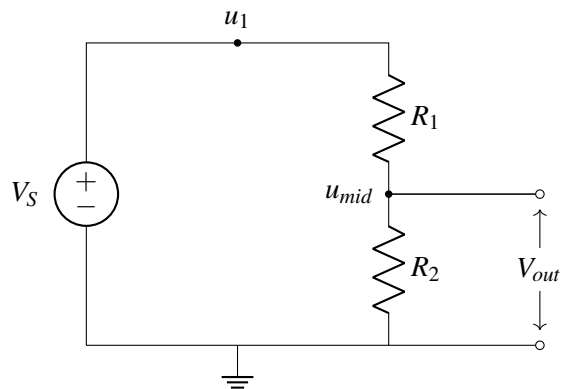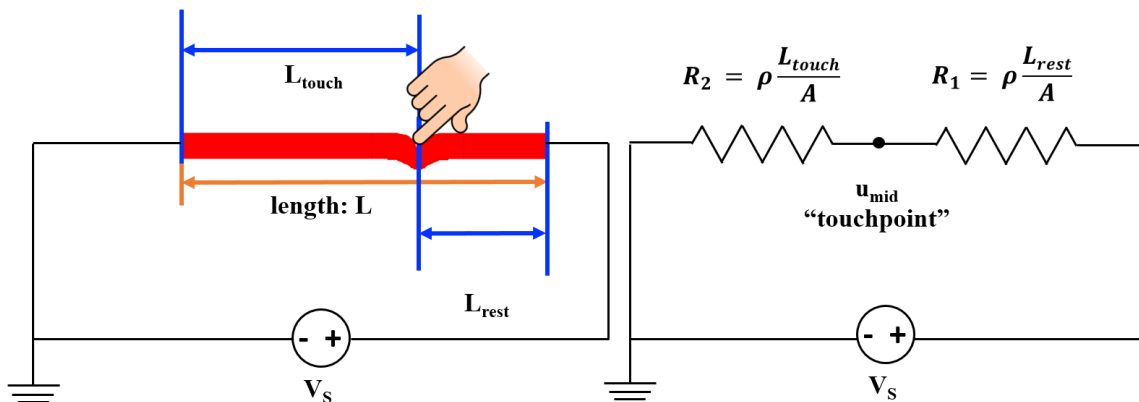## 14.1   Voltage Divider Circuit Review

Before modeling the 2D touchscreen, let's review the most important concept that has enabled 1D touch-screen modeling: the voltage divider circuit.

$$V_{out} = \frac{R_2}{R_1 + R_2} \times V_s. \tag{1}$$

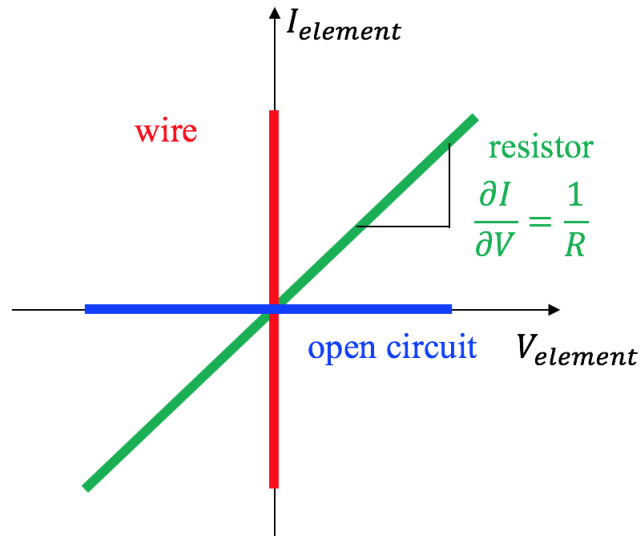By using a voltage divider circuit, we can map u_{mid} to L_{touch}. A relationship between u_{mid} and L_{touch} exists such that:

$$u_{mid} = \frac{R_2}{R_1 + R_2} \times V_s = \frac{L_{touch}}{L} \times V_s. \tag{2}$$

14.2 EE16A Physics Revisited

Before we dive into the modeling of 2D resistive touchscreen, let's review the I-V characteristics for some basic circuit elements.

