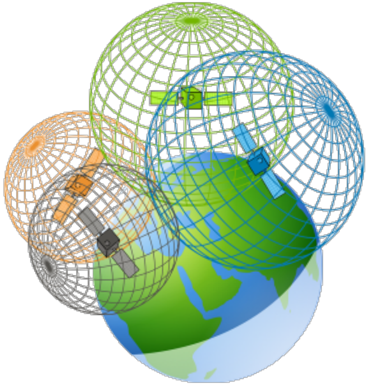
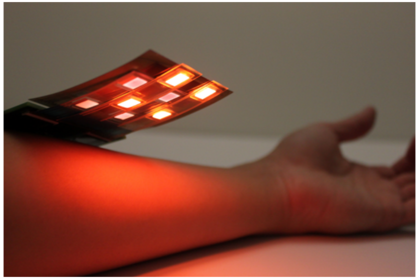




Welcome to EECS 16A!

Designing Information Devices and Systems I

Ana Claudia Arias and Miki Lustig

Fall 2021

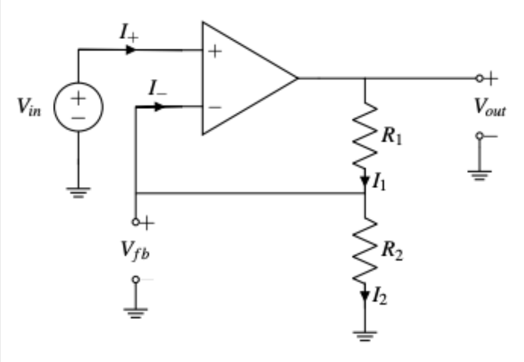
Module 2
Lecture 11
Op-amp circuit analysis (Note 19)

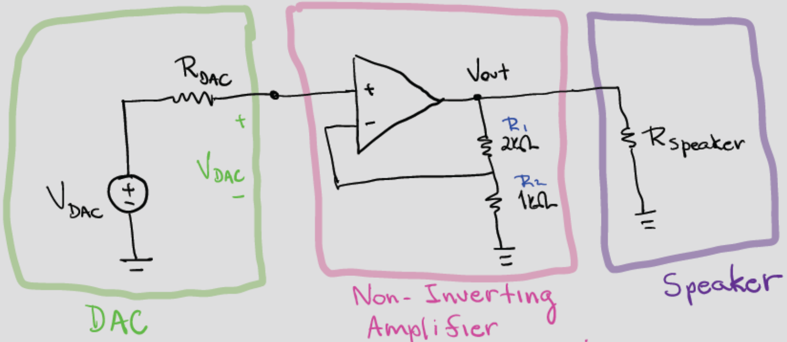
Last Lecture…

Toolbox

- Resistors
- Capacitors
- Open-circuits
- Voltage Dividers/Summers
- Op-Amps
- Thevenin and Norton Equivalence
- KCL/KVL
- Element Definitions
- DAC
- Negative Feedback
- Op-Amp in Negative Feedback
- “Golden Rules” for Op-Amps

