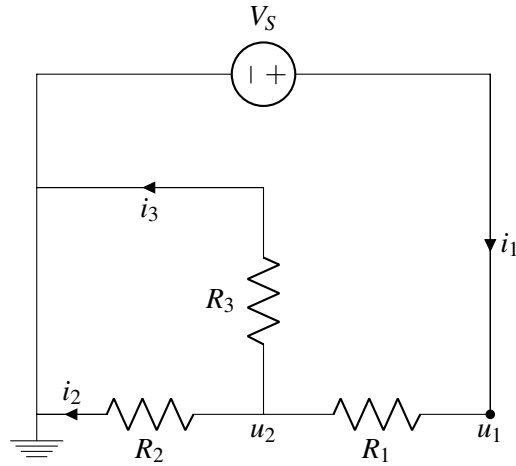


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3. Seven Steps of Highly Resistive Circuits (10 points)

- (a) (3 point) Label the +/- polarity of voltage across each resistor element on the following circuit. Be sure to follow passive sign convention using the labeled currents.



- (b) (7 points) Fill in the following matrix and unknown vector, such that they represent linearly independent equations for the circuit given above. Assume the resistor and voltage source values are known constants. **You do not need to solve this matrix.**

$$\begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

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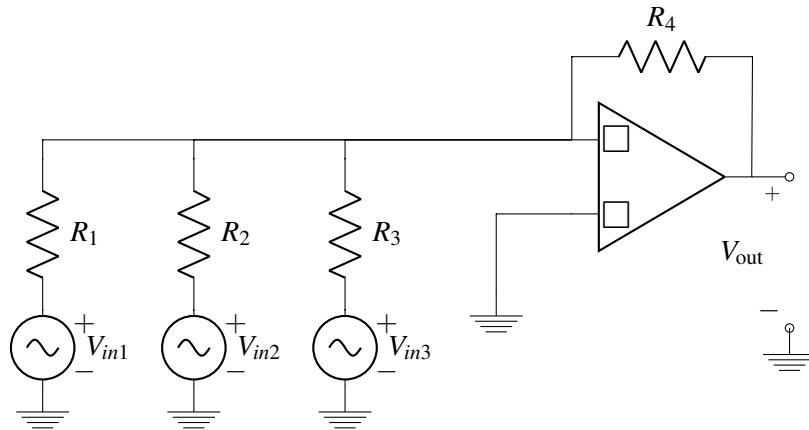
4. "Operational" Amplifiers (12 points)

“As an amplifier so connected can perform the mathematical operations of arithmetic and calculus on the voltages applied to its inputs, it is hereafter termed an ‘operational amplifier.’”

John Ragazzini, Robert Randall and Frederick Russell
Proceedings of IRE, Vol. 35, May 1947

In this problem we will explore some of the mathematical operations an op amp can perform.

(a) (5 points)

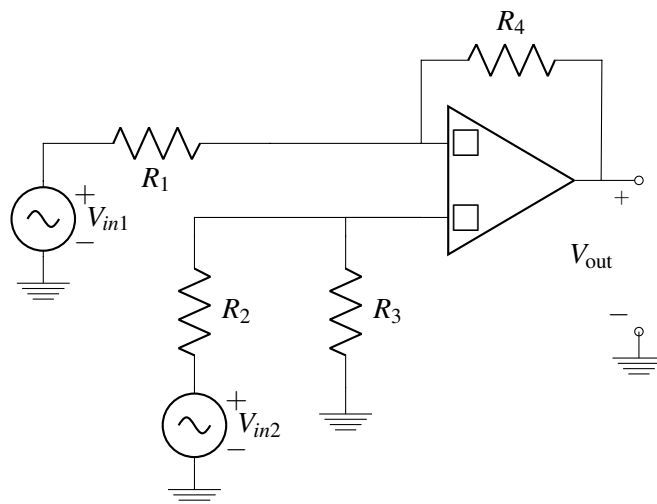


- Label the ‘+’ and ‘-’ terminals of the op amp above so that it is in negative feedback.
- Derive an expression for V_{out} as a function of V_{in1} , V_{in2} , and V_{in3} .

iii. Mark the operation below that is best represented by this configuration.

