EECS16A
Acoustic Positioning System 2

Last Lab! :)

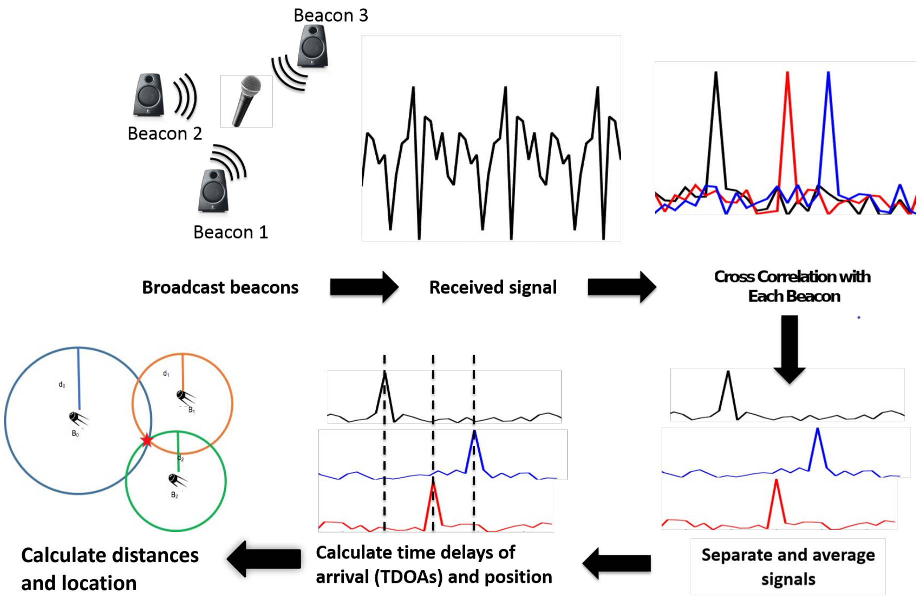
Insert names here

Announcements!

- This is the **last lab!**
- Do APS 1 first if you haven't yet (APS 2 can then be done during buffer)
- Course evaluations: link
- APS buffer labs 12/6-12/10 (RRR week)
  - Sign up here: tiny.cc/aps-buffer-fa21
  - Encouraged to attend a Mon-Wed section
- Good luck on the final!

Last lab: APS 1

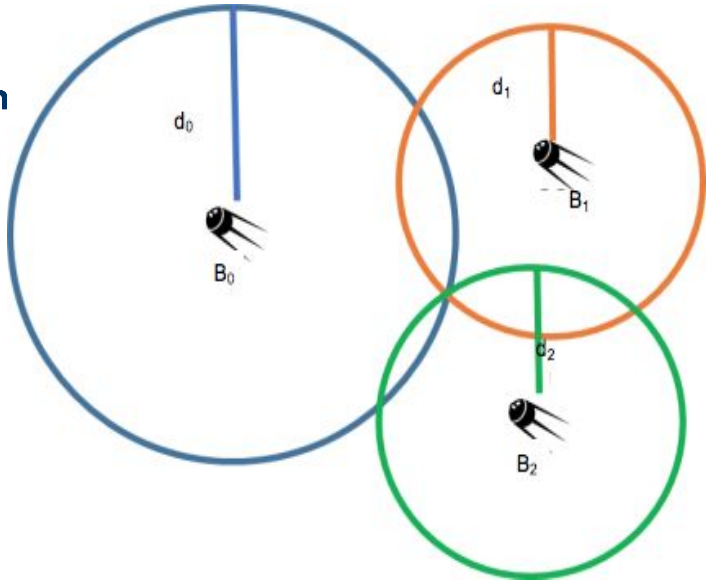
- Cross correlated beacon signals with received signal
- Found the offsets (in samples) between peaks, converted to TDOAs, and calculated distances from each beacon
- **What was the missing piece that we needed to calculate distance?**
  - Hint: we don't have absolute times of arrival for all the beacons, only relative offsets.



