

EECS 16A Designing Information Devices and Systems I

Fall 2021 Discussion 6B

1. Material Resistivity

- (a) Recall the 1D resistive touch screen model introduced in class. In this model, the top layer can be thought of as a resistor, while the bottom layer can be thought of as a wire. When the top layer is touched, it flexes at the touch point and makes contact with the bottom layer. This results in a voltage divider.

Material	Resistivity ρ ($\Omega \text{ cm}$)	Conductivity $1/\rho$ ($\Omega^{-1} \text{ cm}^{-1}$)
Silver	1.6×10^{-5}	6.3×10^4
Aluminum	2.7×10^{-5}	3.7×10^4
Carbon (graphite)	10×10^{-2}	10
Rubber	100×10^{12}	1×10^{-14}

Given the following list of materials and their resistivity/conductivity, which materials would be good to use as a top layer, and which would be good to use as a bottom layer? Why?

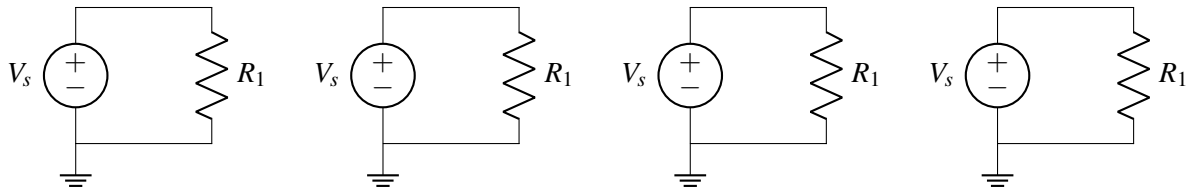
- (b) Let's say you want to make your own **10 cm** long resistor out of **graphite**. You need the resistance to be **1 Ω** . Recall the equation for resistance: $R = \rho \frac{L}{A} = \rho \frac{L}{W*H}$
- What are some possible widths and heights of your resistor?
 - Can you think of advantages of having a wide and thin resistor? How about advantages of a narrow and thick resistor?

Answer:

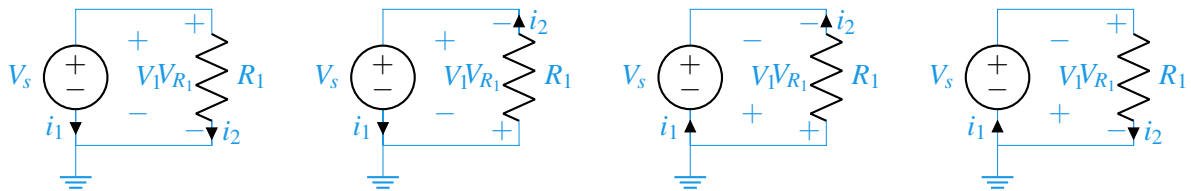
- (a) We want the bottom layer to act like an ideal wire. Thus, it should be made out of highly conductive material. Silver or aluminum would be a good choice for the bottom layer. We want the top layer to act as a resistor. Thus, it should be made out of a material with a high resistivity. Carbon and glass would be a good choice for the top layer.
- (b) Any choice of W and H works for $W \times H = 1 \text{ cm}$ (such as $W=1 \text{ cm}$, $H=1 \text{ cm}$; $W=0.5 \text{ cm}$, $H=2 \text{ cm}$; etc.). A possible advantage for larger width is an increase in surface area, and thus better heat dissipation. Power generates heat which needs to be dissipated from the circuit. However, a smaller surface area as a result of a smaller resistor width can also be advantageous. A resistor with less surface area will take up less space on a circuit.

2. Passive Sign Convention and Power

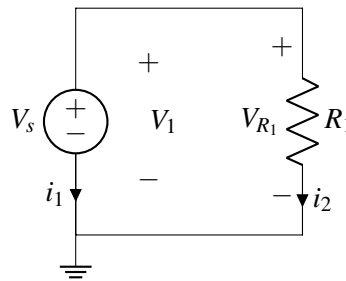
- (a) Below are four copies of a the same single-resistor circuit. On each copy, provide a distinct choice of labels for each circuit's voltage polarities and current directions (there should be 4 possible choices in total!) while keeping with passive sign convention.



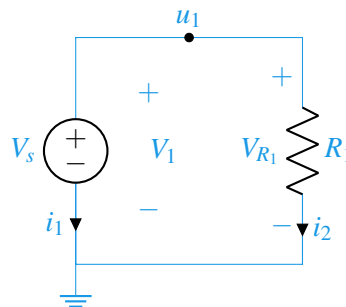
Answer:



(b) Suppose we consider one of the possible labelings you have found above. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\text{V}$ and let $R_1 = 5\Omega$. Recall that the power dissipated is the rate of electric energy converted into other forms and is given by the equation $P = IV$. When the power dissipated by an element is a negative value, it signifies that element is actually supplying electrical power to the circuit.