$$A_V = \frac{V_{out}}{V_{in}}$$



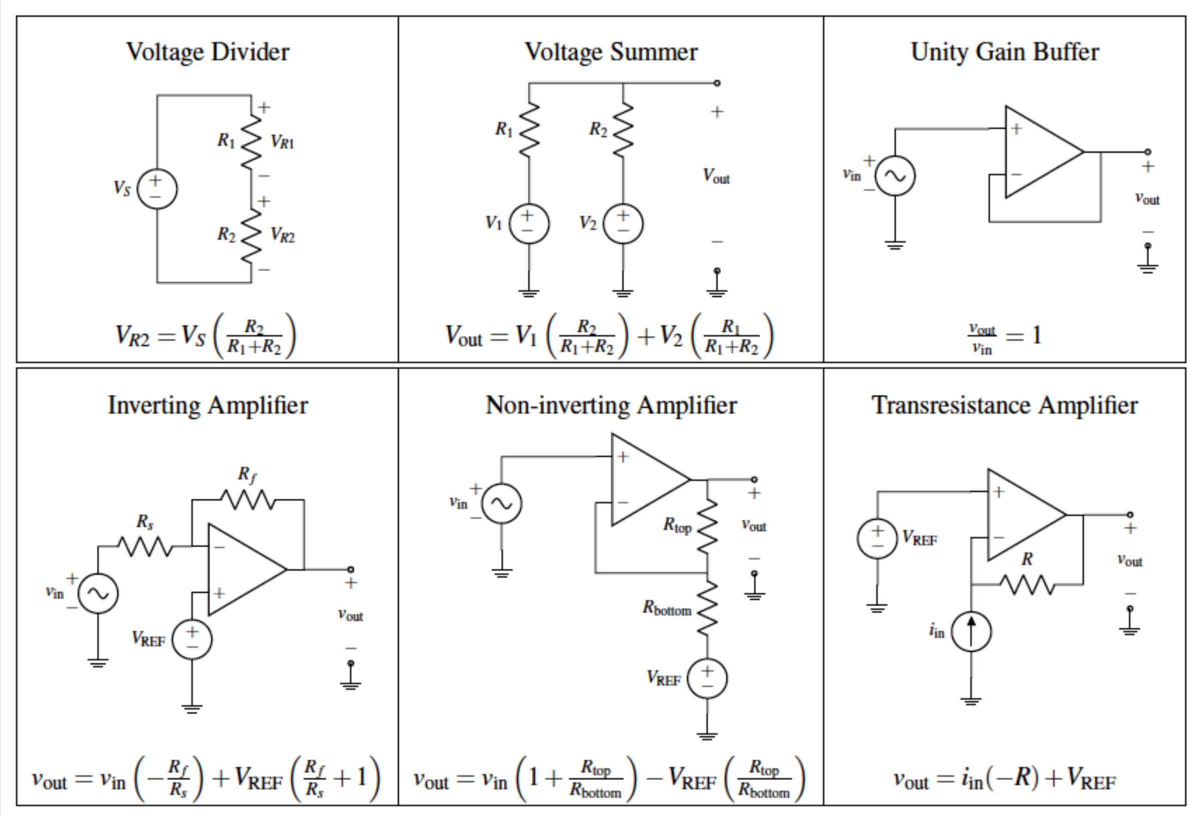


(feedback gain = 3)

$$\text{Gain} = 1 + \frac{R_1}{R_2}$$

Party time!
Yay!

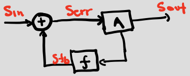
Today



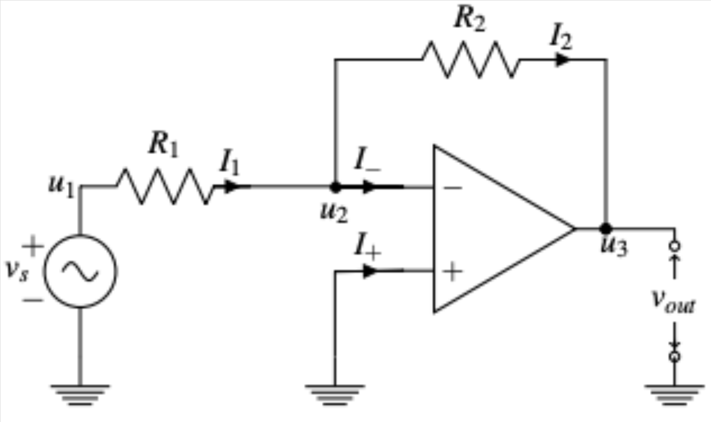
Voltage Divider	Voltage Summer	Unity Gain Buffer
$V_{R2} = V_S\left(\frac{R_2}{R_1+R_2}\right)$	$V_{\text{out}} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$	$\frac{v_{\text{out}}}{v_{\text{in}}} = 1$
Inverting Amplifier	**Non-inverting Amplifier**	**Transresistance Amplifier**
$v_{\text{out}} = v_{\text{in}}\left(-\frac{R_f}{R_s}\right) + V_{\text{REF}}\left(\frac{R_f}{R_s}+1\right)$	$v_{\text{out}} = v_{\text{in}}\left(1+\frac{R_{\text{top}}}{R_{\text{bottom}}}\right) - V_{\text{REF}}\left(\frac{R_{\text{top}}}{R_{\text{bottom}}}\right)$	$v_{\text{out}} = i_{\text{in}}(-R) + V_{\text{REF}}$