3 Beacon Example

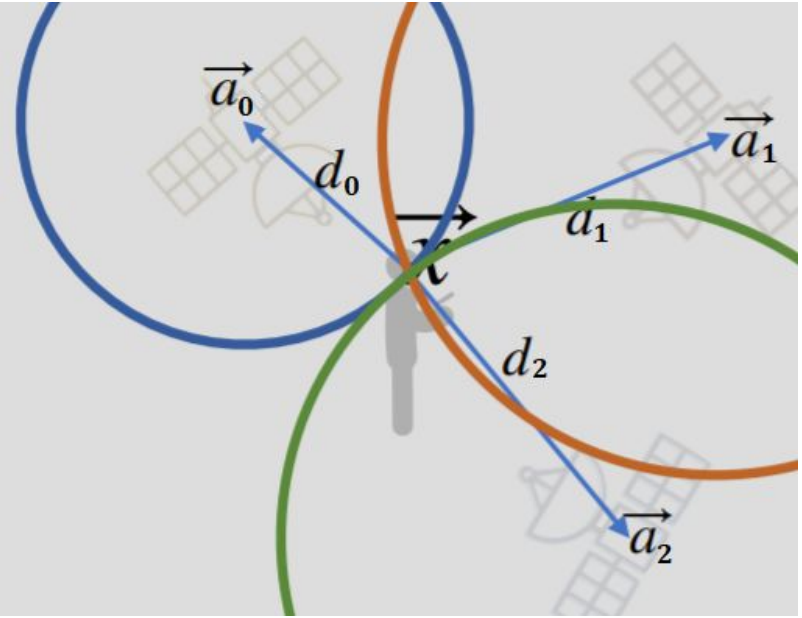
- Let beacon centers be: (x_0, y_0), (x_1, y_1) and (x_2, y_2)
- Time of arrivals: τ_0, τ_1, τ_2
- Distance of beacon m $(m = 0, 1, 2)$ is $d_m = v\tau_m = R_m$ (circle radii)

Circle equations: $(x - x_m)^2 + (y - y_m)^2 = d^2_m$



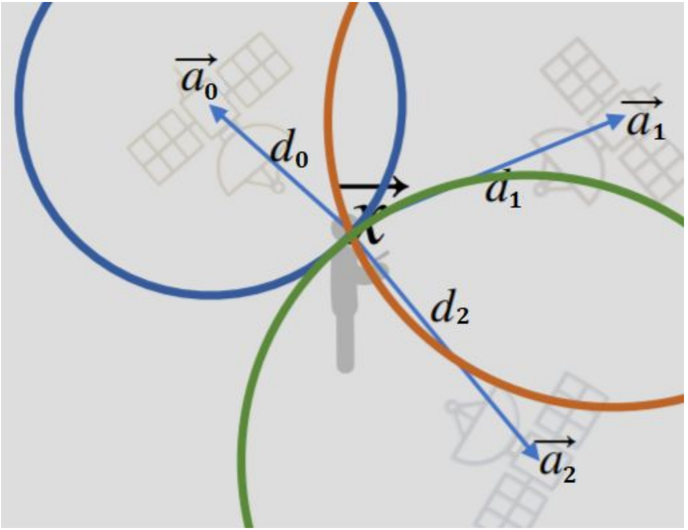
Trilateration

$$||\vec{r} - \vec{a_0}||^2 = d_0^2$$
$$||\vec{r} - \vec{a_1}||^2 = d_1^2$$
$$||\vec{r} - \vec{a_2}||^2 = d_2^2$$