- Subtraction Inner Product Differentiation Integration

(b) (7 points)

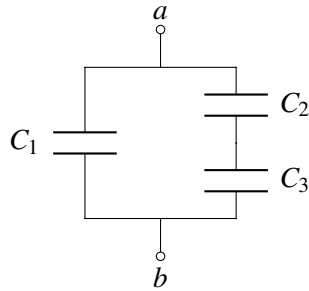


- Label the '+' and '-' terminals of the op amp above so that it is in negative feedback.
- Derive an expression for V_{out} as a function of V_{in1} and V_{in2} .

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5. Equivalent Capacitance (9 points)

- (a) (4 points) Find the equivalent capacitance between terminals a and b of the following circuit in terms of the given capacitors C_1, C_2 , and C_3 . Leave your answer in terms of the addition, subtraction, multiplication, and division operators **only**.



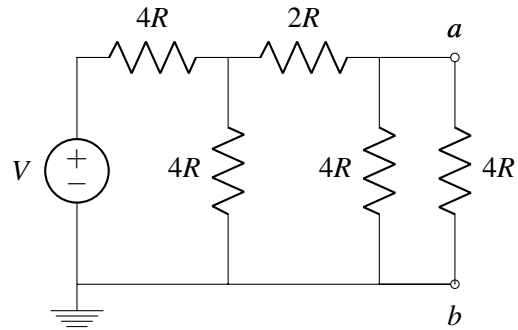
- (b) (5 points) Find and draw a capacitive circuit using three capacitors, C_1, C_2 , and C_3 , that has equivalent capacitance of

$$\frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

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6. Power to Resist (6 points)

Find the power dissipated by the voltage source in the circuit below. Be sure to use passive sign convention.

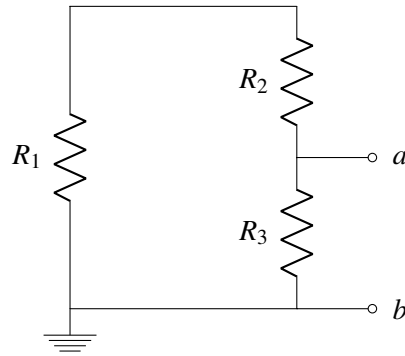


Blank area for student solution.

