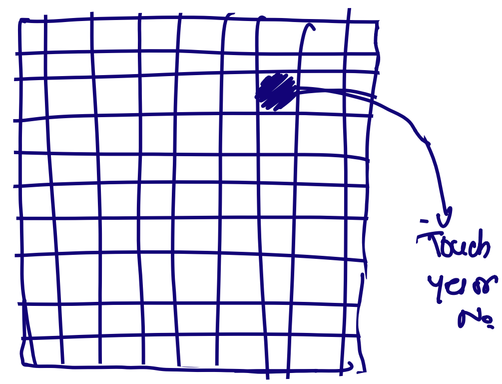
$$\Rightarrow I_{test} \left(\frac{C_1 + C_2}{C_1 C_2} \right) = \frac{dV_{test}}{dt}$$

$$\rightarrow \boxed{I_{test} = \left(\frac{C_1 C_2}{C_1 + C_2} \right) \frac{dV_{test}}{dt}}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \underbrace{C_1 \parallel C_2}_{\text{notations}}$$

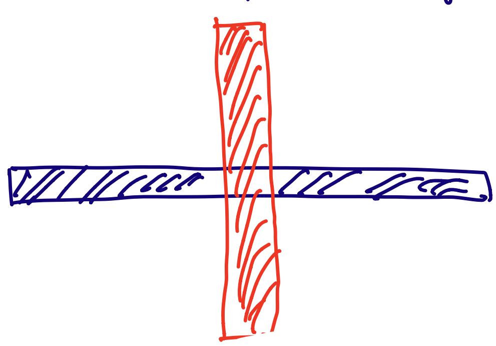
Finally : Building a capacitive touchscreen



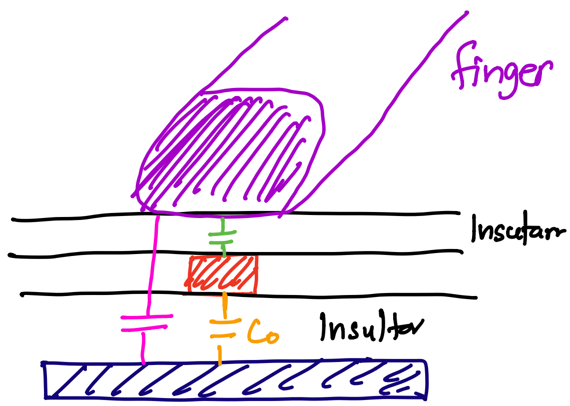


How to detect touch for one pixel.

Top view



Side view:



Office hours:

$$\int_0^t \frac{dV_c}{dt} \cdot dt = \int_{V_c(0)}^{V_c(t)} dV_c$$