$$d_i = v_s \tau_i$$

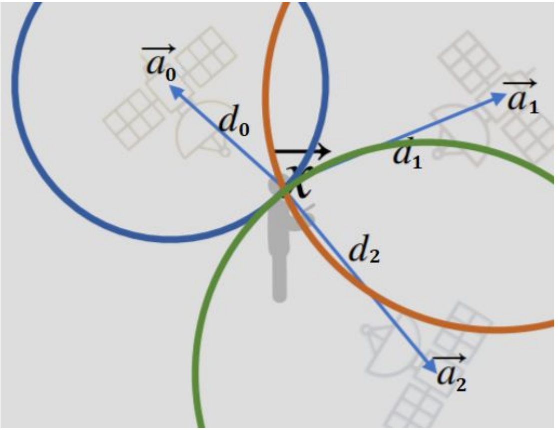
Trilateration



$$||\vec{r}||^2 - 2\vec{a_0}^T\vec{r} + ||\vec{a_0}||^2 = v_s^2\tau_0^2$$
$$||\vec{r}||^2 - 2\vec{a_1}^T\vec{r} + ||\vec{a_1}||^2 = v_s^2\tau_1^2$$
$$||\vec{r}||^2 - 2\vec{a_2}^T\vec{r} + ||\vec{a_2}||^2 = v_s^2\tau_2^2$$

Trilateration

$$||\vec{r}||^2 - 2\vec{a_0}^T\vec{r} + ||\vec{a_0}||^2 = v_s^2\tau_0^2$$
$$||\vec{r}||^2 - 2\vec{a_1}^T\vec{r} + ||\vec{a_1}||^2 = v_s^2\tau_1^2$$
$$||\vec{r}||^2 - 2\vec{a_2}^T\vec{r} + ||\vec{a_2}||^2 = v_s^2\tau_2^2$$



Subtracting the first equation yields:

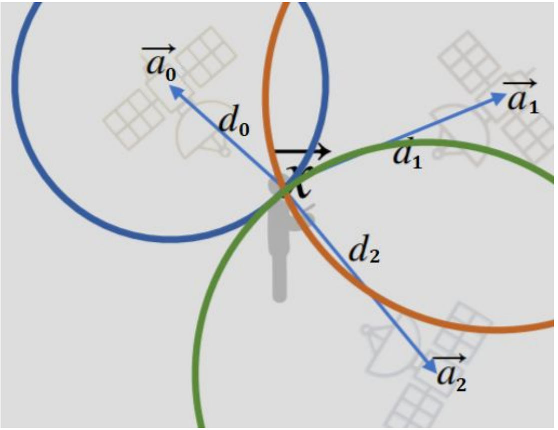
$$-2\vec{a_1}^T\vec{r} + 2\vec{a_0}^T\vec{r} + ||\vec{a_1}||^2 - ||\vec{a_0}||^2 = v_s^2(\tau_1^2 - \tau_0^2)$$
$$\implies 2(\vec{a_0} - \vec{a_1})^T\vec{r} = ||\vec{a_0}||^2 - ||\vec{a_1}||^2 + v_s^2(\tau_1^2 - \tau_0^2)$$

and,

$$2(\vec{a_0} - \vec{a_2})^T\vec{r} = ||\vec{a_0}||^2 - ||\vec{a_2}||^2 + v_s^2(\tau_2^2 - \tau_0^2)$$

Trilateration

$$2(\vec{a_0} - \vec{a_1})^T \vec{r} = ||\vec{a_0}||^2 - ||\vec{a_1}||^2 + v_s^2(\tau_1^2 - \tau_0^2)$$
$$2(\vec{a_0} - \vec{a_2})^T \vec{r} = ||\vec{a_0}||^2 - ||\vec{a_2}||^2 + v_s^2(\tau_2^2 - \tau_0^2)$$



We want to write this in terms of TDOAs and unknowns!

$$(\tau_i^2 - \tau_0^2) = (\tau_i - \tau_0)(\tau_i + \tau_0) = (\tau_i - \tau_0)(\tau_i - \tau_0 + 2\tau_0) = \Delta\tau_i(\Delta\tau_i + 2\tau_0)$$

$$\Longrightarrow$$

$$2(\vec{a_0} - \vec{a_1})^T \vec{r} - 2(v_s^2 \Delta\tau_1)\tau_0 = ||\vec{a_0}||^2 - ||\vec{a_1}||^2 + v_s^2 \Delta\tau_1^2$$
$$2(\vec{a_0} - \vec{a_2})^T \vec{r} - 2(v_s^2 \Delta\tau_2)\tau_0 = ||\vec{a_0}||^2 - ||\vec{a_2}||^2 + v_s^2 \Delta\tau_2^2$$

