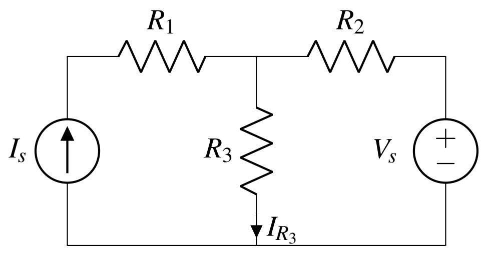
# EECS 16A Designing Information Devices and Systems I
# Fall 2021 Discussion 12A

## 1. Superposition

Consider the following circuit:




(a) With the current source turned on and the voltage source off, find the current $I_{R_3}$.

**Answer:**

We note that that $I_s$ is split between $R_2$ and $R_3$ and therefore we can use the current divider relation (here the notation $I_{R_3,I_s}$ represents the current across $R_3$ with only $I_s$ turned on.):

$$I_{R_3,I_s} = \frac{I_s R_2}{R_2 + R_3}$$

(b) With the voltage source turned on and the current source turned off, find the voltage drop across $R_3$, $V_{R_3}$. **Answer:**

We note that when the current source is turn off it becomes an open circuit. Thus, we are left with a voltage divider.

$$V_{R_3,V_s} = \frac{V_s R_3}{R_2 + R_3}$$

(c) Find the power dissipated by $R_3$.

**Answer:**

We first find the missing quantities. The voltage drop across $R_3$ when the current source is on is given by:

$$V_{R_3,I_s} = I_{R_3,I_s} R_3 = \frac{I_s R_2 R_3}{R_2 + R_3}$$

$$I_{R_3,V_s} = \frac{V_{R_3,V_s}}{R_3} = \frac{V_s}{R_2 + R_3}$$

Thus, we have:

$$V_{R_3} = V_{R_3,V_s} + V_{R_3,I_s} = \frac{V_s R_3 + I_s R_2 R_3}{R_2 + R_3}$$

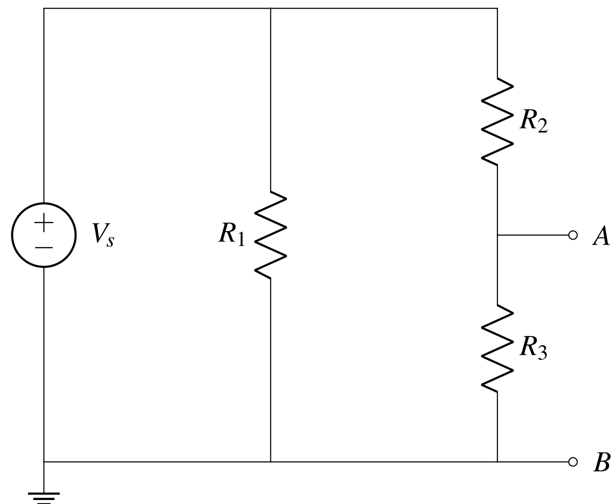
$$I_{R_3} = I_{R_3,V_s} + I_{R_3,I_s} = \frac{I_s R_2 + V_s}{R_2 + R_3}$$

Thus, the power dissipated is:

$$P_{R_3} = I_{R_3} V_{R_3} = \frac{R_3 (I_s R_2 + V_s)^2}{(R_2 + R_3)^2}$$

**2. Thévenin/Norton Equivalence**

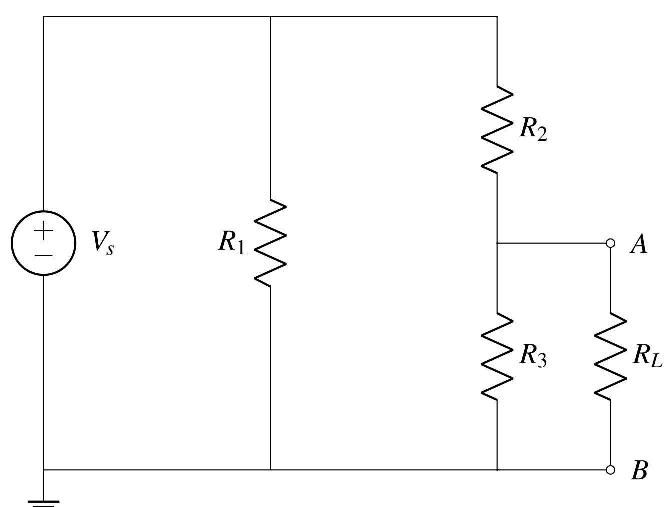
(a) Find the Thévenin resistance $R_{th}$ of the circuit shown below, with respect to its terminals $A$ and $B$.