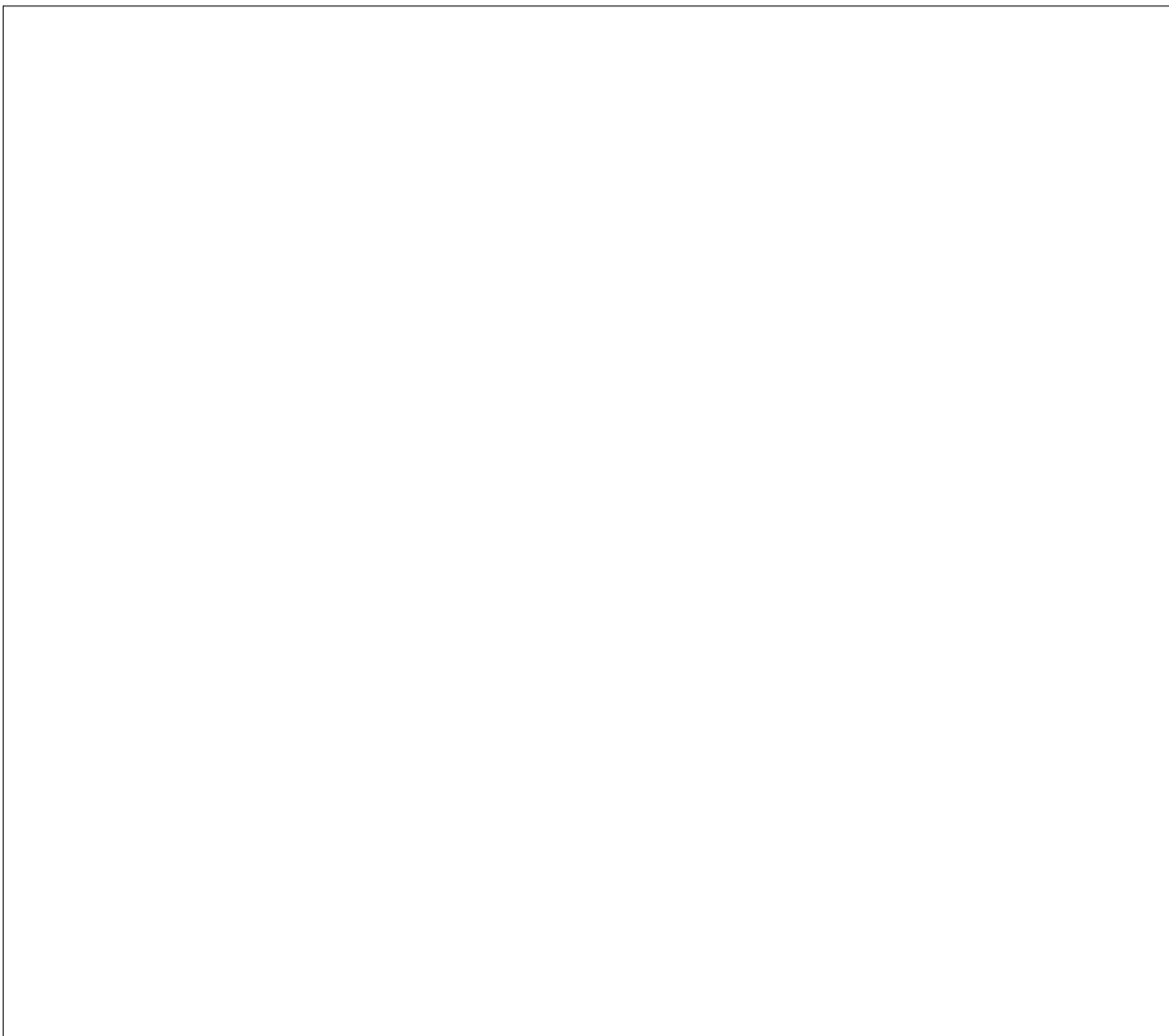
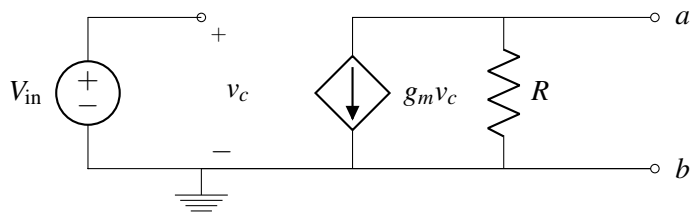
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7. Mechanical Thevenin and Norton Equivalent (10 points)

- (a) (4 points) Find and draw the **Thevenin** equivalent circuit between terminals a and b in the circuit below. Clearly label the Thevenin equivalent voltage, V_{th} , and the Thevenin equivalent resistance R_{th} .



- (b) (6 points) Find and draw the **Norton** equivalent circuit between terminals a and b in the circuit below. Clearly label the Norton equivalent current, I_{no} , and the Norton equivalent resistance R_{no} .

