EECS 16A Dis 6A Notes

Sunday, October 3, 2021 12:39 PM

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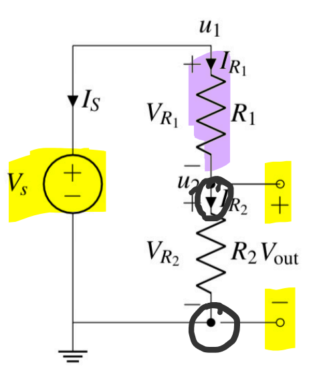
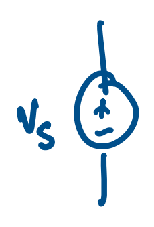
1. Voltage Divider

For the circuit below, your goal will be to find the voltage V_{out} in terms of the resistances R_1, R_2, and V_s, using NVA (Node Voltage Analysis) and Gaussian elimination. The labeling steps (steps 1-4) have already been done for you.



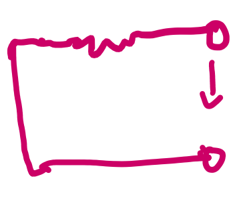


$I_s=?$

$V=V_S$

Ohm's Law:

$V_R=I_R \cdot R$



$I=0$

Here is a reminder of the labeling steps followed to get the circuit diagram above:

- **Step 1:** Select a reference (ground) node. Any node can be chosen for this purpose. We will measure all of the voltages in the rest of the circuit relative to this point.
- **Step 2:** Label all nodes with voltage set by voltage sources.
- **Step 3:** Label remaining nodes.
- **Step 4:** Label element voltages and currents, following **Passive Sign Convention**.

Our goal is to ***find*** V_{out}. In order to do this, we can use NVA to find equations describing our circuit, write our equations in the form $\mathbf{A}\vec{x}=\vec{b}$, and use Guassian elemination to solve for $\vec{x}$. The following steps will walk you through this process:

Step 5: Write out $\mathbf{A}\vec{x}=\vec{b}$, leaving the entries for **A** and $\vec{b}$ blank. Next, fill in the enteries for $\vec{x}$. Recall that $\vec{x}$ is a vector of your unknown currents and voltages.

Step 6: Write KCL equations for all nodes with unknown voltages. Using these equations, fill in as many linearly indepedent rows in **A** and $\vec{b}$ as possible.

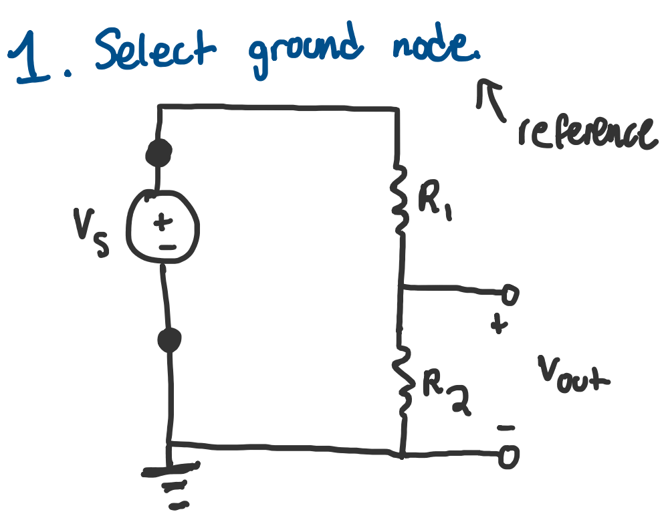
$\sum$ curr @ node $=0$

Step 7: Write down the IV relationships (Ohm's Law) of each of the non-wire elements. Use these equations to fill in the remaining rows in **A** and $\vec{b}$. (Hint: how many equations do you need to write?)

Step 8: Use Gaussian elimination or substition to solve for $u_2=V_{out}$.

