

EECS 16A
Fall 2023

Designing Information Devices and Systems I

Midterm 2 Solutions

Midterm 2 Solution

PRINT your student ID: _____

PRINT AND SIGN your name: _____, _____, _____
(last name) (first name) (signature)

PRINT your discussion section and GSI(s) (the one you attend): _____

Name and SID of the person to your left: _____

Name and SID of the person to your right: _____

1. (0 Points) Honor Code

Acknowledge that you have read and agree to the following statement by signing your name below:
As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. I will follow the rules and do this exam on my own.

If you do not sign your name, you will get a 0 on the exam.

[Empty box for signature]

2. (2 Points) When the exam starts, write your SID at the top of every page.

No extra time will be given for this task.

3. (1 Point) Tell us about something that makes you happy.

Any written answer will be awarded full credit.

[Empty box for answer]

4. (1 Point) What is one course you're excited for next semester?

Any written answer will be awarded full credit.

[Empty box for answer]

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

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5. 16A (Taylor's Version) (22 points)

After attending Taylor Swift's Eras Tour, you are excited to make your own LED-based synchronized wristbands. For the course of this problem, we will model our LEDs as resistors.

(a) (2 points) Examine the following layout of LEDs in your first bracelet model, shown in Figure 5.1:

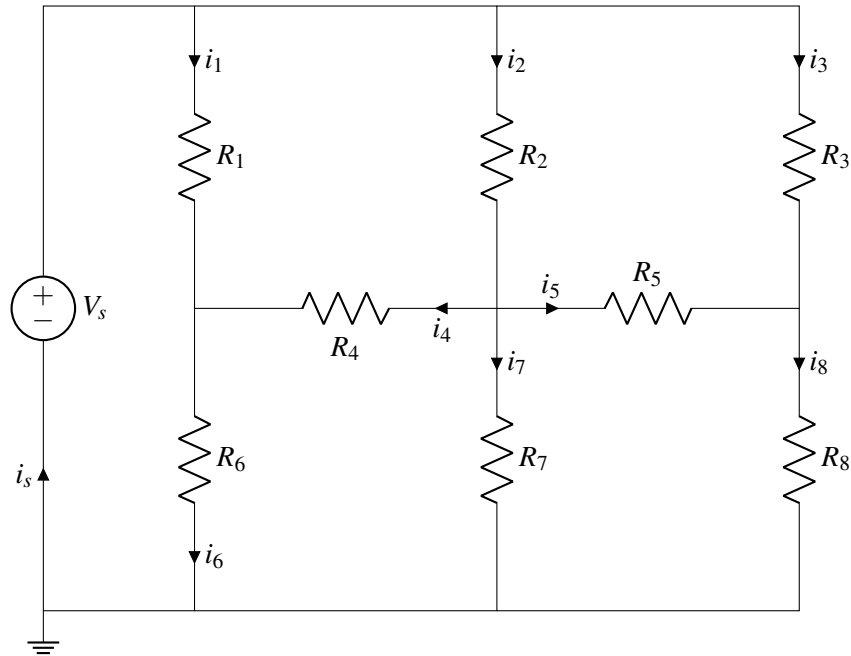


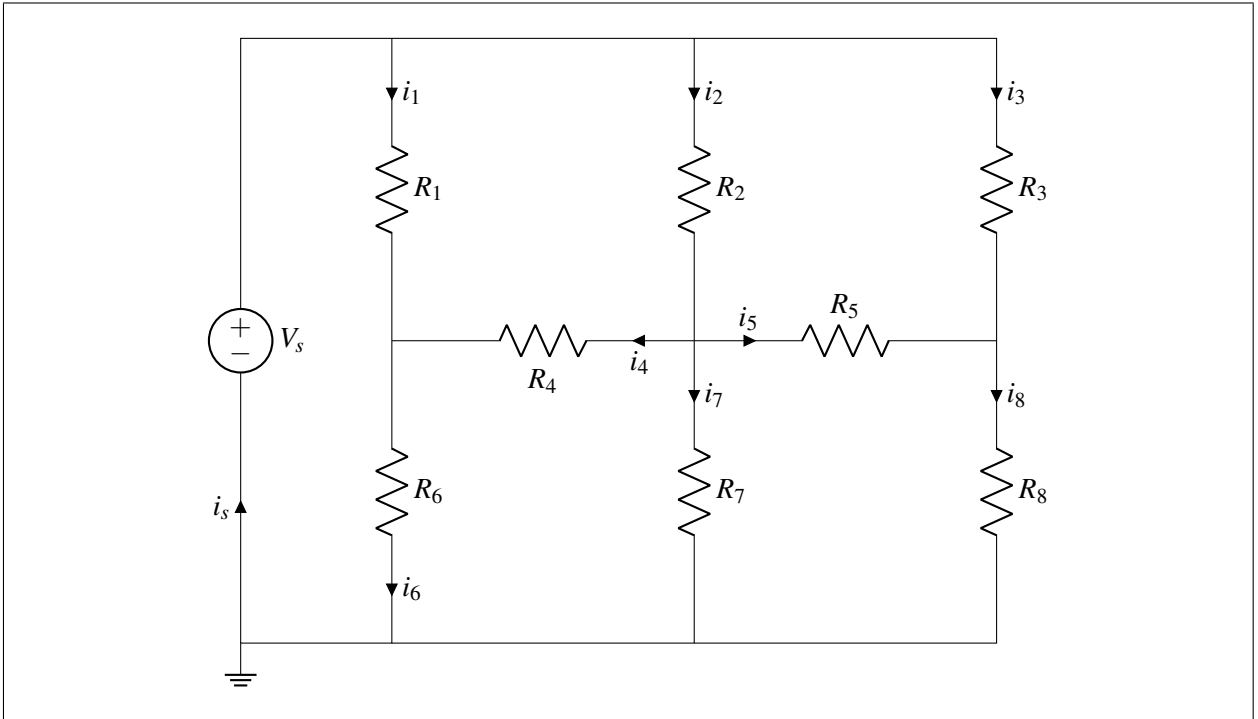
Figure 5.1: Initial Bracelet Model

How many nodes are there in this circuit?