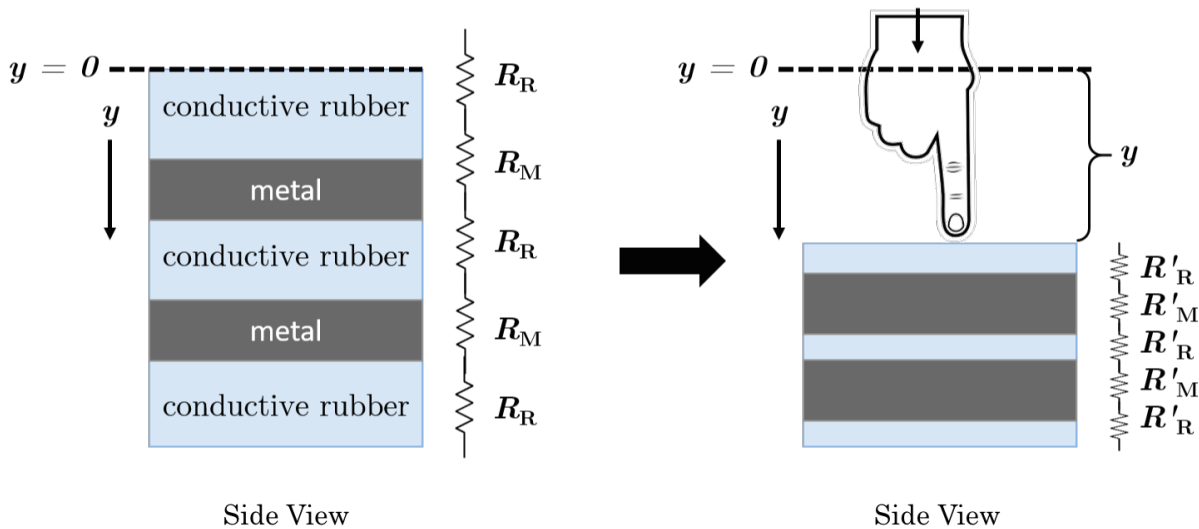


8. Midterms are a lot of Pressure (16 points)

In our labs, we used our resistive touchscreen to figure out where we were pressing in a 2D space. Now we'll use a new setup to determine how hard we're pressing! For this we'll use something called "pressure sensitive rubber," which incorporates conductive rubber and metal into one system. As the rubber is pressed, the conductive rubber portions are compressed, which changes the resistance. The metal plates do not change, but they assist in conduction through the material.

The pressure sensitive rubber system is shown below, with a resistive model next to the diagram. The resistivity of rubber and metal are represented by ρ_R and ρ_M respectively. When the system is at rest (no touch), the resistances of the rubber and metal are represented by R_R and R_M . The area of the sensor, as seen from above, is A .

To use the material, a finger presses on top of the system, compressing the rubber regions, creating a change in resistance, also shown below. Please answer the following questions related to the system.



(a) (2 point) Is the resistor model implementing resistors in series or parallel?

(b) (3 points) If the values are $R_R = 1 \text{ k}\Omega$ and $R_M = 10 \Omega$, what is the total resistance before pressing the system?

- (c) (4 points) During the press, the length of each rubber portion is reduced by a factor of 5. (Its length is now 1/5 of its original value.) The size of the metal plates does not change. What is the new total resistance during a press?

- (d) (5 points) The force required to compress the rubber is $F = ky$, where k is a constant and y is the distance compressed (from the origin). Derive an expression for the resistance as a function of the pressing force F . Write your answer in terms of the initial resistances (R_R and R_M), the resistivities (ρ_R and ρ_M), the area of the sensor, A , and the constant, k . Assume all rubber layers compress the same amount and uniformly.

- (e) (2 points) For a particular sensor, we find that the resistance is:

$$R(F) = \frac{8k\Omega \cdot \text{mm}^2}{A} - \left(100 \frac{\Omega \cdot \text{m}^2}{\text{N}}\right) \frac{F}{A}$$

We define the sensitivity of the sensor, S , to be the change in resistance per unit of force:

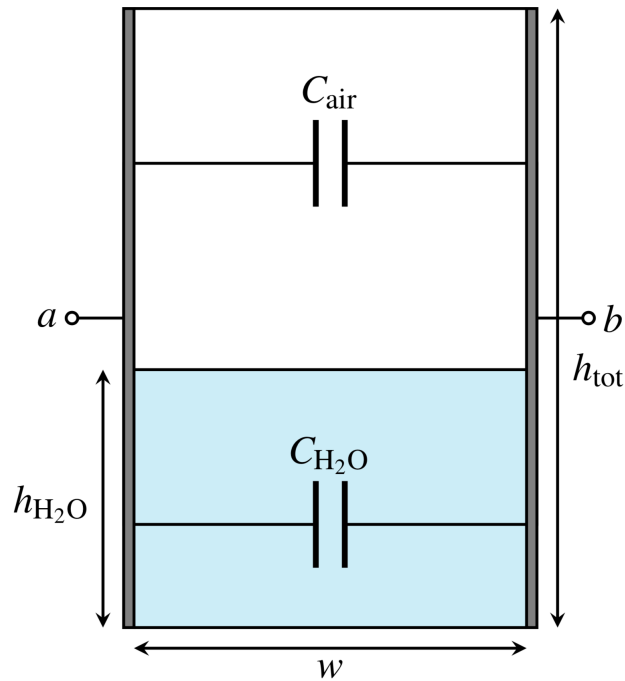
$$S = \left| \frac{dR}{dF} \right|$$

If we want to increase sensitivity, how should we change the area of the sensor? Justify your answer in 1-2 sentences.

