EECS 16A Dis 8B Notes

Tuesday, October 19, 2021 8:12 AM

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1. Capacitance Equivalence

For the structures shown below, assume that the plates have a depth L into the page and a width W and are always a distance d apart. The dielectric between the plates has absolute permitivity ε. For the following calculations, assume the capacitance is purely parallel plate, i.e. ignore fringing field effects.

(a) What is the capacitance of the structure shown below?

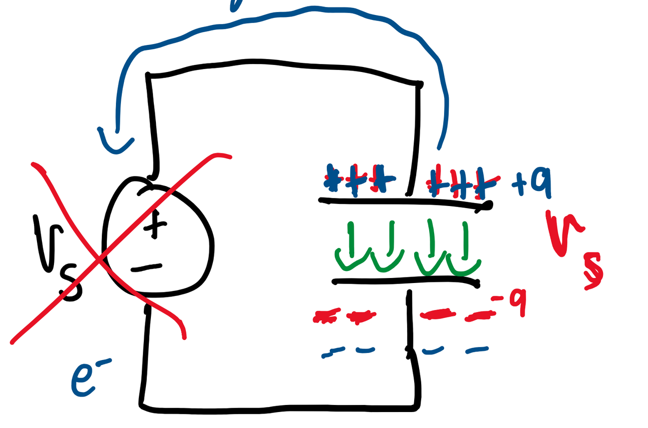
$+q$

$-q$

$$\boxed{C = \frac{\varepsilon \cdot A}{d} = C_a}$$

$A = w \cdot l$

Capacitors: Devices that store charge



$Q = CV$

charge capacitance voltage

$R = \frac{\rho \cdot L}{A}$ $\boxed{C = \frac{\varepsilon \cdot A}{d}}$

ε_0 ε

A

d

(b) Suppose that we take two such structures and put them next to each other as shown below. What is the capacitance of this new structure?

$$\boxed{C_b = \frac{\varepsilon \cdot 2A}{d} = 2 \cdot \frac{\varepsilon A}{d} = 2 \cdot C_a}$$

(c) Now suppose that rather than connecting the together as shown above, we connect them with an ideal wire as shown below. What is the capacitance of this structure?

$+q$ $-q$ C C $+q$ $-q$

$$\boxed{C_c = \frac{\varepsilon \cdot 2A}{d} = 2 \cdot C_a}$$

$= C_a + C_a$

Parallel Cap: $C_{eq} = C_1 + C_2$ $\longleftrightarrow$ Series Res. $R_{eq} = R_1 + R_2$

Series Cap: $C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \frac{C_1 C_2}{C_1 + C_2}$ $\longleftrightarrow$ Parallel Res. $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

(d) Suppose that we now take two capacitors and connect them as shown below. What is the capacitance of the structure?

d d $2d$ $2d$ d d d

$$\boxed{C_d = \frac{\varepsilon \cdot A}{2d} = \frac{1}{2} \cdot C_a}$$

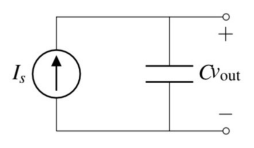
(e) What is the capacitance of the structure shown below?

$+q$ $-q$ $2d$

$$\boxed{C_e = \frac{\varepsilon \cdot A}{2d} = \frac{1}{2} \cdot C_a}$$

2. Current Sources And Capacitors

Given the circuit below, find an expression for $v_{out}(t)$ in terms of I_s, C, V_0, and t, where V_0 is the initial voltage across the capacitor at $t = 0$.



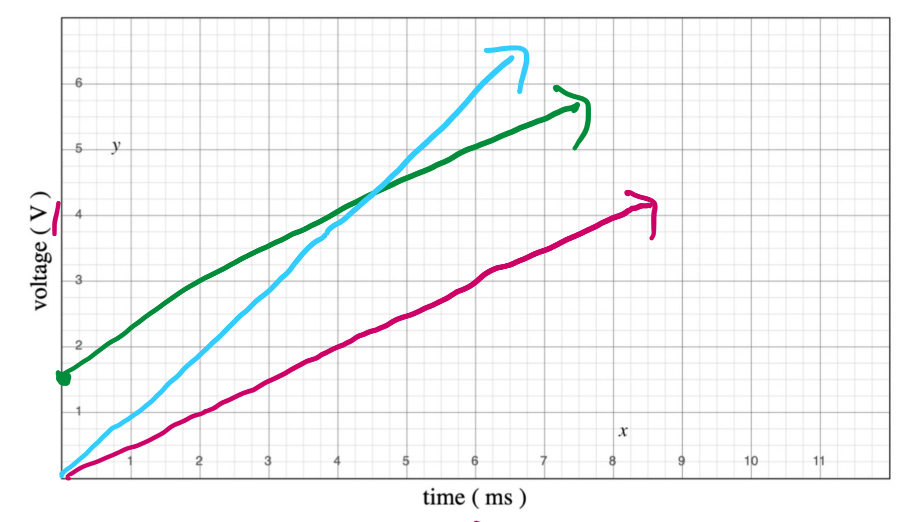
Then plot the function $v_{out}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\mu\text{F}$.

(a) Capacitor is initially uncharged $V_0 = 0$ at $t = 0$.

(b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.

(c) **Practice:** Swap this capacitor for one with half the capacitance $C = 1\mu\text{F}$, which is initially uncharged $V_0 = 0$ at $t = 0$.

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.



$I = \frac{dQ}{dt}$

$Q = CV$

$\frac{dQ}{dt} = C \cdot \frac{dv_{out}}{dt}$

$\boxed{\frac{dQ}{dt}} \frac{1}{C} = \frac{dv_{out}}{dt}$

$\int_0^t \frac{I_s\, dt}{C} = \int_0^t dv_{out}(t)$

$\frac{I_s}{C} t = v_{out}(t) - v_{out}(0)$

$\star\ v_{out}(t) = \frac{I_s}{C} t + v_{out}(0)$

μV mV

10^{-6} 10^{-3}

1. a) $v_0 = 0$ @ $t = 0$

$v_{out}(t) = \frac{1mA}{2\mu F} t = \frac{1V}{2ms} t$

$[A] = \frac{[C]}{[s]}$

$Q = CV$

$[C]$ $[F]$ $[V]$

$[F] = \frac{[C]}{[V]}$

$C = \frac{Q}{V}$ $\frac{[C]}{[V]}$

$\frac{A}{F} = \frac{\cancel{C}}{s} \cdot \frac{V}{\cancel{C}} = \frac{V}{s}$

b) $v_0 = 1.5V$ @ $t = 0$

$v_{out}(t) = \frac{1mA}{2\mu F} t + 1.5V$

$= \frac{1V}{2ms} t + 1.5V$

c) $v_0 = 0V$ @ $t = 0$ $C = 1\mu F$

$v_{out}(t) = \frac{1mA}{1\mu F} t$

$= \frac{1V}{1ms} t$