From the I-V plots, although a resistor, a wire and an open circuit can behave quite differently, their behaviors are exactly the same at $(0, 0)$. This means that at $(0, 0)$, these three circuit elements can be replaced by one another and the same behavior ($I = 0, V = 0$) is still expected.

14.3 An Interesting Circuit

Let's look at an example of different circuit elements behaving in the same way. The circuit we will analyze next (and the corresponding thought process) will be important when we analyze our 2D resistive touchscreen at the end of this note. Consider the following circuit:
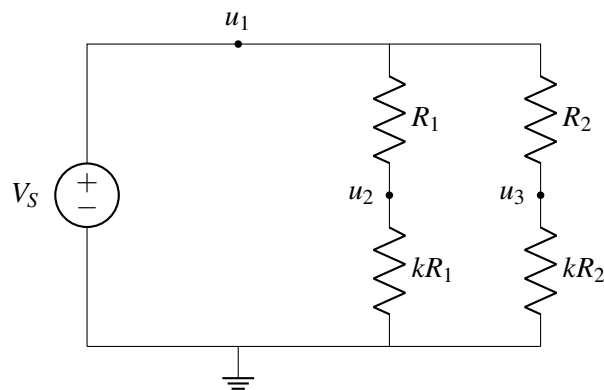
If we want to solve for u_1, u_2, u_3, we could use our general analysis procedure. However, we can simplify

our analysis by noticing that this circuit is very similar to the voltage divider we already analyzed. In fact, it is two voltage dividers — one consisting of resistors R_1, kR_1 and another consisting of R_2, kR_2.

Therefore, we can apply our voltage divider equation twice to find u_2 and u_3. Note that the total voltage drop over both voltage dividers is V_s.
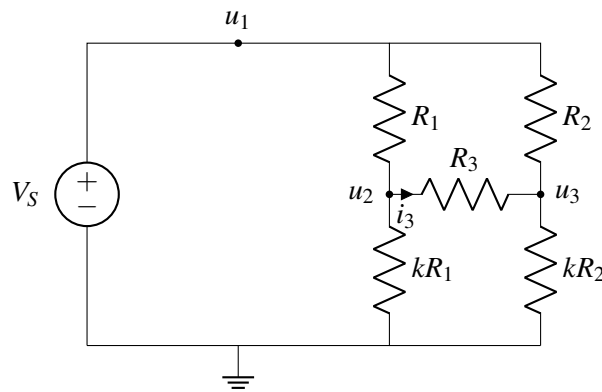
$$u_2 = \frac{kR_1}{R_1 + kR_1} V_s = \frac{k}{1+k} V_s.$$

$$u_3 = \frac{kR_2}{R_2 + kR_2} V_s = \frac{k}{1+k} V_s.$$

We see that regardless of the resistances R_1 and R_2, the potentials u_2 and u_3 are the same! This holds as long as k is constant.

$$u_2 = u_3 = \frac{k}{1+k} V_s$$

Now, let's add another resistor R_3 to the circuit.

Once again, we could analyze this circuit from scratch using our circuit analysis procedure, but maybe we can simplify the analysis. Let's make a bold assumption that adding R_3 will not affect circuit operation and therefore $u_2 = u_3$ from our analysis above. We can determine if this assumption is true by analyzing the circuit and seeing if there are any contradictions that arise. If there are no contradictions, then we know that this bold assumption is true.

First, analyze the current flow through R_3.

$$R_3 i_3 = u_2 - u_3 \tag{3}$$

Under the assumption that R_3 does not affect circuit operation, then $u_2 = u_3$. Plugging this in tells us that the current flowing through R_3 is zero. In addition, we can calculate that the voltage drop across R_3 is $(u_2 - u_3)$ which is also zero. This means that R_3 is at the special $(0, 0)$ point on the I-V plot, where it behaves the same way as a wire or open circuit. This means that there is no contradiction from our bold assumption since a resistor and open circuit have the same current and voltage at this point! Our bold assumption that R_3 will not affect circuit operation is correct!