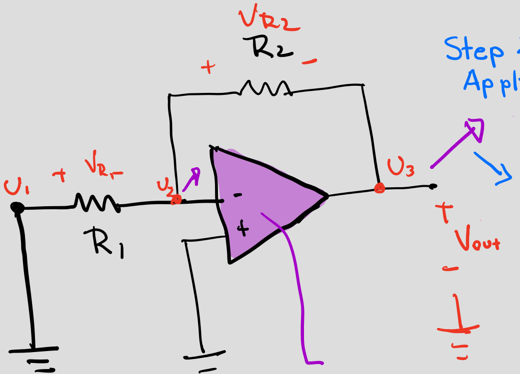
Checking for Negative Feedback (Determining the polarity of NFB)

Step 1 – Zero out all independent sources : replacing voltage sources with wires and current sources with open circuits as in superposition

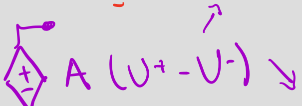


Step 2 – Wiggle the output and check the loop – to check how the feedback loop responds to a change.
- if the error signal decreases, the output must also decrease. The circuit is in negative feedback
- if the error signal increases, the output must also increase. The circuit is in positive feedback



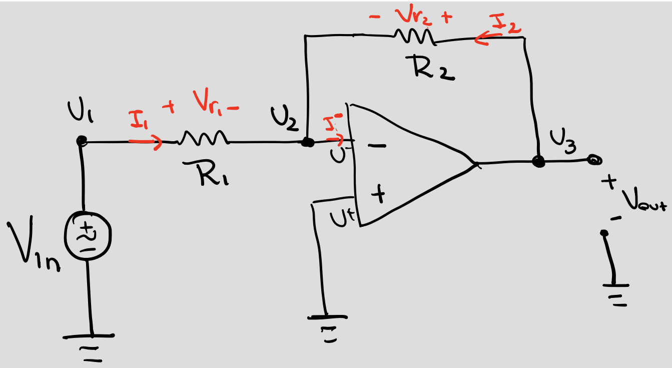


Step 2 :
Apply disturbance
U_3 increases
U_2 increases (U^-)
U_3 decreases
(NFB) ✓



(when U^- increase
$A(U^+ - U^-)$ decreases)

Now lets solve it...



NFB ⇒ GR#2 applies
$U^+ = U^-$

①
$U_1 = V_{in}$
$U_3 = V_{out}$
$U_2 = 0$ (circuit in NFB ⇒ GR#2 applies $U^+ = U^-$
↳ $U_2 = U^-$ we know $U^+ = 0 \Rightarrow U^- = 0$
$U^- = U_2 \Rightarrow U_2 = 0$)

② Element Definitions:
$V_{R1} = I_1 R_1$
$V_{R2} = I_2 R_2$

Voltage Def.
$V_{R1} = U_1 - \cancel{U_2}^{\to 0} = U_1 = V_{in}$
$V_{R2} = U_3 - \cancel{U_2}^{\to 0} = U_3 = V_{out}$

③ (KCL)
$I_1 + I_2 = \cancel{I^-}^{\to 0}$ (GR#1)

$V_{in} = U_1 = I_1 R_1$
$V_{out} = U_3 = I_2 R_2$

$I_1 + I_2 = 0$

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$$

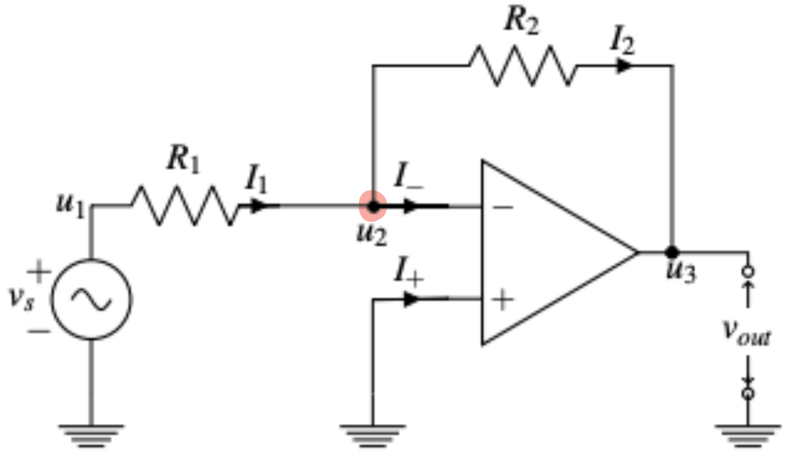
$$V_{out} = R_2 \cdot \left(-\frac{V_{in}}{R_1}\right)$$