Answer: We'll start by solving the circuit for the unknown node potentials and currents.



The KCL equation for the one node in this circuit is:

$$i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$u_1 - 0 = V_1 = V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\text{ V}$ and $R_1 = 5\ \Omega$:

$$\begin{aligned}u_1 &= 5\text{ V} \\i_1 &= -1\text{ A} \\i_2 &= 1\text{ A}\end{aligned}$$

From above, we can solve for the power dissipated across the resistor:

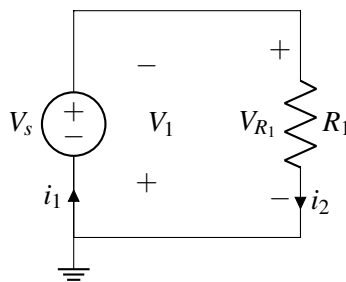
$$P_{R_1} = i_2 V_{R_1} = 1\text{ A} \cdot 5\text{ V} = 5\text{ W}$$

Next we can solve for the power dissipated across the voltage source:

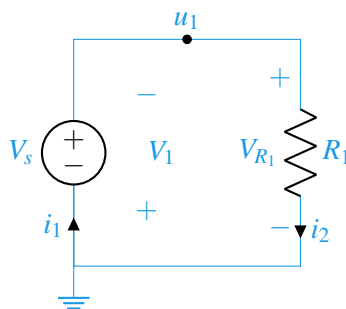
$$P_{V_s} = i_1 V_1 = i_1 V_s = -1\text{ A} \cdot 5\text{ V} = -5\text{ W}$$

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is adding power to the circuit.

- (c) Suppose we choose a second labeling of the circuit as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\text{ V}$ and let $R_1 = 5\ \Omega$.



Answer: We'll solve the circuit the same way as last time.



The KCL equation for the one node in this circuit is:

$$-i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$0 - u_1 = V_1 = -V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\text{ V}$ and $R_1 = 5\ \Omega$:

$$u_1 = 5\text{ V}$$

$$i_1 = 1\text{ A}$$

$$i_2 = 1\text{ A}$$

From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = i_2 V_{R_1} = 1\text{ A} \cdot 5\text{ V} = 5\text{ W}$$

Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = i_1 V_1 = i_1 (-V_s) = 1\text{ A} \cdot -5\text{ V} = -5\text{ W}$$

Notice here that the circuit has the same power dissipated by all the elements. This is because with both labeling of currents, we followed the passive sign convention.

- (d) Did the values of the element voltages and element currents change with the different labeling? Did the power for each circuit element change? Did the node voltages change? If a quantity didn't change with a difference in labeling, discuss what would have to change for quantity to change.

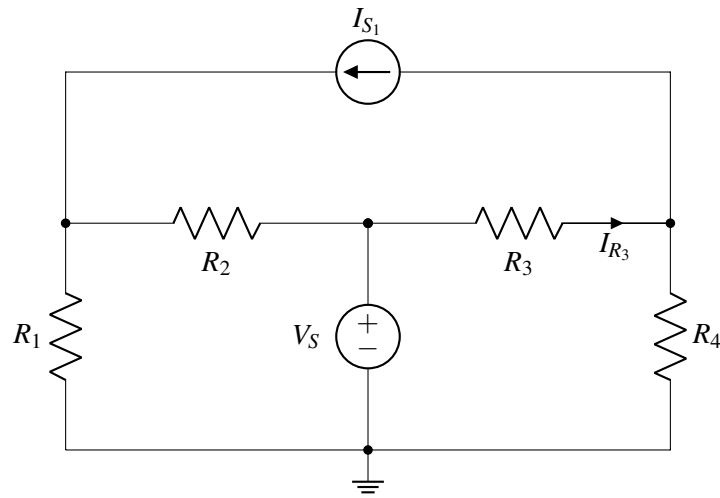
Answer: With a different labeling, element voltages and element currents will change. The quantities were $V_1 = 5\text{ V}$ and $i_1 = -1\text{ A}$ in (b) and in (c) $V_1 = -5\text{ V}$ and $i_1 = 1\text{ A}$. Flipping the direction of a labeled current or the polarity of a labeled voltage will lead to negation of the value.

