1. Solving Systems of Equations

While we'd love every system of linear equations to have a unique solution, in reality we can either have (a) one unique solution, (b) an infinite number of solutions, or (c) no solution at all. We are going to walk through some examples to see what sort of equations create these types of solutions.

(a) Let's consider the system (where each $a, b \in \mathbb{R}$ can be any real number)

$$ax + y = 3$$
$$-x + 2y = b$$

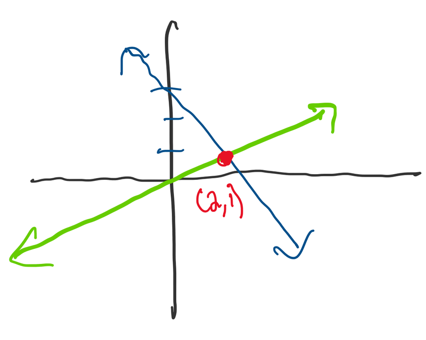
For each of the selected values for a, b sketch or plot out the lines $y(x)$ each of these equations form. Can you conclude which values result in a unique solution? Infinite solutions? No solutions?

- i. $a = 1$, $b = 0$ ii. $a = 0$, $b = 2$
- iii. $a = -1/2$, $b = 6$ iv. $a = -1/2$, $b = 4$

$ax + y = 3$
$-x + 2y = b$

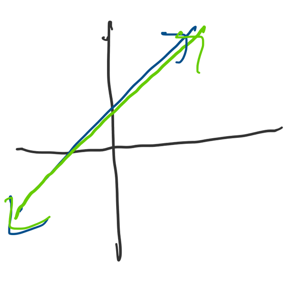
$\begin{cases} y = 3 - ax \\ y = \frac{1}{2}b + \frac{1}{2}x \end{cases}$

i) $a = 1$, $b = 0$ $y = 3 - ax$ $\begin{cases} y = 3 - x \\ y = \frac{1}{2}x \end{cases}$ one unique sol



iii) $a = -\frac{1}{2}$ $b = 6$

$y = 3 - ax$ $y = 3 + \frac{1}{2}x$
$y = \frac{1}{2}b + \frac{1}{2}x$ $y = 3 + \frac{1}{2}x$



same line → ∞ sol

(b) Now, assume we are using the tomography imaging technique described in lecture to image a 2x2 grid, as shown below.

2x2 Tomography Example

x_1	x_2
x_3	x_4

i. We record the following measurements:

$$x_1 + x_2 = 2$$
$$x_1 + x_3 = 2$$
$$x_2 + x_4 = 2$$
$$x_3 + x_4 = 3$$

Solve this system using any method. Is this a valid set of measurements? Why or why not?

$x_1 + x_2 + x_3 + x_4 = 2 + 3 = 5$
$x_1 + x_2 + x_3 + x_4 = 4$

NOT VALID

ii. We are led to believe our measurements might be faulty, so we record the new following measurements:

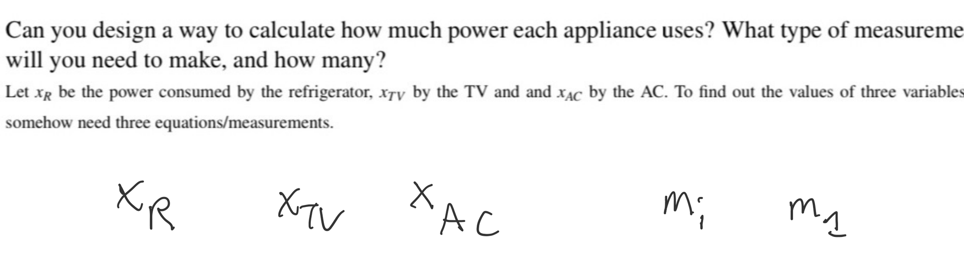
$$x_1 + x_2 = 2$$
$$x_1 + x_3 = 3$$
$$x_2 + x_4 = 2$$
$$x_3 + x_4 = 3$$

$x_1 + x_2 + x_3 + x_4 = 5$
$x_1 + x_2 + x_3 + x_4 = 5$

VALID

2. Energy Disaggregation

Recently, energy companies (like PG&E) have invested heavily into a problem called *energy disaggregation*. This is where you take measurements of the total electricity drawn from a home and then try to deduce which appliances are running in that house. Energy companies value this information since it helps them predict the amount of electricity they will need to produce on a given day, which can be particularly useful during heat waves or power cuts due to wildfire danger. They can also offer their customers energy-saving recommendations.



Let's try solving the *energy disaggregation* problem ourselves!
Suppose you live in a home with just these **3 appliances; an air conditioning unit (AC), a television (TV), and a refrigerator (R)**. Now say you want to find the amount of electricity these appliances use individually, but the only measurement you can take is of the total power your home draws using your meter outside (this is often mounted on the side of the house and shows a running total of your electricity usage).

To do this you will turn some appliances on and off and then read different total measurements. You can turn off the TV at any time, but you **can't unplug the fridge** since the food would spoil. We keep the air conditioner off throughout the morning, but then it must stay on during the afternoon. However, the breaker trips (meaning the electricity suddenly shuts off) if all three are running, so **the TV and AC cannot run at the same time**.