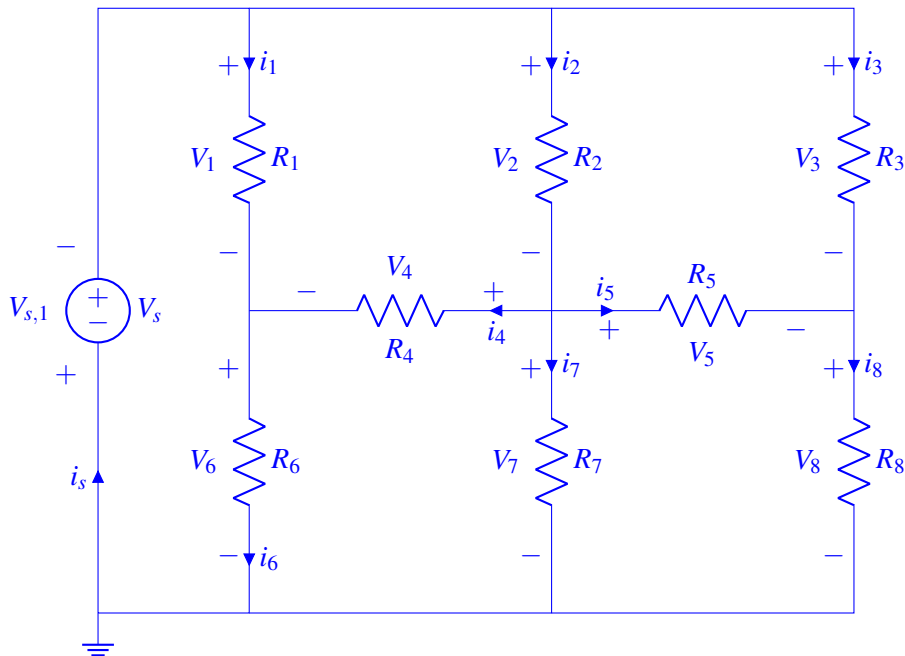
Solution: 5

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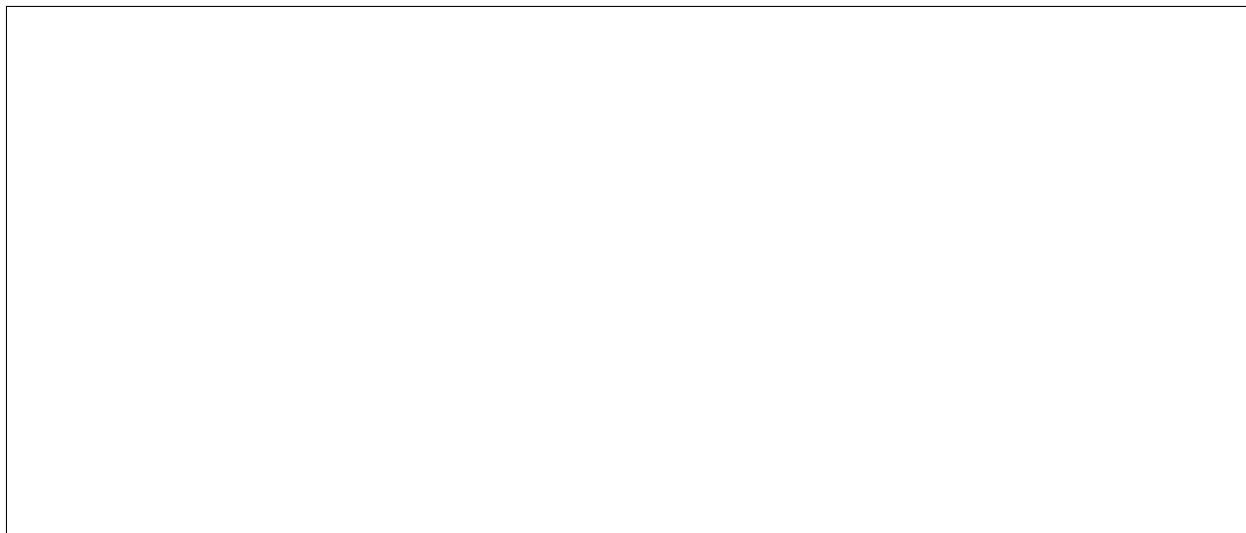
- (b) (4 points) **Label the element voltages on V_s , R_4 , R_5 , and R_6** in the circuit below (identical to Figure 5.1) such that the current flow abides by **passive sign convention**.



Solution:



- (c) (3 points) Once again, consider the circuit in Figure 5.1 (also repeated above in part (b)). Let $V_s = 12\text{V}$. **Write Kirchhoff's Current Law (KCL) equations for all nodes with *unknown* node potentials.**

**Solution:**

$$i_6 - i_1 - i_4 = 0$$

$$i_4 + i_5 + i_7 - i_2 = 0$$

$$i_8 - i_5 - i_3 = 0$$

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- (d) (4 points) To create a light pattern, we want to turn off the LEDs corresponding to R_4 and R_5 in the circuit in Figure 5.1, repeated below. An LED is off if there is zero current flowing through it. The values of R_4 and R_5 are fixed at $10\ \Omega$.

Select values for $R_1, R_2, R_3, R_6, R_7,$ and R_8 such that the current flowing through R_4 and R_5 is zero. Due to component availability, these resistors can be **either $1\ \Omega$ or $10\ \Omega$.** List the values, with units, on the appropriate lines. Use the box to justify your answer.

R_1 : _____

R_2 : _____

R_3 : _____

R_6 : _____

R_7 : _____

R_8 : _____

Circuit reproduced for your reference.