Trilateration

We can expand our equations by writing our vectors in component form!

$$\vec{r} = \begin{bmatrix} r_x \\ r_y \end{bmatrix} \qquad \vec{a_i} = \begin{bmatrix} a_{i,x} \\ a_{i,y} \end{bmatrix} \qquad \vec{a_0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\implies$$

$$2a_{1,x}r_x + 2a_{1,y}r_y + 2v_s^2\Delta\tau_1\tau_0 = a_{1,x}^2 + a_{1,y}^2 - v_s^2\Delta\tau_1^2$$

$$2a_{2,x}r_x + 2a_{2,y}r_y + 2v_s^2\Delta\tau_2\tau_0 = a_{2,x}^2 + a_{2,y}^2 - v_s^2\Delta\tau_2^2$$

Trilateration

$$2a_{1,x}r_x + 2a_{1,y}r_y + 2v_s^2\Delta\tau_1\tau_0 = a_{1,x}^2 + a_{1,y}^2 - v_s^2\Delta\tau_1^2$$

$$2a_{2,x}r_x + 2a_{2,y}r_y + 2v_s^2\Delta\tau_2\tau_0 = a_{2,x}^2 + a_{2,y}^2 - v_s^2\Delta\tau_2^2$$

What are our unknowns in this system?

Trilateration

$$2a_{1,x}r_x + 2a_{1,y}r_y + 2v_s^2\Delta\tau_1\tau_0 = a_{1,x}^2 + a_{1,y}^2 - v_s^2\Delta\tau_1^2$$
$$2a_{2,x}r_x + 2a_{2,y}r_y + 2v_s^2\Delta\tau_2\tau_0 = a_{2,x}^2 + a_{2,y}^2 - v_s^2\Delta\tau_2^2$$

What are our unknowns in this system?

$$r_x, r_y, \tau_0$$

Trilateration

$$2a_{1,x}r_x + 2a_{1,y}r_y + 2v_s^2\Delta\tau_1\tau_0 = a_{1,x}^2 + a_{1,y}^2 - v_s^2\Delta\tau_1^2$$
$$2a_{2,x}r_x + 2a_{2,y}r_y + 2v_s^2\Delta\tau_2\tau_0 = a_{2,x}^2 + a_{2,y}^2 - v_s^2\Delta\tau_2^2$$

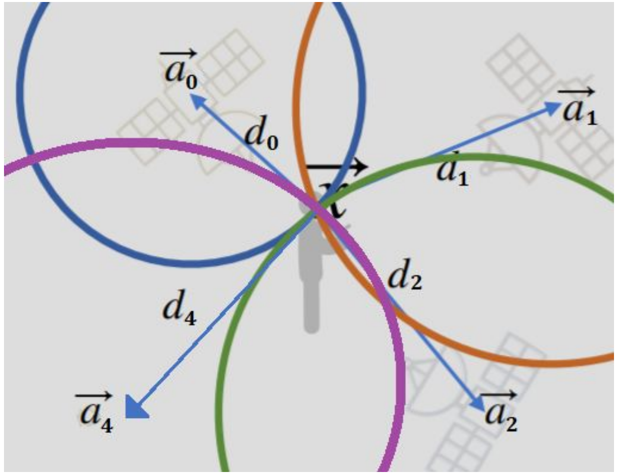
What are our unknowns in this system?

$$r_x, r_y, \tau_0$$

Problem: 3 unknowns and 2 equations!

Solution: add another beacon to produce a third equation!