To complete our analysis, we can replace R_3 with an open circuit:
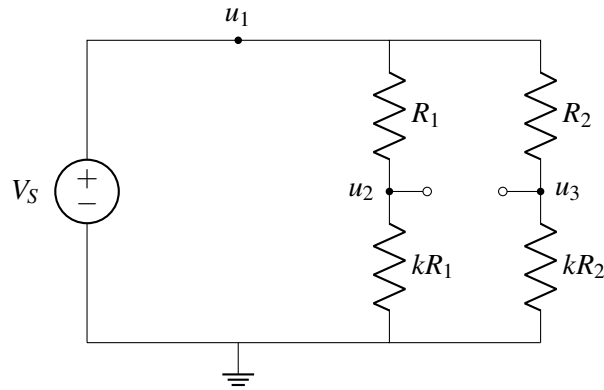
Now, we can write u_2, u_3 directly, using the voltage divider equation as we did above:

$$u_2 = u_3 = \frac{k}{1+k}V_s. \tag{4}$$

When is it ok to replace R_3 with an open circuit? If R_1, kR_1, R_2, kR_2 were four arbitrary resistors, could we still replace R_3 with an open circuit? No, this simplification is only possible because we have set our resistor values so that $u_2 = u_3$. This means that the voltage drop over R_3 and current flowing in R_3 are both zero. In this case, the resistor R_3 is operating at $(0, 0)$ on the I-V plot, so we can replace it without affecting circuit operation.

Next, we will introduce the 2D resistive touchscreen — and we'll see a very similar circuit appear in our model!

14.4   2D Resistive Touchscreen

Now, let's introduce the physical structure of a 2D touchscreen: it consists of a top red plate and a bottom black plate. When a finger touches the screen, the top red plate is pushed into contact with the bottom black plate at the touch point.
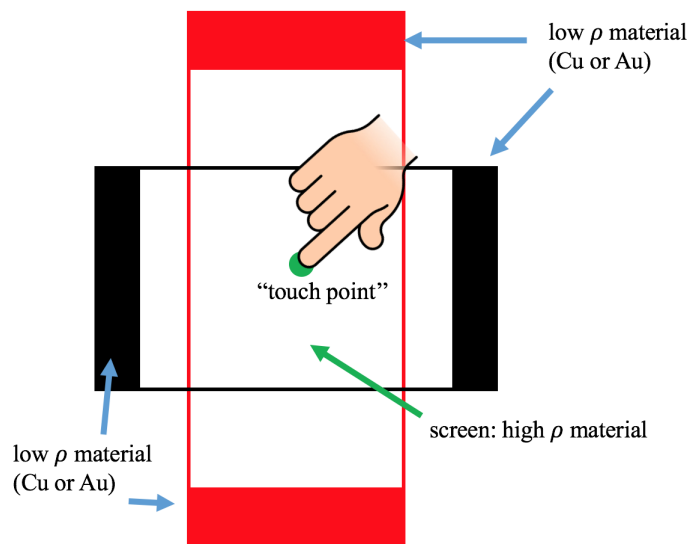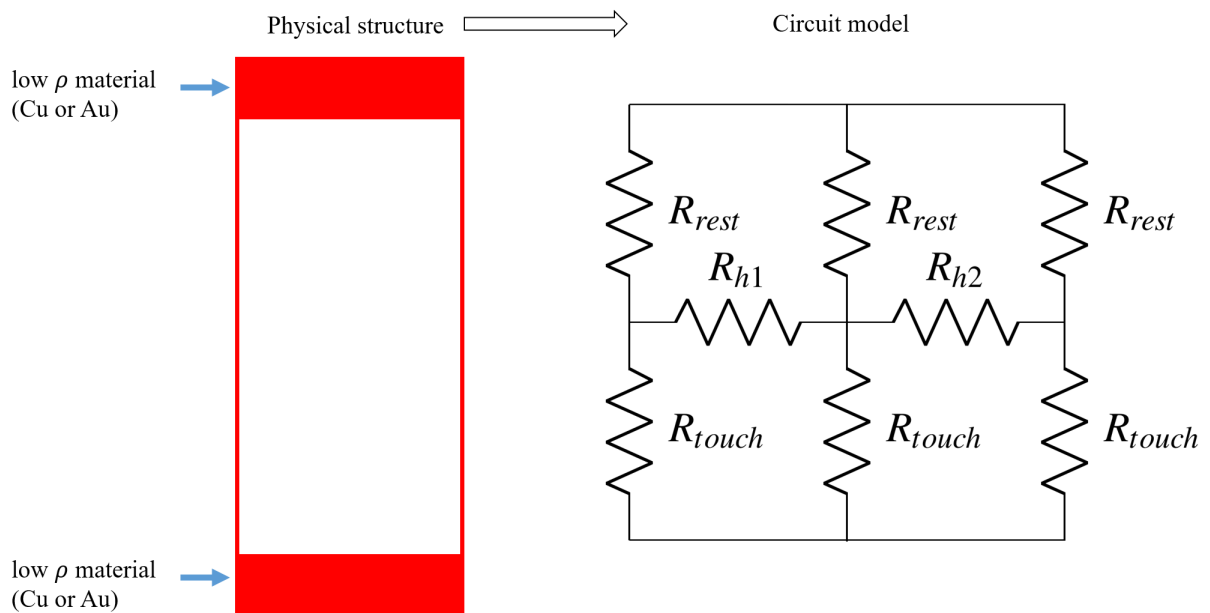
The top and bottom ends of the top red plate as well as the left and right ends of the bottom black plate are made of materials that have very low resistivities ρ, we can treat them as ideal wires ($\rho = 0$). The materials of the transparent screen that we touch in the middle have much higher resistivity.