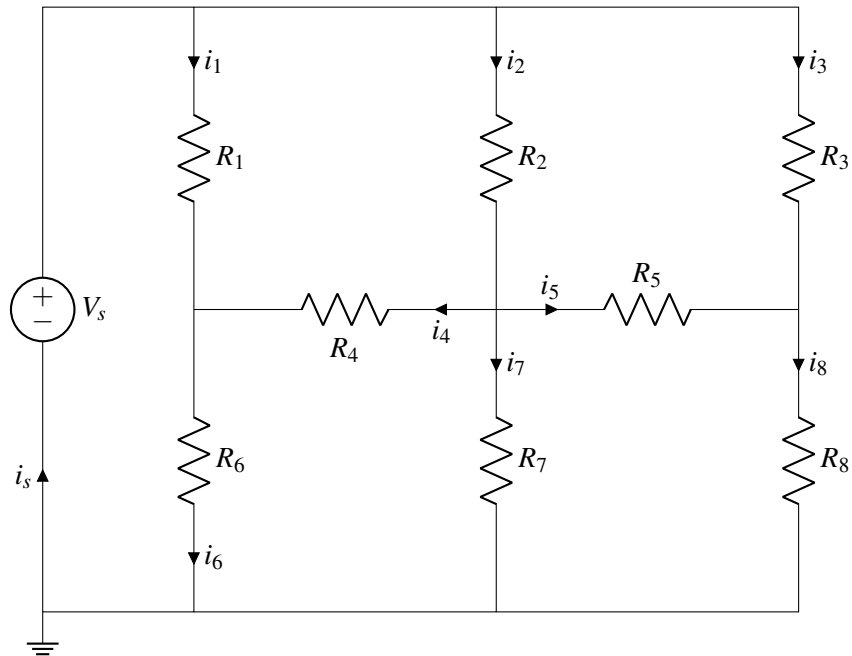


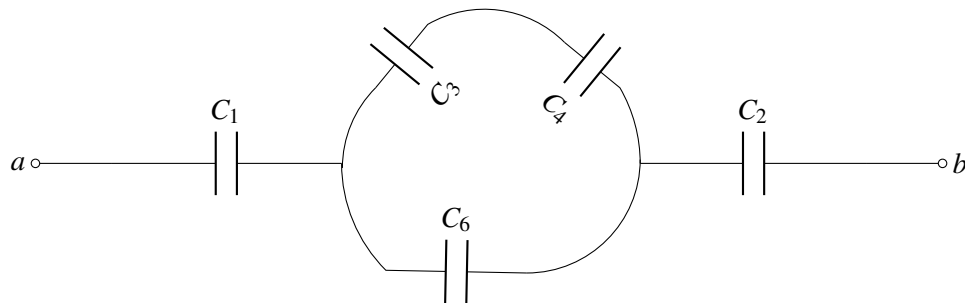
Figure 5.1

Solution:

In order for there to be zero current across R_4 and R_5 , there needs to be no voltage *drop* across them (in other words, the nodes on either side must be at the same potential). You may recognize this circuit as a network of voltage dividers, quite similar to the setup from the touch lab. To have the same voltage along all the middle nodes, the ratio of top resistor to (top resistor + middle resistor) must be the same. This means that any answer where $R_1/R_6 = R_2/R_7 = R_3/R_8$ is acceptable. One example would be all resistors set to 10Ω .

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- (e) (4 points) Kanav suggests a different LED bracelet that uses a network of capacitors to store and release energy for the lights, as shown below:



Write the expression for the equivalent capacitance between nodes a and b . You may use the \parallel operator in your answer. You do not need to simplify or justify your expression.

Solution: C_3 and C_4 are in series, and their combination is in parallel with C_6 . This combination is in series with C_1 and C_2 . Combining yields:

$$C_1 \parallel ((C_3 \parallel C_4) + C_6) \parallel C_2$$

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- (f) (5 points) Manooshree is at the Taylor Swift concert, and is wearing a different bracelet, as shown in Figure 5.2. It has six LEDs (modeled as resistors) — two red, two yellow, and two blue. You want to identify which color is dissipating the most power (i.e. shining the brightest). Each color corresponds to a different Era that Taylor is singing, as in the following table. **Identify which color (pair of resistors) is dissipating the most power**, and which Era this corresponds to. **Justify your answer.**

Era	Brightest Color
Fearless	Yellow
Red	Red
1989	Blue

Note: You can rigorously justify which color is dissipating the most power without explicitly calculating all the powers, though you may calculate them if you prefer.