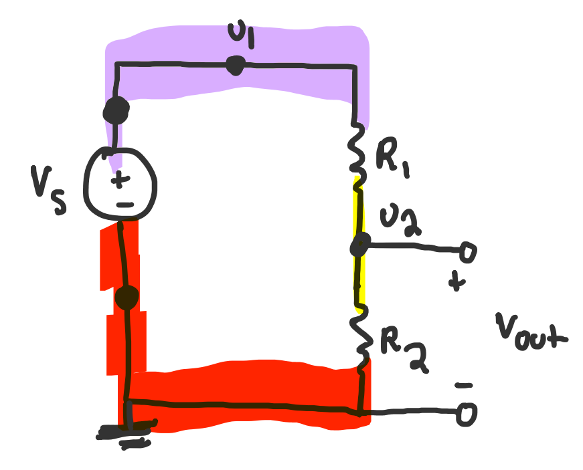
$V_{out}=u_2-0=u_2$

1. Select ground node.

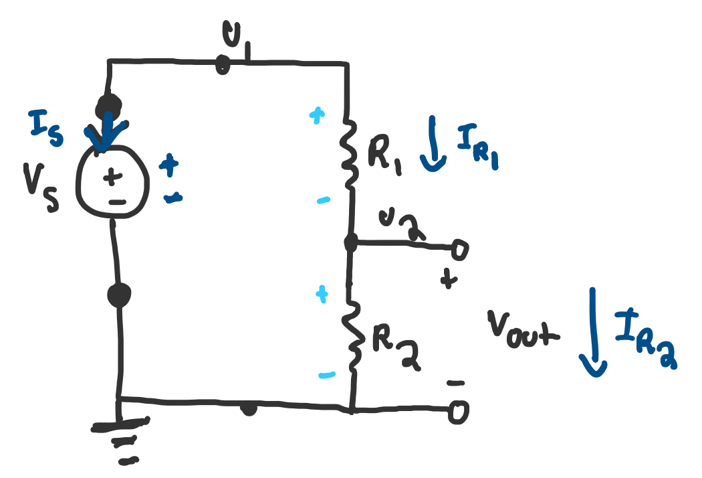


reference point (abs 0)