**Answer:** To find the Thévenin resistance, we null out the voltage source (which shorts out $R_1$) and find the equivalent resistance which is simply:

$$R_{th} = R_2 \parallel R_3$$

(b) Now a load resistor, $R_L = R$, is connected across terminals $A$ and $B$ as shown in the circuit below. Find the power dissipated in the load resistor in terms of given variables.



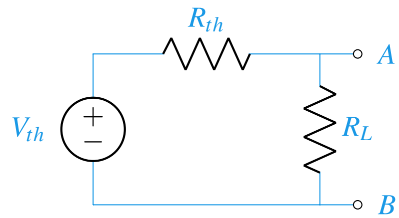

 

**Answer:**

To help simplify the analysis, we replace the circuit by its Thévenin equivalent circuit. In order to do so, we first need to finds the Thévenin voltage. That is the open circuit voltage, $V_{AB}$, in the original circuit, which is simply a voltage divider:

$$V_{th} = V_s \frac{R_3}{R_2 + R_3}$$

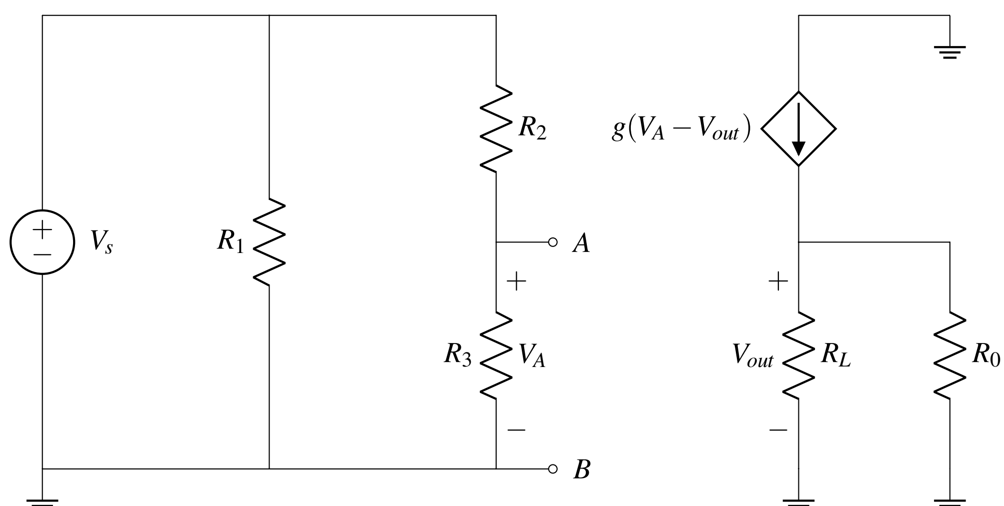
Thus, the circuit can be simplified to:




The power through the load resistor is given by:

$$P_{R_L} = \left(\frac{V_{th}}{R_L + R_{th}}\right)^2 R_L = \left(V_s \frac{R_3}{R_2 + R_3} \cdot \frac{1}{R_L + R_{th}}\right)^2 R_L$$

(c) We modify the circuit as shown below, where $g$ is a known constant:




Find a symbolic expression for $V_{out}$ as a function of $V_s$.
Hint: Redraw the left part of the circuit using with its Thévenin equivalent.