In a 2D touchscreen, we want to figure out the vertical position and the horizontal position of the touch point: $L_{\text{touch, vertical}}$, $L_{\text{touch, horizontal}}$.

Let's first analyze the physical structure of the top red plate. We can treat the red plate as a bunch of vertical resistor strips, where each vertical strip is connected to the strips next to it by horizontal resistors as well. When we touch the plate, we split it into a top and bottom half, or R_{rest} and R_{touch}.

To simplify analysis, rather than considering many vertical strips, we will divide the red plate into just three vertical segments (of equal width) represented by resistors, which are connected by horizontal resistors R_{h1} and R_{h2}. By the end of this analysis you will see that increasing the number of vertical strips does not change the result.


As we did with the 1D touchscreen, we connect a voltage supply V_s to the top and bottom ends of the top red plate:

Physical structure ⟹ Circuit model
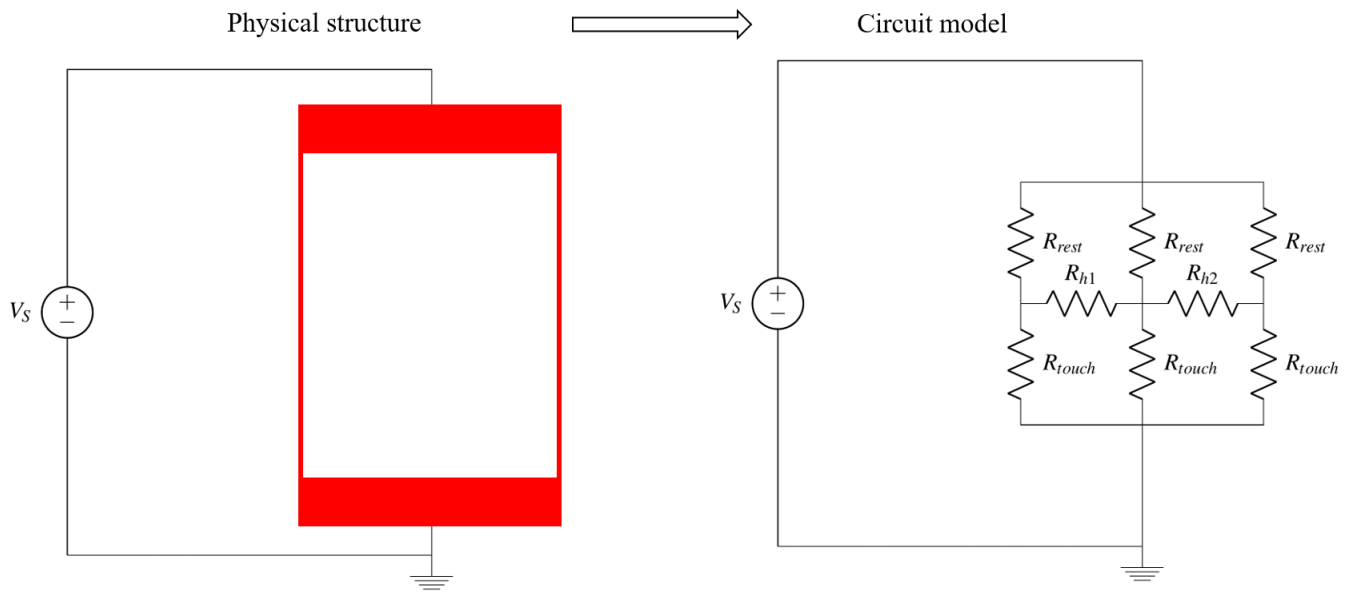
Let's analyze the circuit we just built with the top red plate and a voltage supply V_s:
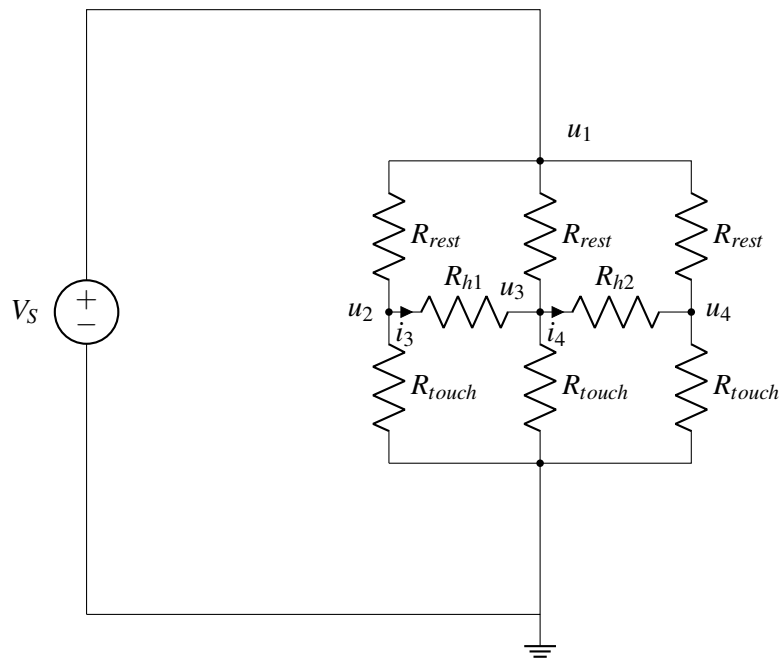
Does this circuit remind you of the "interesting circuit" we analyzed in the previous section? Since R_{rest} and R_{touch} are the same for each segment, we know that $u_2 = u_3 = u_4$. As with the "interesting circuit" can replace horizontal resistors R_{h1}, R_{h2} with open circuits.
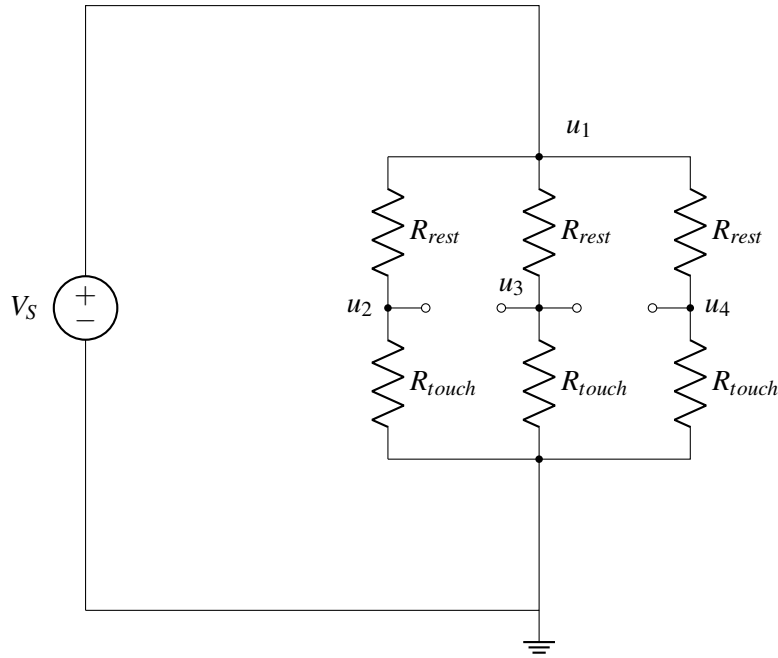
After replacing horizontal resistors with open circuit, we can use a voltmeter and measure u_3. Once again, using the voltage divider equation, we get:

$$u_3 = \frac{R_{touch}}{R_{rest} + R_{touch}} \times V_s. \tag{5}$$

Given $R_{touch} = \rho \frac{L_{touch}}{A}$, $R_{rest} = \rho \frac{L_{rest}}{A}$, u_3 can be further simplified:

$$u_3 = \frac{L_{touch}}{L} \times V_s \quad \text{where} \quad L_{touch} = L_{\text{touch, vertical}} \tag{6}$$

This means that u_3 is mapped to the vertical position touched in the same way as the 1D touchscreen. When measuring the vertical position touched ($L_{\text{touch, vertical}}$), the bottom black plate connects to a voltmeter and measures u_3, the same way it did in the 1D touchscreen. Note that, although we have represented the top red plate by three segments of equal width in the circuit model we built, the value of u_3 will remain the same if we choose to represent the top red plate by an infinite number of segments.

Now that we found $L_{\text{touch, vertical}}$, how can we find $L_{\text{touch, horizontal}}$? We know from linear algebra that if we want to find two values (i.e. vertical and horizontal position), we will need two measurements. What's another useful measurement that we can take? Well, the black bottom plate is rotated $90°$ compared to the red plate, so we can repeat this procedure on the black plate to get the horizontal touch position.

To do this, we connect the supply voltage source V_s to the bottom black plate, and connect the top red plate to a voltmeter. As before, we choose to represent the bottom black plate by three segments of equal width which are connected in between by vertical resistors R_{v1}, R_{v2}.