$$V_{out} = -\frac{R_2}{R_1} \cdot V_{in}$$

Inverting Amplifier

$$A_V = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

A faster way…



GR2: $U^+ = U^-$

$U_2 = U^-$

$U^+ = 0 \Rightarrow \boxed{U_2 = 0}$

GR1 + KCL $\left(I_1 = I_2 + I^- \right)$

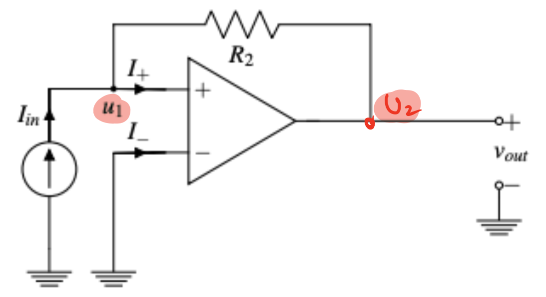
$$\frac{\overset{\to 0}{\cancel{U_2}} - U_1}{R_1} = \frac{U_3 - \overset{\to 0}{\cancel{U_2}}}{R_2} + \overset{\to 0}{\cancel{I^-}}$$

$$-\frac{\overset{V_{in}}{\cancel{U_1}}}{R_1} = \frac{\overset{V_{out}}{\cancel{U_3}}}{R_2}$$

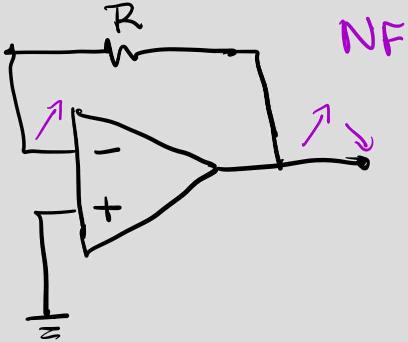
$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

Example circuit 2 (trans-resistance amplifier)

$I^+ = 0 \Rightarrow U_1 = U_2$

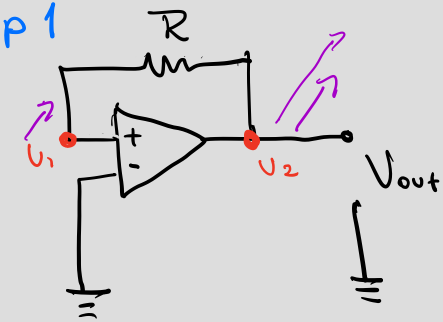


Invert polarity $\Rightarrow$



NFB

Step 1



Step2: Check for NFB

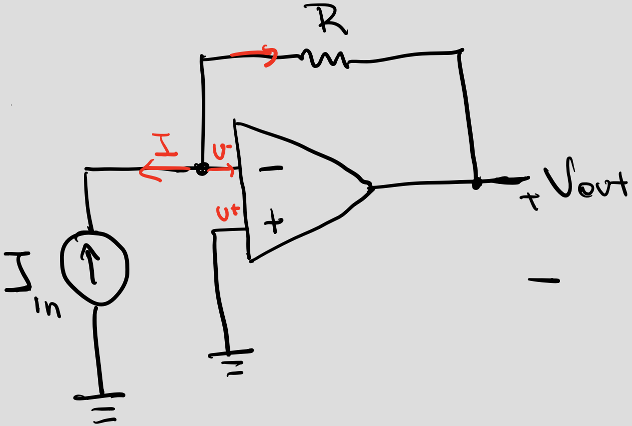
Increase output →

\+ Moves up

output increases

by a lot

X Not in NFB



NFB : $U^+ = U^-$

$U^+ = 0 \rightarrow U^- = 0$

GR # 2

$$\frac{\overset{\text{GR2}\,\to 0}{\cancel{U^-}} - V_{out}}{R} + (-I_{in}) + \overset{\text{GR1}\,\to 0}{\cancel{I^-}} = 0$$