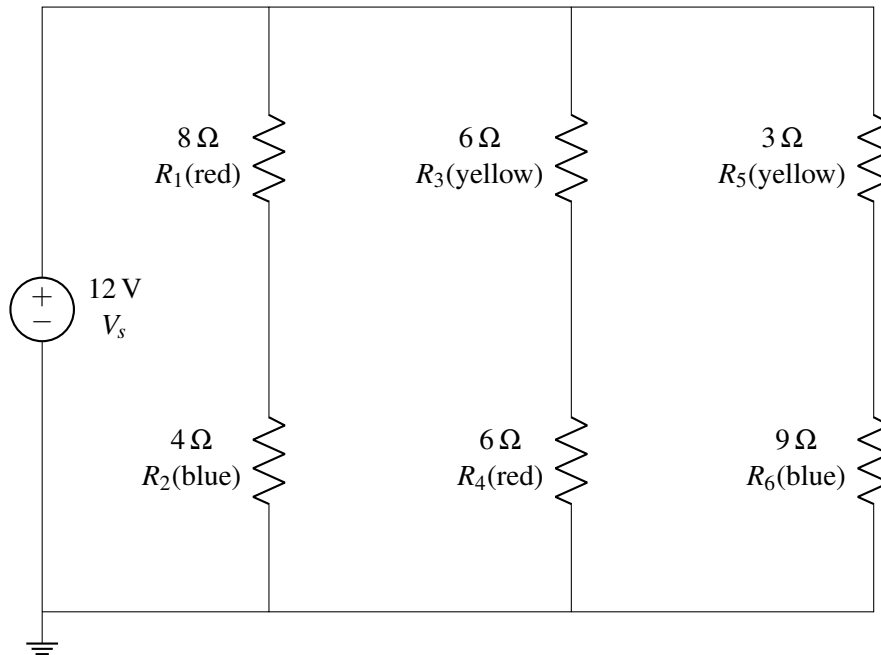


Figure 5.2: Manooshree's Bracelet Model

Solution:

Note that because each parallel set of two LEDs in series has the same equivalent resistance (12Ω), the current through each parallel set is the same. So, each LED (i.e., resistor) has the same current $I = 1\text{ A}$ through it. The total power dissipated by each color is then proportional to the total resistance of that color (more specifically, $P = I^2R$), meaning the color with highest total resistance in the circuit will be dissipating the most power and thus shining most brightly.

$$R_{\text{red}} = R_1 + R_4 = 8 + 6 = 14\Omega$$

$$R_{\text{yellow}} = R_2 + R_5 = 6 + 3 = 9\Omega$$

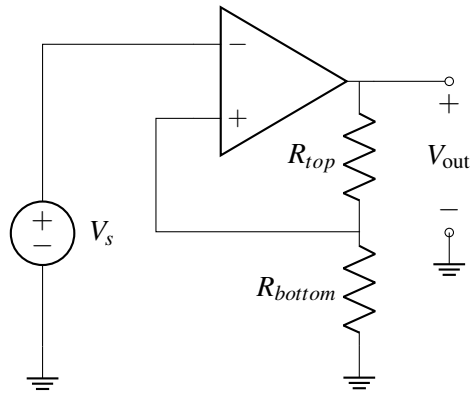
$$R_{\text{blue}} = R_3 + R_6 = 4 + 9 = 13\Omega$$

So, the red LEDs are dissipating the most power, and the bracelet is in the Red Era!

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6. Anti-Hero (2 points)

Is the circuit below in negative feedback? Select your answer by completely filling in the appropriate circle. No justification is required.



In negative feedback

Not in negative feedback

Solution: The circuit is not in negative feedback. We can observe that there is a voltage divider from V_{out} to u_+ to ground. This is because there is no current into u_+ , from the first Golden Rule. If we raise the value of V_{out} , the value of u_+ also increases. Recalling that $V_{out} = A(u_+ - u_-)$, increasing u_+ results in an increase in V_{out} . In other words, the feedback of the op amp *increases* the error signal (change is in the same direction as our manipulation), and hence the circuit is in positive feedback.

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7. Mastermind (11 points)

Sunash is studying neurons. The outer and inner layers of the neuron's cell membrane can be modeled as resistors with resistances R_2 and R_3 , respectively. Current is driven by an ion-pump protein that can be modeled as a voltage source V_s with internal resistance R_1 . The full model for the neuron cell membrane is shown below.

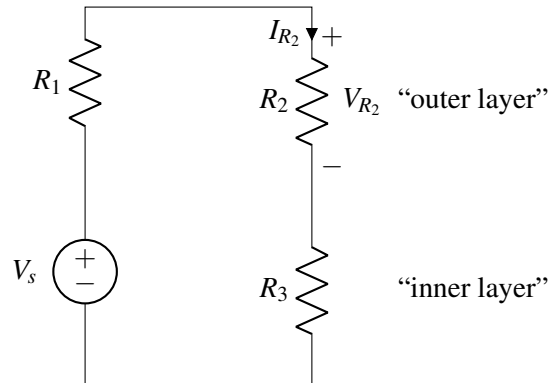
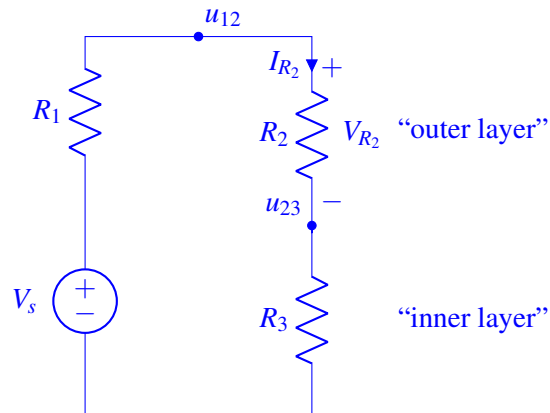