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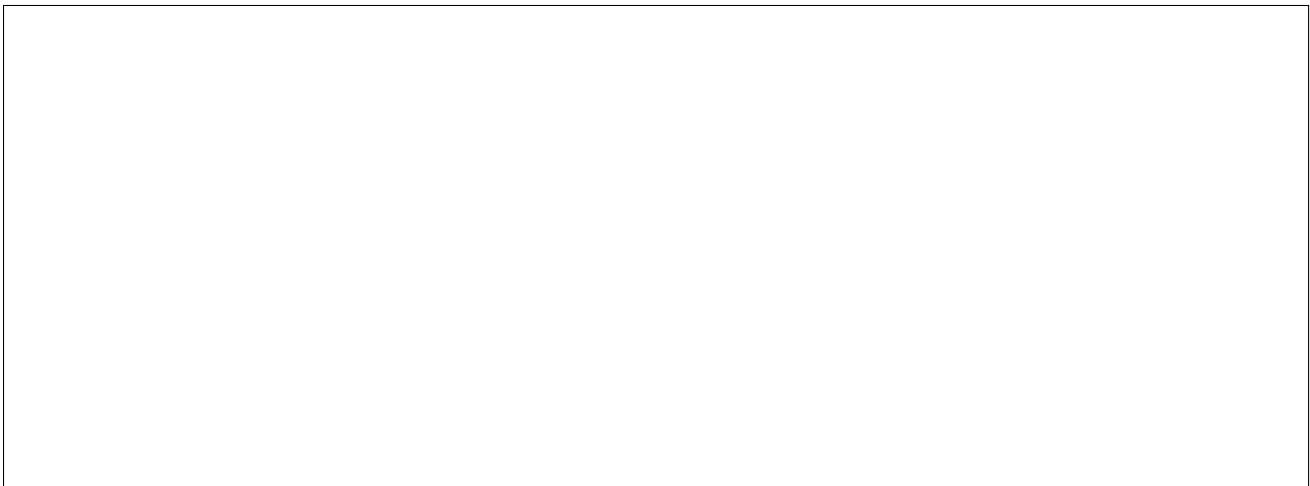
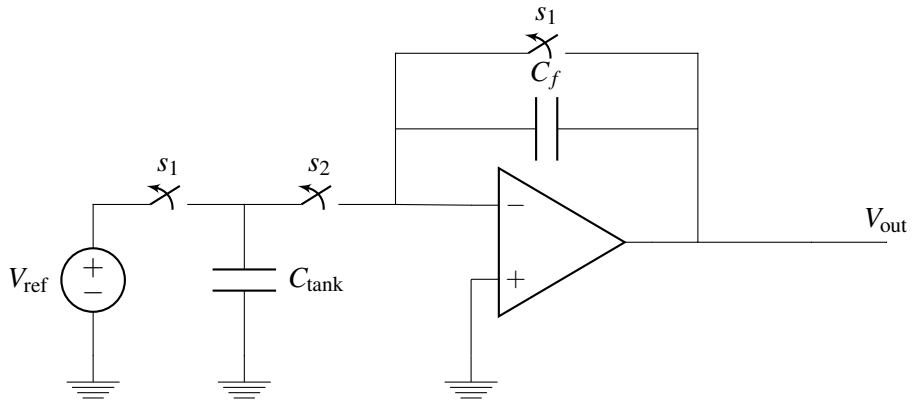
9. Rain Sensor (20 points)

A capacitive sensor can be used to measure how much rain water is in a tank. To do this, two capacitor plates are attached to a rectangular tank. The idea is to make a capacitor whose capacitance varies with the amount of water inside. The width and length of the tank are both w (i.e. the base is square), and the height of the tank is h_{tot} . The permittivity of air is ϵ , and the permittivity of rainwater is 81ϵ .