2-3. Label all nodes



4. Label element voltages & currents



5. Write out $A\vec{x}=\vec{b}$

$x=?$ $x:$ $\begin{bmatrix}I_{R_1}\\I_{R_2}\\I_S\\u_1\\u_2\end{bmatrix}$

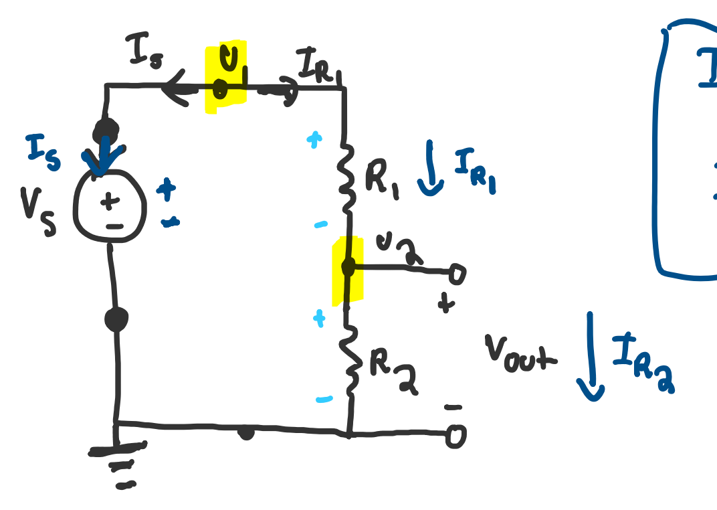
5x5 matrix

$A\vec{x}=\vec{b}$

5x1 vector

5x1 vector

6. Use KCL to fill in $A\vec{x}=\vec{b}$



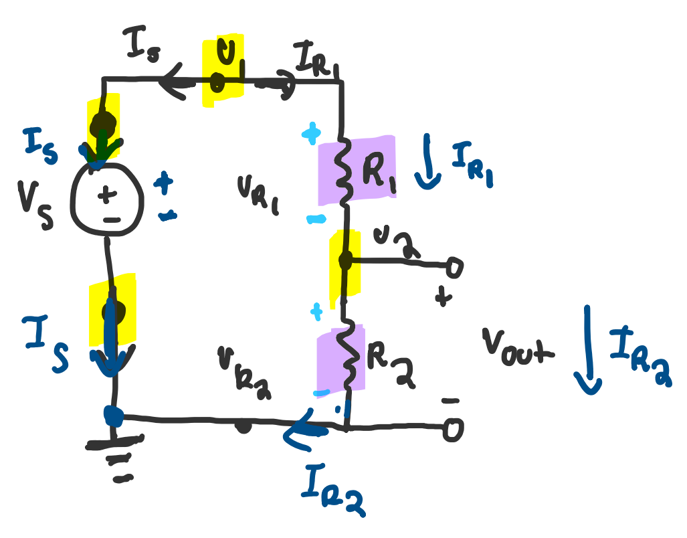
$$\boxed{\begin{aligned}I_S+I_{R_1}&=0\\ I_{R_1}&=I_{R_2}\end{aligned}}$$

$I_S+I_{R_1}=0$ (1)

$I_{R_1}-I_{R_2}=0$ (2)

$$\begin{bmatrix}1&0&1&0&0\\1&-1&0&0&0\\ &&&&\\ &&&&\\ &&&&\end{bmatrix}\begin{bmatrix}\boxed{I_{R_1}}\\I_{R_2}\\\boxed{I_S}\\u_1\\\boxed{u_2}\end{bmatrix}=\begin{bmatrix}0\\0\\ \\ \\ \end{bmatrix}$$

7. Use Ohm's Law to fill in $A\vec{x}=\vec{b}$



$V_{R_1}=I_{R_1}\cdot R_1$

$V_{R_2}=I_{R_2}\cdot R_2$

$I_{R_1}=\frac{V_{R_1}}{R_1}=\frac{u_1-u_2}{R_1}$

$I_{R_2}=\frac{V_{R_2}}{R_2}=\frac{u_2}{R_2}$

$I_{R_1}-\frac{u_1}{R_1}+\frac{u_2}{R_1}=0$

$I_{R_2}-\frac{u_2}{R_2}=0$

$V_S=u_1-0$

$V_S=u_1$

$V_{R1}=u_1-u_2$

$V_{R_2}=u_2-GND$
$=u_2$

I_S: $I_S+I_{R_2}=0$

$I_S+I_{R_1}=0$

since $I_{R_1}=I_{R_2}$ these 2 other eqs make sense ⇒ just need one of them (choose $I_{R_1}=I_{R_2}$ and $I_S+I_{R_2}=0$)

$$\begin{bmatrix}1&0&1&0&0\\1&-1&0&0&0\\1&0&0&-\frac{1}{R_1}&\frac{1}{R_1}\\0&1&0&0&-\frac{1}{R_2}\\0&0&0&1&0\end{bmatrix}\begin{bmatrix}I_{R_1}\\I_{R_2}\\I_S\\u_1\\u_2\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\V_S\end{bmatrix}$$