Trilateration



3 equations and 3 unknowns, so we have a solvable system!

$$2a_{1,x}r_x + 2a_{1,y}r_y + 2v_s^2\Delta\tau_1\tau_0 = a_{1,x}^2 + a_{1,y}^2 - v_s^2\Delta\tau_1^2$$
$$2a_{2,x}r_x + 2a_{2,y}r_y + 2v_s^2\Delta\tau_2\tau_0 = a_{2,x}^2 + a_{2,y}^2 - v_s^2\Delta\tau_2^2$$
$$2a_{3,x}r_x + 2a_{3,y}r_y + 2v_s^2\Delta\tau_3\tau_0 = a_{3,x}^2 + a_{3,y}^2 - v_s^2\Delta\tau_3^2$$

Multilateration

We can produce overdetermined system with M beacons!

$$2\begin{bmatrix} a_{1,x} & a_{1,y} & v_s^2\Delta\tau_1 \\ a_{2,x} & a_{2,y} & v_s^2\Delta\tau_2 \\ & \vdots & \\ a_{M-1,x} & a_{M-1,y} & v_s^2\Delta\tau_{M-1} \end{bmatrix} \begin{bmatrix} r_x \\ r_y \\ \tau_0 \end{bmatrix} = \begin{bmatrix} a_{1,x}^2 + a_{1,y}^2 - v_s^2\Delta\tau_1^2 \\ a_{2,x}^2 + a_{2,y}^2 - v_s^2\Delta\tau_2^2 \\ \vdots \\ a_{M-1,x}^2 + a_{M-1,y}^2 - v_s^2\Delta\tau_{M-1}^2 \end{bmatrix}$$

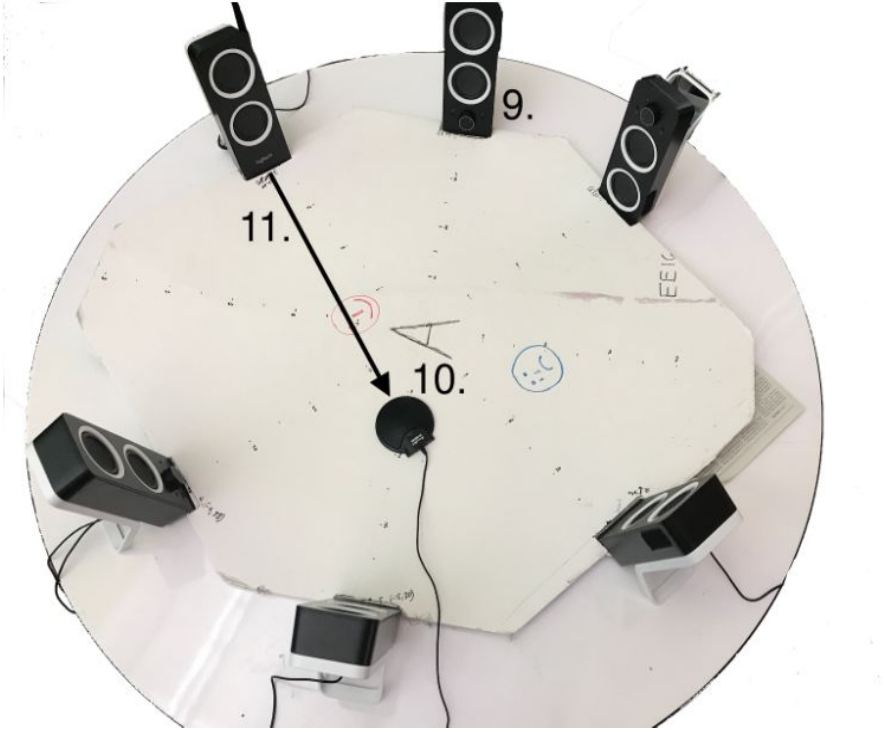
“Solving” an Overdetermined System

- After simplifying, we have more equations than unknowns (x,y)
- Can do least-squares regardless of number of beacons
- Best estimate of location if measurements are inconsistent
- If there is no exact point of intersection because of error or noise

$$Ax = b$$

$$A^T A x = A^T b$$

Setup Looks Like:



Important Notes

- Read over the math carefully, we’ll be asking you about it!
- Stay safe and good luck with the rest of the semester!