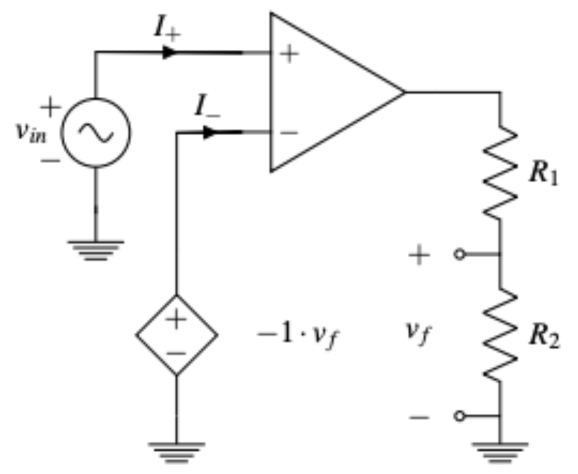
$$-\frac{V_{out}}{R} = I_{in}$$

$$V_{out} = -I_{in} R$$

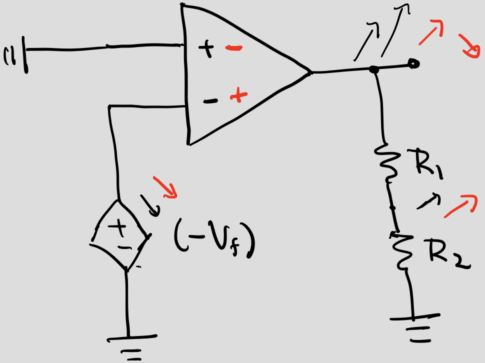
$$\boxed{\frac{V_{out}}{I_{in}} = -R}$$

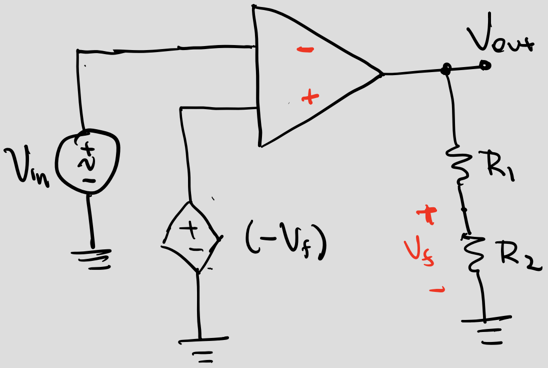
The input is current; output is Voltage: we use this model in the lab for photo sensors!

Example circuit 3 -



Check NFB:





Voltage Divider

$$V_f = \frac{R_2}{R_1+R_2} \cdot V_{out}$$

NFB (GR#2) $V^- = V^+$

$$\underset{V^-}{\cancel{V_{in}}} = -\underset{V^+}{\cancel{V_f}}$$

$$V_{in} = -\frac{R_2}{R_1+R_2} V_{out} \Rightarrow \frac{V_{in}}{V_{out}} = -\frac{R_2}{R_1+R_2}$$

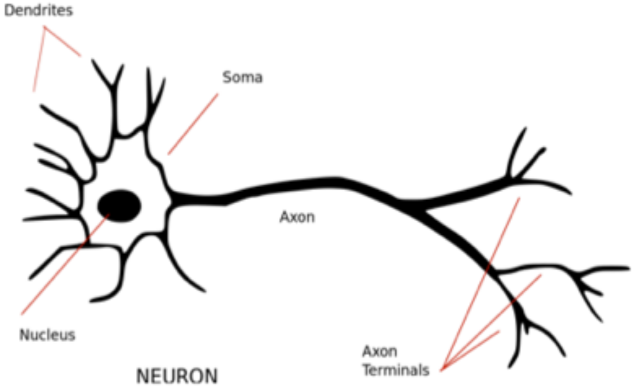
$$\boxed{A_v = \frac{V_{out}}{V_{in}}} = -\frac{R_1+R_2}{R_2} = \boxed{-\left(1+\frac{R_1}{R_2}\right)}$$

Artificial Neuron

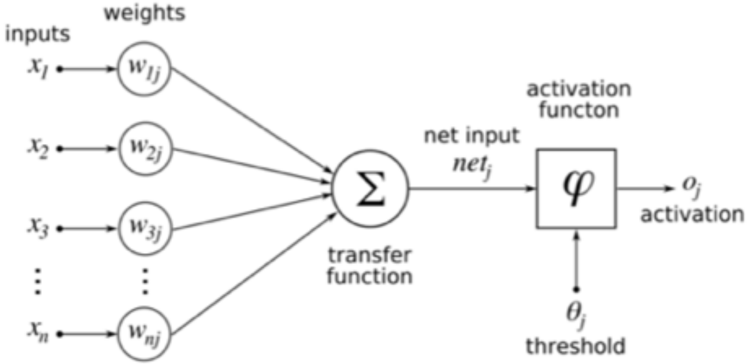
(Energy Efficient Neural Networks) — Yes we can!