8. Solve $A\vec{x}=\vec{b}$ using Gaussian elimination (or substitution)

$V_{R_1}=I_{R_1}\cdot R_1$

$V_{R_2}=I_{R_2}\cdot R_2$

$I_{R_1}=\frac{V_{R_1}}{R_1}=\frac{u_1-u_2}{R_1}$

$I_{R_2}=\frac{V_{R_2}}{R_2}=\frac{u_2}{R_2}$

$I_{R_1}-\frac{u_1}{R_1}+\frac{u_2}{R_1}=0$

$I_{R_2}-\frac{u_2}{R_2}=0$

$V_S=u_1-0$

$V_S=u_1$

$V_{R1}=u_1-u_2$

$V_{R_2}=u_2-GND$
$=u_2$

I_S: $I_S+I_{R_2}=0$

$I_{R_1}=I_{R2}$

$\frac{u_1-u_2}{R_1}=\frac{u_2}{R_2}$

$\frac{V_S-u_2}{R_1}=\frac{u_2}{R_2}$

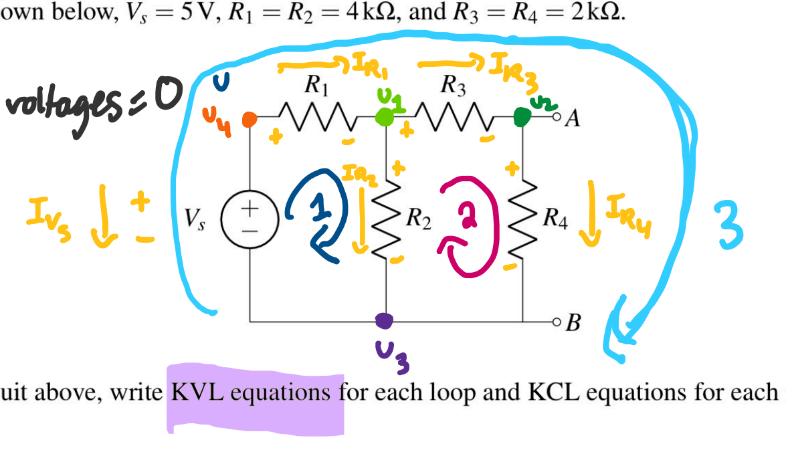
$\frac{V_S}{R_1}-\frac{u_2}{R_1}=\frac{u_2}{R_2}$

$$\boxed{V_{out}=u_2=\left(\frac{R_2}{R_1+R_2}\right)\cdot V_S}$$

2. KVL and KCL

For the circuit shown below, $V_s=5\,\text{V}$, $R_1=R_2=4\,\text{k}\Omega$, and $R_3=R_4=2\,\text{k}\Omega$.

KVL: $\sum^{loop}$ voltages $=0$



(a) For the circuit above, write KVL equations for each loop and KCL equations for each node.

Loop 1: $V_S-V_{R_1}-V_{R_2}=0$

Loop 2: $V_{R_2}-V_{R_3}-V_{R_4}=0$

Loop 3: $V_S-V_{R_1}-V_{R_3}-V_{R_4}=0$

@u_1: $I_{R_1}=I_{R_2}+I_{R_3}$

@u_2: $I_{R_3}=I_{R_4}$

@u_3: $I_{V_S}+I_{R_2}+I_{R_4}=0$

@u_4: $I_{V_S}+I_{R_1}=0$

7 eqs.

(b) Solve for the voltage between A and B using the equations from part (a).

Loop 1: $V_S-I_{R_1}R_1-I_{R_2}R_2=0$

Loop 2: $I_{R_2}R_2-I_{R_3}R_3-I_{R_4}R_4=0$

Loop 3: $V_S-I_{R_1}R_1-I_{R_3}R_3-I_{R_4}R_4=0$

@u_1: $I_{R_1}=I_{R_2}+I_{R_3}$

@u_2: $I_{R_3}=I_{R_4}$

@u_3: $I_{V_S}+I_{R_2}+I_{R_4}=0$

@u_4: $I_{V_S}+I_{R_1}=0$

$\boxed{V_{R_4}=.833\,V}$

Use Gaussian elimination or substitution to solve

Using Gaussian Elimination:

$$\begin{bmatrix}1&1&0&0&0\\0&-1&1&1&0\\0&0&0&-1&1\\-1&0&-1&0&-1\\0&R_1&R_2&0&0\\0&R_1&0&R_3&R_4\\0&0&R_2&-R_3&-R_4\end{bmatrix}\begin{bmatrix}I_S\\I_{R_1}\\I_{R_2}\\I_{R_3}\\I_{R_4}\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\V_S\\V_S\\0\end{bmatrix}$$