The power dissipated by a circuit element will not change because we follow passive sign convention. Passive sign convention requires that if we flip the direction of an element current we also flip the polarity of the corresponding element voltage, so there is a double negation in the computation of power. The only way to get a different value of power would be to change the component values or the circuit diagram itself by removing or adding more circuit elements. A physical system will only have one behavior as governed by the laws of physics - how we compute our answer should not change how it behaves. Our labeled voltage polarities and current directions are more akin to measurement choices which can change what we see.

The node voltages too, did not change. The top node voltage in both labelings were 5 V and the bottom node voltages were 0 V . What would have to change to alter these values is one of three things: either the location of ground, the circuit component values, or the circuit diagram itself.

3. Circuit Analysis

- Use nodal analysis to solve for all node voltages.
- Practice writing out your expressions in matrix vector form. (Recall the form $\mathbf{A}\vec{x} = \vec{b}$, where \vec{x} is your vector of unknown voltages or/and currents).
- Find current I_{R_3} flowing through resistor R_3 .

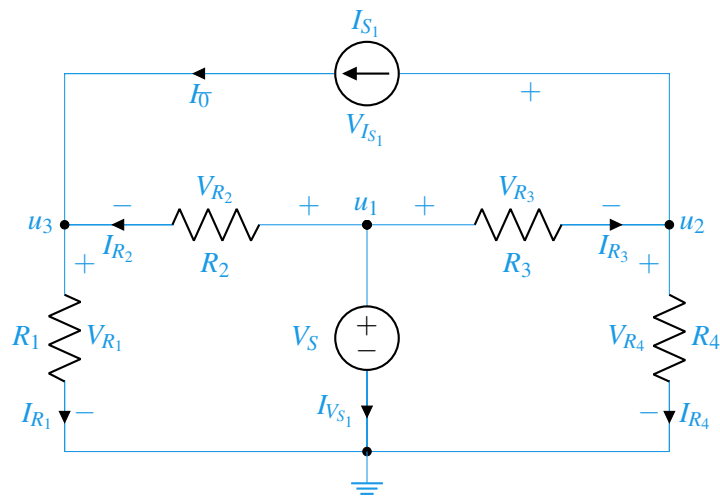
**Answer:**

(a) Let us follow the procedure illustrated in lecture. Step 1 (selecting a ground node) has already been done for us, we will go through steps 2-4 in the following circuit diagram. As a reminder the steps are the following:

Step 2: Label all known voltages determined by voltage sources (denoted below as u_1),

Step 3: Label all remaining unknown nodes (denoted below as u_2, u_3),

Step 4: Label element voltages and currents (V_{R_1}, I_{R_1} , etc.)



Note: You can pick any direction you want for the currents but once you have done so you the voltages you choose have to abide by the passive sign convention!! Vice versa, you can pick the element voltage sign first but then the current has to abide by the passive sign convention.

Step 5: Write KCL equations for all nodes (u_1, u_2, u_3):

$$-I_{R_3} + I_0 + I_{R_4} = 0$$

$$-I_{R_2} - I_0 + I_{R_1} = 0$$

$$I_{R_3} + I_{V_{s1}} + I_{R_3} = 0$$

Step 6: Find expressions for all element currents in terms of element voltages and characteristics,

$$\begin{aligned} I_0 &= I_{S1} \\ u_3 - 0 &= I_{R1}R_1 \\ V_s - u_3 &= I_{R2}R_2 \\ u_2 - 0 &= I_{R4}R_4 \\ V_s - u_2 &= I_{R3}R_3 \\ V_s &= u_1 \end{aligned}$$

Step 7: Substitute expressions found in 6 into the KCL equations from step 5,

$$\begin{aligned} -\frac{V_s - u_2}{R_3} + I_{S1} + \frac{u_2}{R_4} &= 0 \\ -\frac{V_s - u_3}{R_2} - I_{S1} + \frac{u_3}{R_1} &= 0 \end{aligned}$$

Notice that we can now directly solve for u_2 and u_3 and then substitute into the equations of Step 6 to find the currents through all resistors. The node voltages are given by:

$$\begin{aligned} u_2 &= \frac{V_s R_4 - I_{S1} R_3 R_4}{R_3 + R_4} \\ u_3 &= \frac{V_s R_1 + I_{S1} R_1 R_2}{R_1 + R_2} \end{aligned}$$

Finally, KCL in node u_1 will give us the current through the voltage source, V_s .

(b) Using the expressions in steps 6 and 7:

$$\begin{bmatrix} 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -R_1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -R_2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -R_3 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -R_4 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{V_{s1}} \\ I_0 \\ I_{R1} \\ I_{R2} \\ I_{R3} \\ I_{R4} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_{S1} \\ 0 \\ 0 \\ V_s \\ 0 \\ 0 \end{bmatrix}$$

(c)

$$I_{R3} = \frac{V_s - u_2}{R_3}$$