Figure 7.1

Sunash's primary interest is in the protein behavior in the outer layer, so he aims to measure the voltage drop across this layer, V_{R_2} .

- (a) (4 points) **Find an expression for V_{R_2} in terms of V_s and the resistances R_1 , R_2 , and R_3 , in Figure 7.1.**

Solution:

The voltage at the node between R_1 and R_2 , which we will denote u_{12} , can be found using the voltage divider equation, with $R_2 + R_3$ as the resistance in the numerator and $R_1 + R_2 + R_3$ as the total resistance in the denominator:

$$u_{12} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \times V_s$$

The voltage at the node between R_2 and R_3 , which we will denote u_{23} , can be found using the voltage divider equation, with R_3 as the resistance in the numerator and $R_2 + R_3$ as the total resistance in the denominator. However, we must use u_{12} as the “source” voltage in this case.

$$\begin{aligned} u_{23} &= \frac{R_3}{R_2 + R_3} \times u_{12} \\ &= \frac{R_3}{R_2 + R_3} \times \frac{R_2 + R_3}{R_1 + R_2 + R_3} \times V_s \\ &= \frac{R_3}{R_1 + R_2 + R_3} \times V_s \end{aligned}$$

Finally, the voltage drop across R_2 , V_{R_2} , is the difference $u_{12} - u_{23}$:

$$\begin{aligned} V_{R_2} &= u_{12} - u_{23} \\ &= \frac{R_2 + R_3}{R_1 + R_2 + R_3} \times V_s - \frac{R_3}{R_1 + R_2 + R_3} \times V_s \\ &= \frac{R_2}{R_1 + R_2 + R_3} \times V_s \end{aligned}$$

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 Circuit repeated for your reference.

