EECS 16A Designing Information Devices and Systems I
Fall 2021 Discussion 4A

1. Mechanical Inverses

For each sub-part below, determine whether or not the inverse of $\mathbf{A}$ exists.
If it exists, compute the inverse using Gauss-Jordan method.

(a) $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 1 & 5 & 3 \\ 2 & -2 & 4 \end{bmatrix}$

(d) $\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$

2. Identifying a Subspace: Proof

Is the set

$$V = \left\{ \vec{v} \;\middle|\; \vec{v} = c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ where } c, d \in \mathbb{R} \right\}$$

a subspace of $\mathbb{R}^3$? Why/why not?

3. Exploring Column Spaces and Null Spaces

- The **column space** is the **span** of the column vectors of the matrix.
- The **null space** is the set of input vectors that output the zero vector.

For the following matrices, answer the following questions:

i. What is the column space of $\mathbf{A}$? What is its dimension?

ii. What is the null space of **A**? What is its dimension?

iii. Are the column spaces of the row reduced matrix **A** and the original matrix **A** the same?

iv. Do the columns of **A** span $\mathbb{R}^2$? Do they form a basis for $\mathbb{R}^2$? Why or why not?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 1 & 1 & 3 & -3 \end{bmatrix}$

4. Exploring Dimension, Linear Independence, and Basis

In this problem, we are going to talk about the connections between several concepts we have learned about in linear algebra – linear independence, dimension of a vector space/subspace, and basis.

Let's consider the vector space $\mathbb{R}^k$ and a set of n vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ in $\mathbb{R}^k$.

(a) For the first part of the problem, let $k > n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ form a basis for $\mathbb{R}^k$? Why/why not? What conditions would we need?

(b) Let $k = n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ form a basis for $\mathbb{R}^k$? Why/why not? What conditions would we need?

(c) Now, let $k < n$. Can $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ form a basis for $\mathbb{R}^k$? What vector space could they form a basis for?

Hint: Think about whether the vectors can be linearly independent.