- Neurons in our brain are interconnected.
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

$$\begin{bmatrix} a_1 & a_2 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = a_1 V_1 + a_2 \cdot V_2$$



A biological Neuron



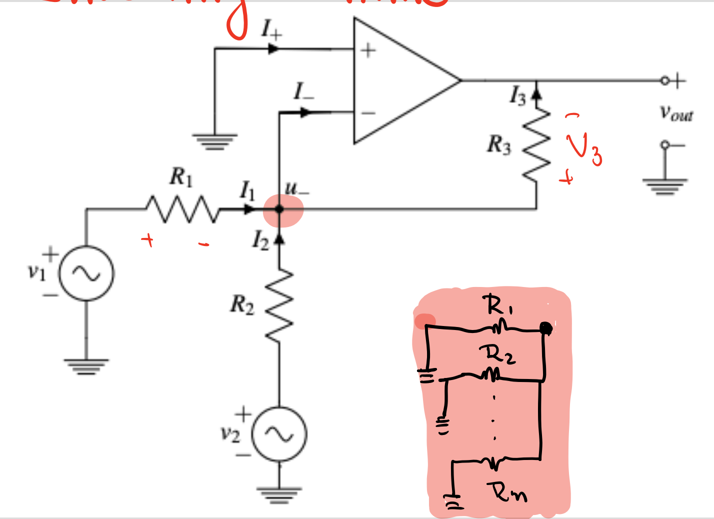
An Artificial Neuron

Artificial Neuron

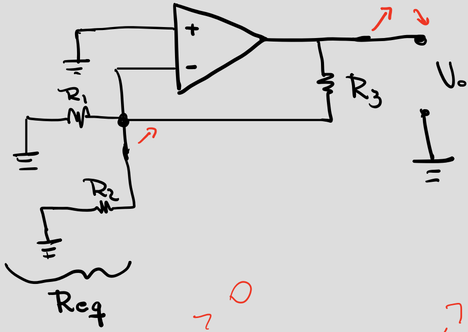
- Neurons in our brain are interconnected.
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

Inverting summer

$V_3 = V_{out} - V^-$



Check for NFB:



$V^+ = V^-$: GR2

$V^+ = 0 \Rightarrow V^- = 0$

$$KCL: \frac{\cancel{V^-}^{\to 0} - V_1}{R_1} + \frac{\cancel{V^-}^{\to 0} - V_2}{R_2} = \cancel{I^-}^{\to 0} + \frac{V_{out} - \cancel{V^-}^{0}}{R_3}$$