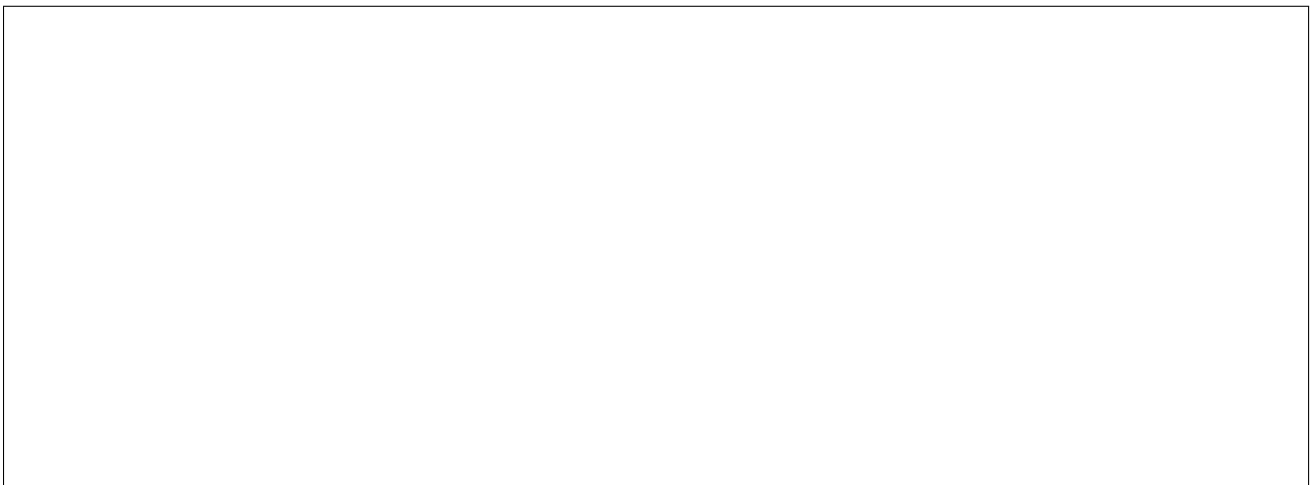


- (a) (5 points) Derive an expression for the total capacitance, C_{tank} , between terminals a and b as a function of h_{tot} , w , $h_{\text{H}_2\text{O}}$, and ϵ .

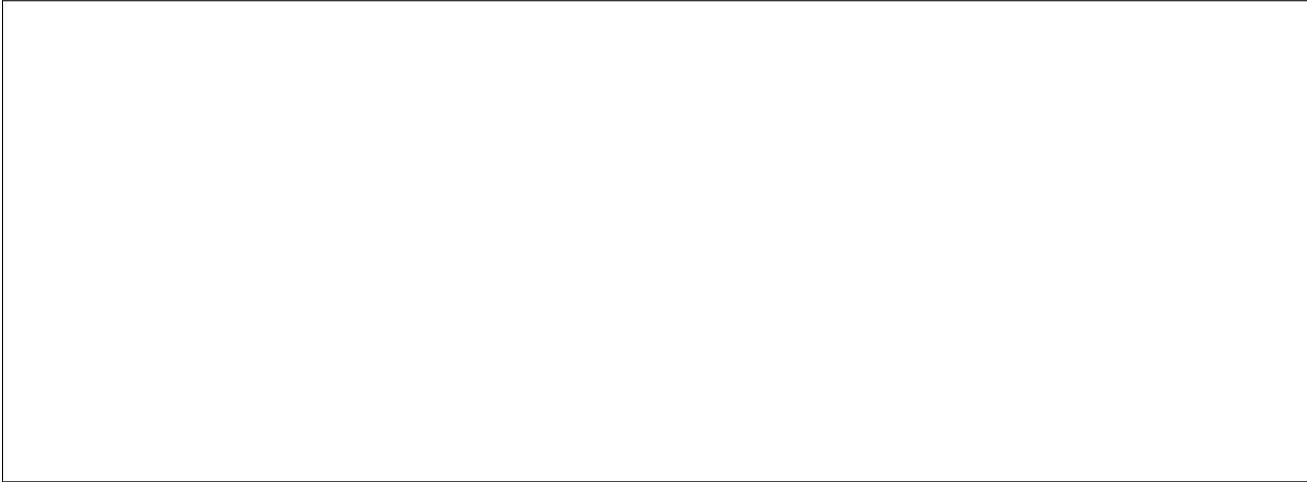
(b) (2 points) You would like to measure changes in C_{tank} as changes in voltage, so you design the following circuit. Draw the equivalent circuit when s_1 is on and s_2 is off.