**Answer:**

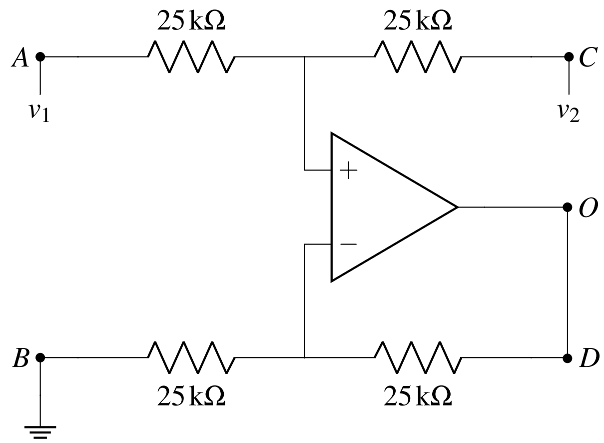
We note that $V_{AB}$ simply equals $V_{th} = \frac{R_3}{R_2+R_3} V_s$. Then, noting that $R_0$ and $R_L$ are in parallel, we have that $V_{out} = g(V_{th} - V_{out})(R_0 \parallel R_L)$. Solving for $V_{out}$, we get:

$$V_{out} = \frac{R_2}{R_2 + R_3} V_s \frac{gR_L \parallel R_0}{1 + gR_L \parallel R_0}$$

**3. A Versatile Opamp Circuit**

For each subpart, determine the voltage at $O$.

(a) Configuration 1:



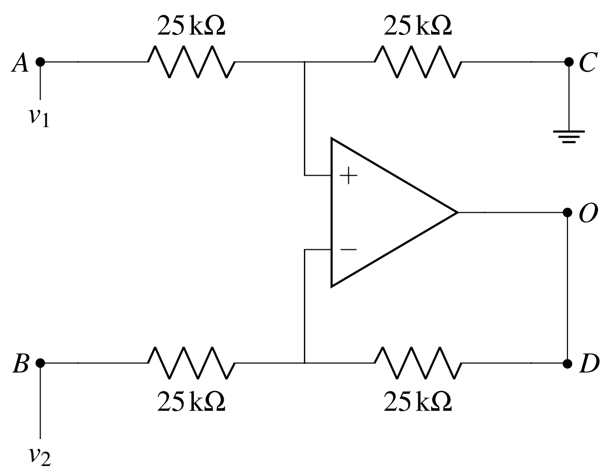

**Answer:**
By superposition, we note that the voltage at $v^+$ is given by:

$$v^+ = \frac{v_1 + v_2}{2}$$

The rest of the circuit looks like a non-inverting amplifier with a gain of $1 + \frac{25\,\text{k}\Omega}{25\,\text{k}\Omega} = 2$. Therefore, the output voltage is:

$$v_O = 2v^+ = v_1 + v_2$$

(b) Configuration 2:

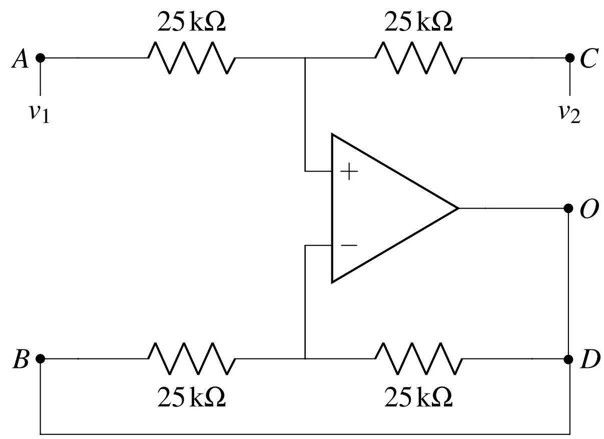



**Answer:**
Through the voltage divider equation, we note that the voltages at the input terminals of the op-amp are given by:

$$v^+ = \frac{v_1}{2}$$
$$v^- = \frac{v_2 + v_O}{2}$$

By the Golden Rules, these must be equal to each other since the op-amp is in negative feedback. Therefore, $v_O = v_1 - v_2$.

(c) Configuration 3:




**Answer:**

Like in part (i), by superposition, we note that the voltage at $v^+$ is given by:

$$v^+ = \frac{v_1 + v_2}{2}$$

We note that the resistors connected to $B$ and $D$ do not affect the circuit as no current is flowing through them. Therefore, $v_O = v^- = v^+ = \frac{v_1+v_2}{2}$.