Physical structure $\Longrightarrow$ Circuit model
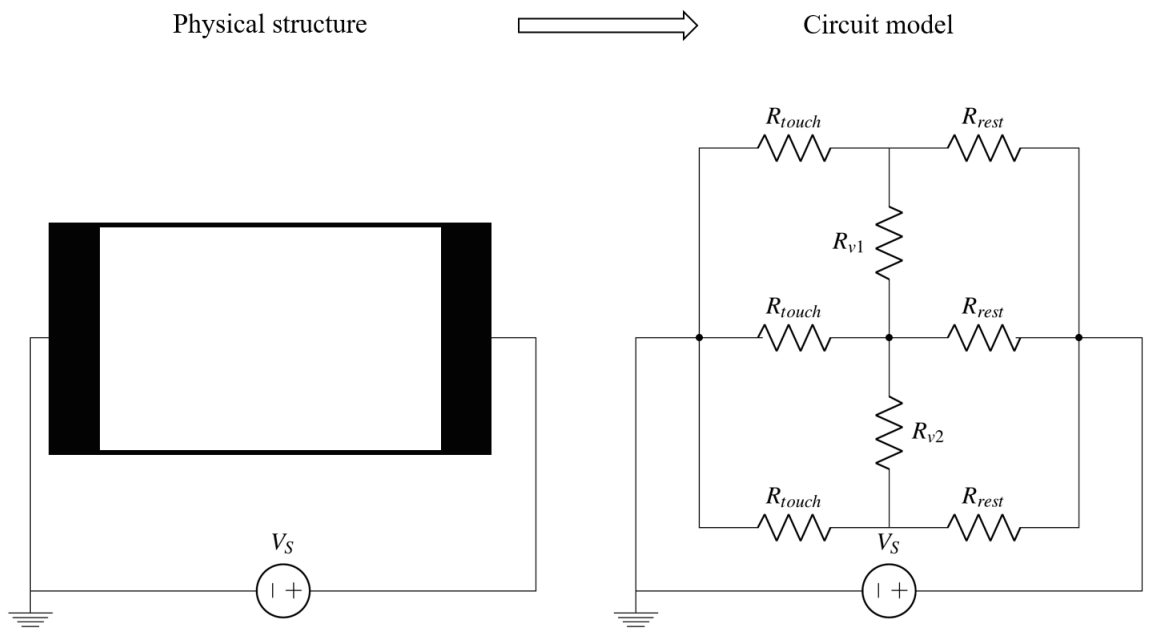
Let's analyze the circuit model for the bottom black plate:
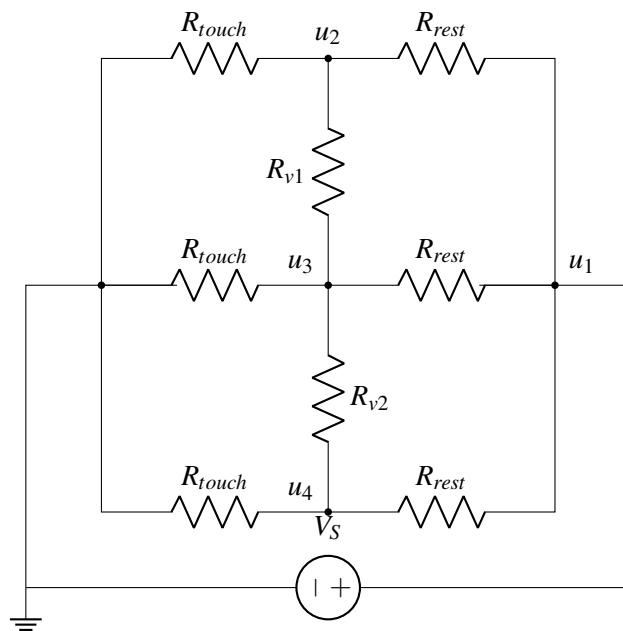
Once again, we see that this is very similar to the "interesting circuit" and we can replace R_{v1} and R_{v2} with open circuits. Then we perform the same analysis as for top red plate, and we can derive u_3, which is:

$$u_3 = \frac{R_{touch}}{R_{touch} + R_{rest}} \times V_s \qquad (7)$$

Here, $R_{touch} = \rho \frac{L_{\text{touch, horizontal}}}{A}$ in which $L_{\text{touch, horizontal}}$ is the horizontal position touched.

The measurement of vertical and horizontal positions ($L_{\text{touch, vertical}}$, $L_{\text{touch, horizontal}}$) for a 2D touchscreen can be summarized as follows:

- **Vertical Position Measurement**
  We connect a voltage source V_s to the top red plate and connect a voltmeter to the bottom black plate. We can map the voltage measured to the vertical position touched:

$$V_{out} = \frac{L_{\text{touch, vertical}}}{L} \times V_s. \tag{8}$$

- **Horizontal Position Measurement**
  We connect a voltage source V_s to the bottom black plate and connect a voltmeter to the top red plate. We can map the voltage measured to the horizontal position touched:

$$V_{out} = \frac{L_{\text{touch, horizontal}}}{L} \times V_s. \tag{9}$$

The important simplification used is replacing R_{h1}, R_{h2} with open circuits for the $L_{\text{touch, horizontal}}$ measurement and R_{v1}, R_{v2} for the $L_{\text{touch, vertical}}$ measurement. However, this kind of simplification is valid only if the resistor is at (0, 0) on the I-V plot, which means the resistor has zero current flow and therefore zero voltage drop ($IR = V$).