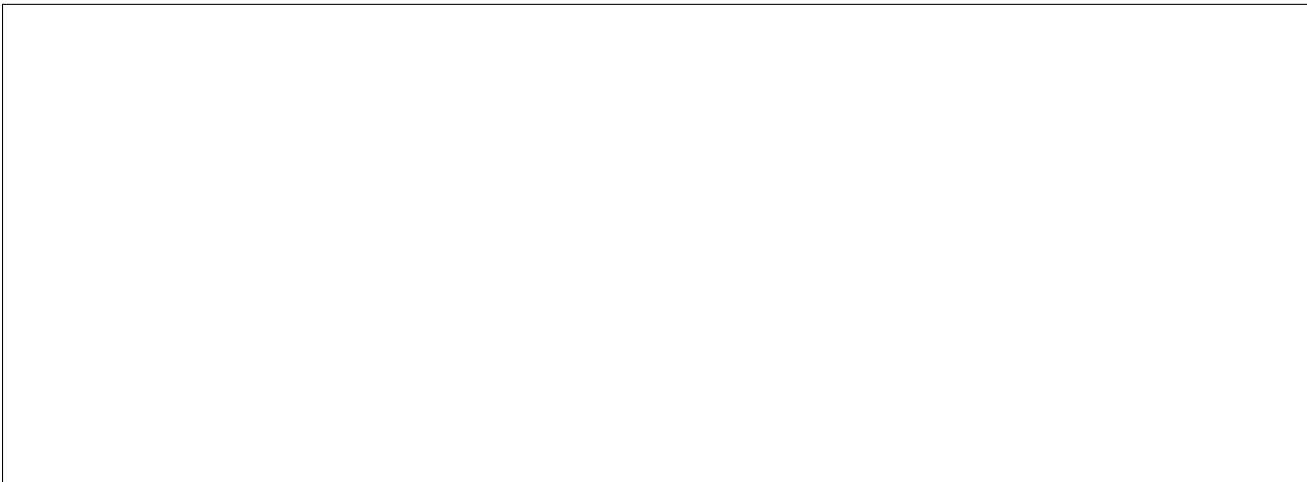
(c) (3 points) Find expressions for charge across each capacitor when s_1 is on and s_2 is off.



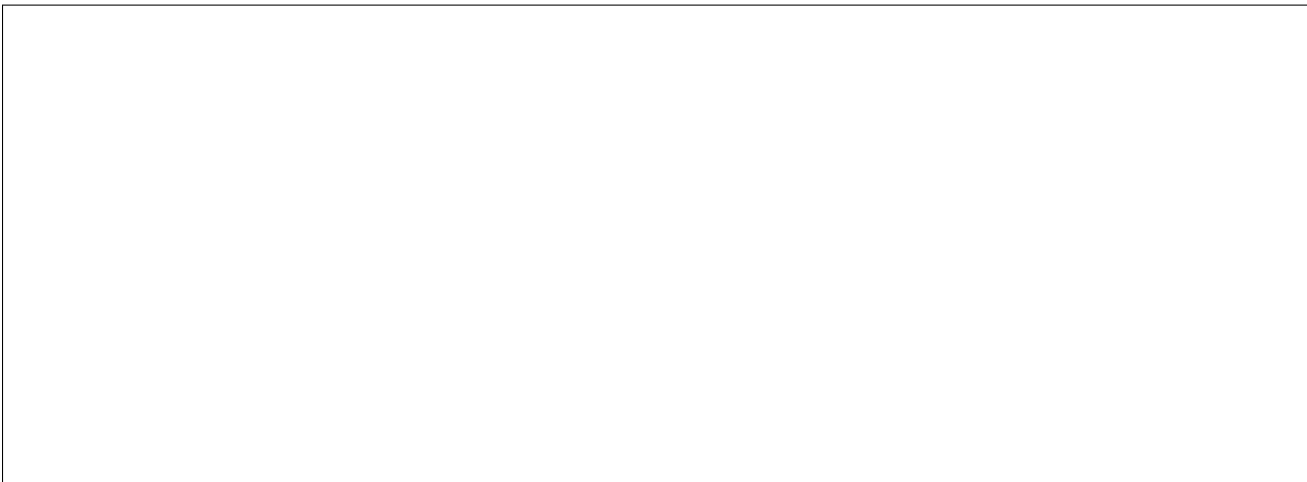
(d) (2 points) Draw the equivalent circuit when s_2 is on and s_1 is off.