(a) Can you design a way to calculate how much power each appliance uses? What type of measurements will you need to make, and how many?

Let x_R be the power consumed by the refrigerator, x_{TV} by the TV and and x_{AC} by the AC. To find out the values of three variables we somehow need three equations/measurements.

x_R x_{TV} x_{AC} m_i m_1

$x_R = m_1$ (1)
$x_R + x_{TV} = m_2$ (2)
$x_R + x_{AC} = m_3$ (3)

(b) Write out a system of equations that describe your measurements. Can you solve this system so that each appliance's power is written in terms of measurements m_i?

For example: If you measure the power m_1 in the afternoon with the AC and refrigerator on but the TV is off, then the equation might look like $x_{AC} + x_R = m_1$

$m_1 + x_{TV} = m_2$ (2) [plug in $x_R = m_1$ into eqs 2 & 3]
$m_1 + x_{AC} = m_3$ (3)

$x_{TV} = m_2 - m_1$
$x_{AC} = m_3 - m_1$
$x_R = m_1$

(c) Let us say the breaker is fixed, so now we can safely run the AC and TV at the same time. Is there another way (or ways) you could create a new system of equations to solve? If so, see if you can solve your new system!

$x_R + x_{TV} + x_{AC} = m_4$ (4)
$- \quad x_R + x_{TV} = m_2$ (2)
$- \quad x_R + x_{AC} = m_3$ (3)

$-1(-x_R) = (m_4 - m_2 - m_3) - 1$

$x_R = m_3 + m_2 - m_4$

$(m_3 + m_2 - m_4) + x_{TV} = m_2$ (2)

$x_{TV} = -m_3 + m_4$

$(m_3 + m_2 - m_4) + x_{AC} = m_3$ (3)

$x_{AC} = m_4 - m_2$

$x_R = m_1$ $m_1 = m_3 + m_2 - m_4$?

(d) Lastly suppose, as a busy Berkeley student, you only get a chance to take two measurements. Can you determine how much power each of the three appliances draw? If not, what combinations of power consumption can you find out?

Possible combos: 1&2, 1&3, 1&4, 2&3, 2&4, 3&4

1&2:

$x_R = m_1$ (1)
$x_R + x_{TV} = m_2$ (2)

$m_1 + x_{TV} = m_2$ (plug $x_R = m_1$ into eq. 2)

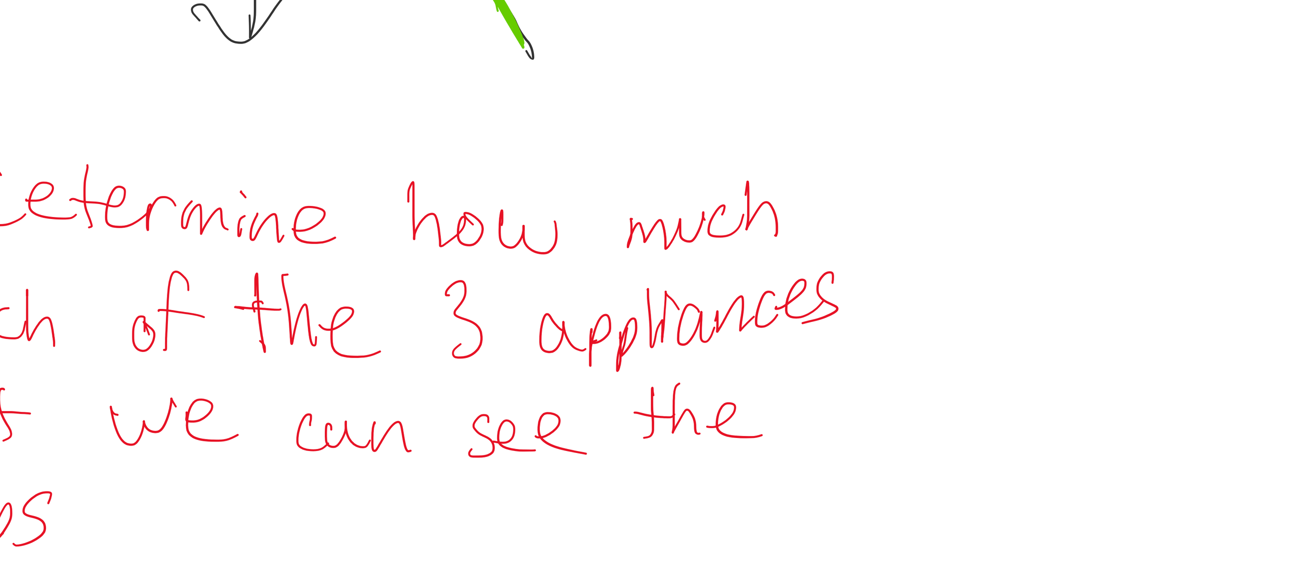
$x_{TV} = m_2 - m_1$
$x_R = m_1$

1&4:

$x_R + x_{TV} + x_{AC} = m_4$ (4)
$x_R = m_1$ (1)

$x_{TV} + x_{AC} = m_4 - m_1$

Solutions to eq. are on the line $x_{AC} = -x_{TV} + m_4 - m_1$



Cannot determine how much power each of the 3 appliances draw but we can see the relationships