$$-\frac{V_1}{R_1} - \frac{V_2}{R_2} = \frac{V_{out}}{R_3}$$

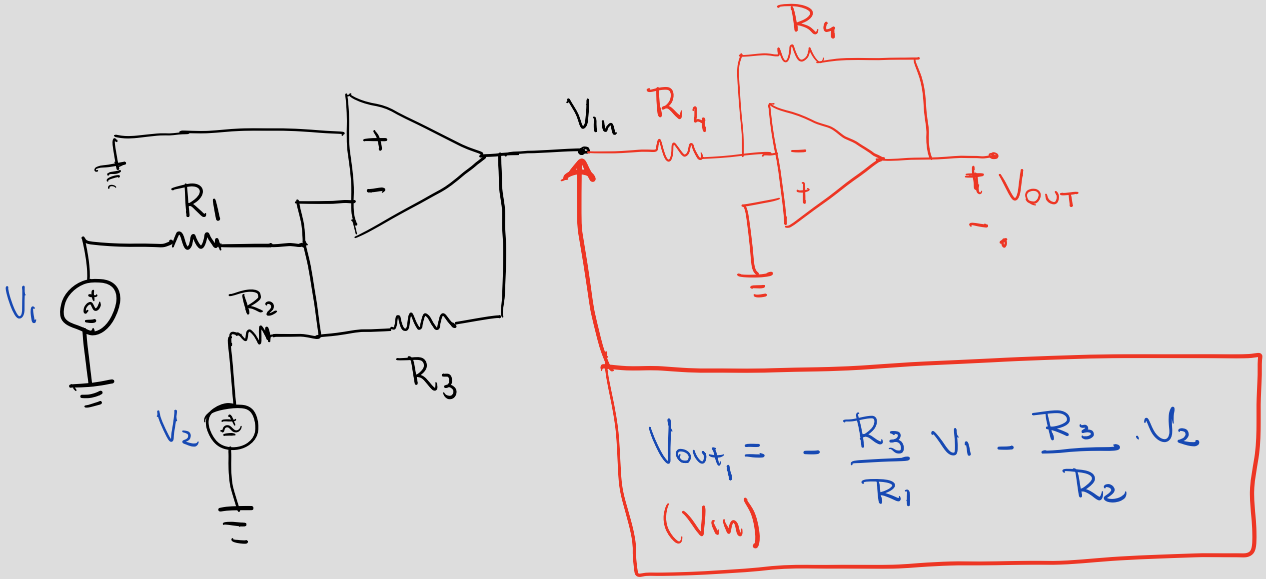
$$V_{out} = -\underbrace{\frac{R_3}{R_1}}_{a_{11}} \cdot \underbrace{V_1}_{v_1} + \left(-\underbrace{\frac{R_3}{R_2}}_{a_{12}} \underbrace{V_2}_{v_2}\right) + \cdots + \left(-\underbrace{\frac{R_3}{R_N}}_{a_{1N}} \underbrace{V_N}_{v_N}\right)$$

only negative coef.

weights

All weights are negative : How can we make a_1 and a_2 positive?

Add another inverting amplifier circuit.



$$V_{out_1} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}\cdot V_2$$

(Vin)

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

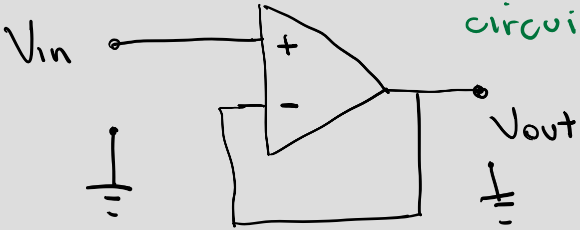
↳ result from Inverting amplifier

$$V_{out} = -\frac{R_2}{R_1}\cdot V_{in}$$

$V_{out} = -V_{in}$ (when R_1 and R_2 are the same)

Unity Gain Buffer

↳ Allows us to isolate circuits



$V^+ = V_{in}$

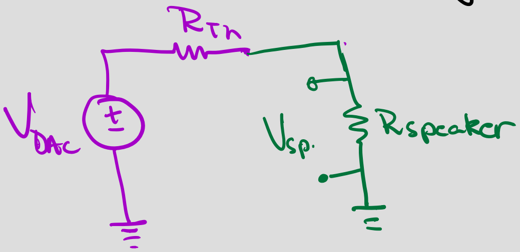
$V^- = V_{out}$

GR2

$V^+ = V^-$

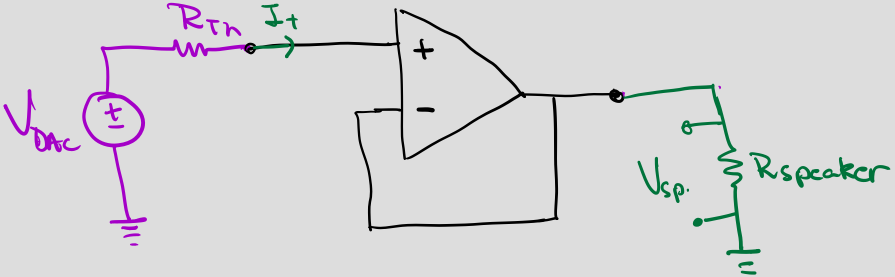
$$\boxed{V_{in} = V_{OUT}}$$

Speaker Design



$V_{speaker} = \frac{V_{DAC}}{126}$

loading



$I^+ = 0 \Rightarrow V^+ = V_{DAC}$

$V_{out} = V_{speaker} = V^- \Rightarrow V^+ = V^-$

$$\boxed{V_{DAC} = V_{speaker}}$$