(e) (3 points) Find expressions for charge across each capacitor when s_2 is on and s_1 is off.



(f) (5 points) Express V_{out} as a function of V_{ref} and the capacitances. What happens to V_{out} when the amount of water in the tank increases?

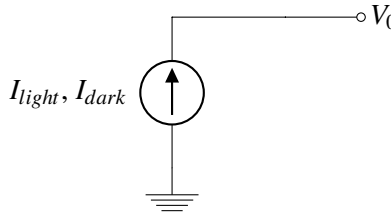


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10. A Light Design Problem, So to Speak... (15 points)

Your pet cactus needs a lot of light every day, and you're not sure your room is sunny enough. You decide to use your knowledge from 16A to build a device to detect if the sun is shining in your room.

- (a) (8 points) Assume you just finished the Imaging Lab, where you build a light sensor, which can be modeled as a current source as shown below. When the light is on, the current source produces 5 mA. When the light is off, 1 mA.



$$I_{light} = 5 \text{ mA}, I_{dark} = 1 \text{ mA}$$

Using this knowledge, **design a circuit that outputs 10 V when the light is on and 0 V when the light is off.** In addition to the model of the light sensor above, you may use the following components (only!):

- **one** op amp
- **one** resistor
- **two** voltage sources

You must explicitly note the power supplies used on the op amp. Clearly label values of resistors and voltage sources that you use.

- (b) (4 points) You realize that the light sensor can only provide a maximum of $P_{max} = 40mW$ of power. Does this affect your design from part (a)? If so, how should it change? If not, why does it not affect it?
(Do not use additional components beyond those specified in the problem statement.)

- (c) (3 points) You notice that there is noise in both I_{light} and I_{dark} (ie. sometimes the currents are higher or lower than expected). Assuming that there is the same amount of noise in both I_{light} and I_{dark} , modify your design to be as robust as possible to these fluctuations. Justify your design choices in 1-2 sentences. If you believe your design does not need modification, explain why.
(Do not use additional components beyond those specified in the problem statement.)

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Extra page for scratchwork.
Work on this page will NOT be graded.

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Doodle page!

Draw us something if you want or give us suggestions, compliments, or complaints.
You can also use this page to report anything suspicious that you might have noticed.

Read the following instructions before the exam.

You have 120 minutes for the exam. There are 10 problems of varying numbers of points. The problems are of varying difficulty, so pace yourself accordingly and avoid spending too much time on any one question until you have gotten all of the other points you can.

There are 100 points possible on this exam. Partial credit will be given for substantial progress on each problem.

Distribution of the points:

Problem	1	2	3	4	5	6	7	8	9	10	Total
Points	1	1	10	12	9	6	10	16	20	15	100

The exam is printed double-sided. Do not forget the problems on the back sides of the pages!

There are 18 pages on the exam, so there should be 9 sheets of paper in the exam. Notify a proctor immediately if a page is missing. **Do not tear out or remove any of the pages. Do not remove the exam from the exam room.**

Write your student ID on each page before time is called. If a page is found without a student ID, we are not responsible for identifying the student who wrote that page.

You may consult TWO handwritten 8.5" × 11" note sheets (front and back). No phones, calculators, tablets, computers, other electronic devices, or scratch paper are allowed. **No collaboration is allowed, and do not attempt to cheat in any way. Cheating will not be tolerated.**

Please write your answers legibly in the spaces provided on the exam; we will not grade outside a problem's designated space. In general, show all of your work in order to receive full credit.

If you need to use the restrooms during the exam, bring your student ID card, your phone, and your exam to a proctor. You can collect them once you return from the restrooms.

Our advice to you: if you can't solve the problem, state and solve a simpler one that captures at least some of its essence. You might get some partial credit, and more importantly, you will perhaps find yourself on a path to the solution.

Good luck!