14.5 Faster Circuit Analysis

In the previous section, we used the fact that all of the segments had equal R_{rest} and R_{touch}. In this section, we'll consider a similar circuit where the resistances are all different! We'll solve the circuit using analysis that's theoretically equivalent to the 7 step procedure, except this time we will do the steps in a smart way that makes everything faster and easier.

We'll consider the following circuit:
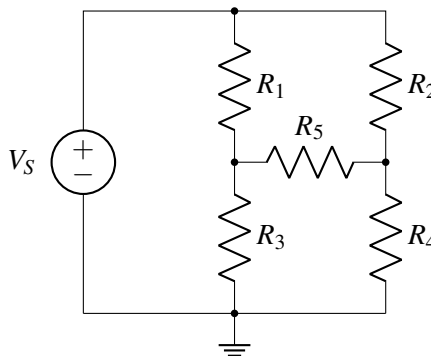
Our goal is to solve for the voltage at each node.

- **Step 1: Label the circuit.** We will label **all unknown node voltages** (u_1, u_2, and u_3).
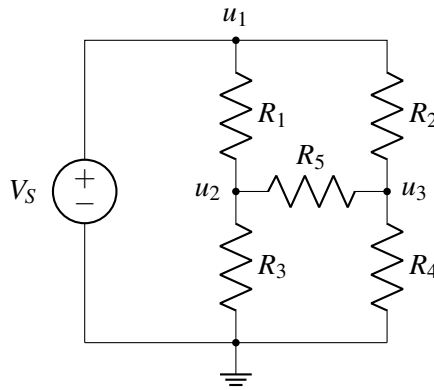
- **Step 2: Write equations for the nodes with voltage sources between them.** Here, the ground node and u_1 have V_s between them, which tells us that

$$u_1 = V_s.$$

- **Step 3: Write KCL for any unknown nodes, using the $V = IR$ relationship, and taking into account any current sources connected to the node.** In this case, our unknown nodes are u_2 and u_3, and we have no current sources (an example with a current source is below). We will treat the current as always flowing **out** of the node.

  Recall that $V = IR$. If we consider node u_2, the sum of the currents flowing out of u_2 through R_1, R_5, and R_3 is 0. Following passive sign convention, since the currents are treated as flowing out of the node, so the voltage drop is $u_2 - u_i$ for some adjacent node u_i. So as an example, the current flowing from u_2 to u_1 through R_1 is $\frac{u_2 - u_1}{R_1}$. Following this pattern for all the currents, and replacing $u_1 = V_s$, we get

$$\frac{u_2 - V_s}{R_1} + \frac{u_2}{R_3} + \frac{u_2 - u_3}{R_5} = 0, \qquad \frac{u_3 - V_s}{R_2} + \frac{u_3}{R_4} + \frac{u_3 - u_2}{R_5} = 0.$$

  You can check that the signs of your voltage drops are correct by checking that the node in question is always positive in the numerators. So in the first equation, u_2 is positive, while in the second equation, u_3 is positive.

  Now, we have just two equations and two unknowns (u_2 and u_3)! So all the remains to be done is to solve the equations by hand.

What if a node has a current source entering? How would we write the KCL equation in that case?
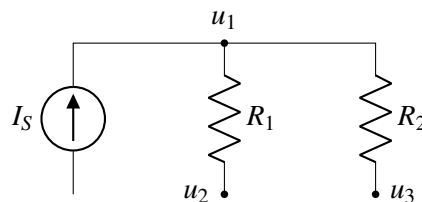
Remember that we treat all currents as flowing out of the node. We should also take the direction of the current source into account. In this case, the left branch of u_1 has a current source I_S flowing in, so $-I_S$ current flows out. So our equation is

$$-I_S + \frac{u_1 - u_2}{R_1} + \frac{u_1 - u_3}{R_2} = 0.$$