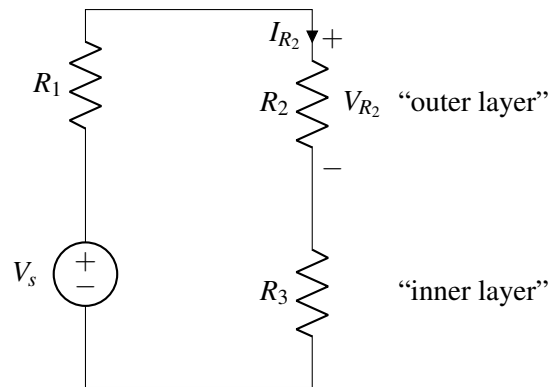
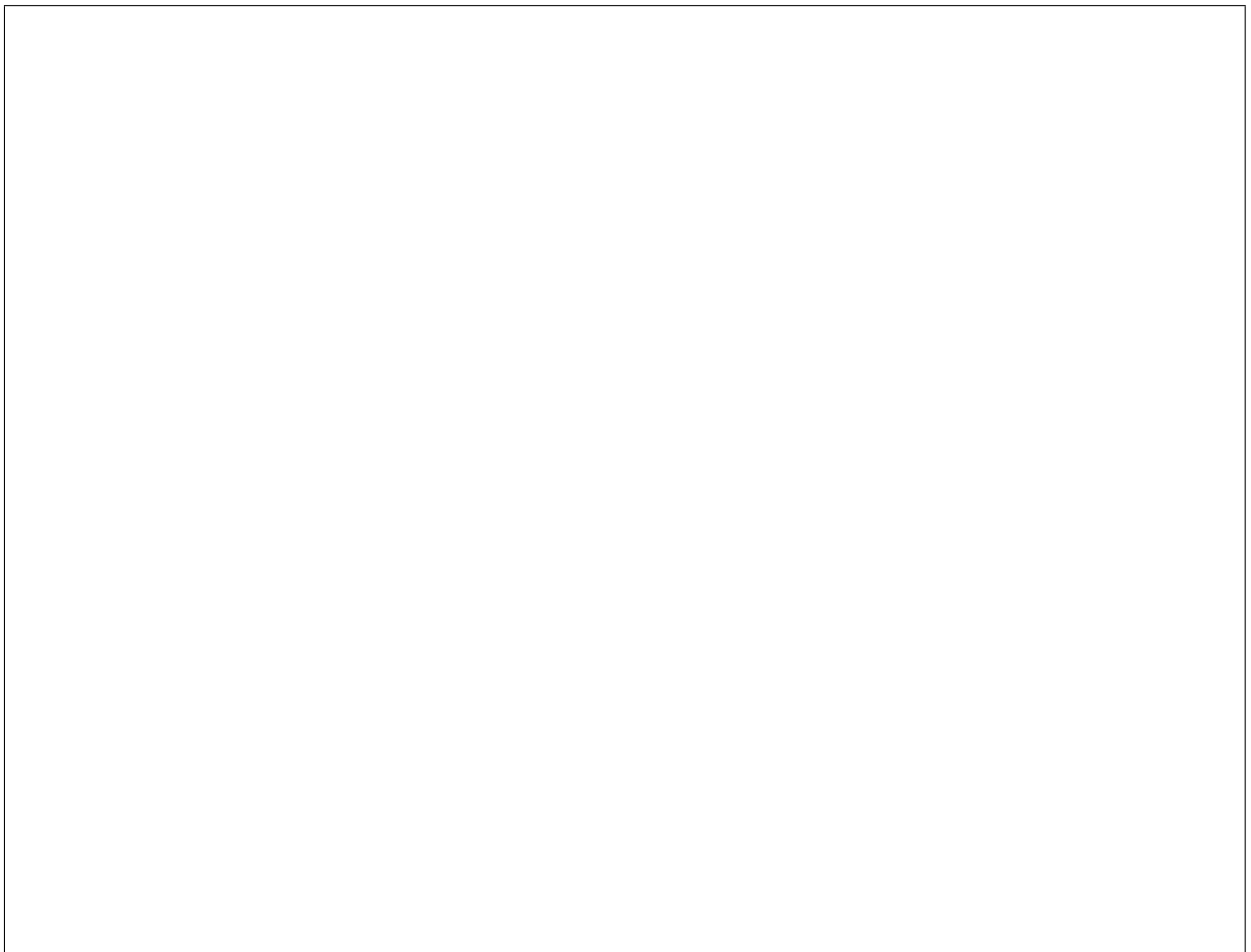
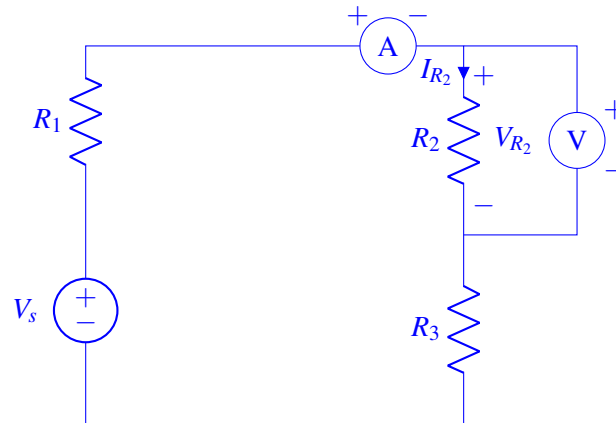


Figure 7.1

- (b) (3 points) We wish to measure V_{R_2} using an ideal voltmeter and I_{R_2} (the current through R_2) using an ideal ammeter. **Redraw the circuit in Figure 7.1 with the ideal voltmeter and ammeter correctly connected.**



Solution:



We connect the ideal voltmeter with infinite internal resistance in parallel to the resistor R_2 , so that it does not draw any current from the circuit, and we connect the ideal ammeter with zero internal resistance in series with the resistor R_2 , so that there is no voltage drop across the ammeter. Connecting the measurement devices in this way ensures they do not alter the circuit behavior.

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- (c) (4 points) Sunash discovers he can modify how the neuron behaves. He can add up to five parallel paths for current, each with a **finite** resistance R_C , as shown in Figure 7.2.

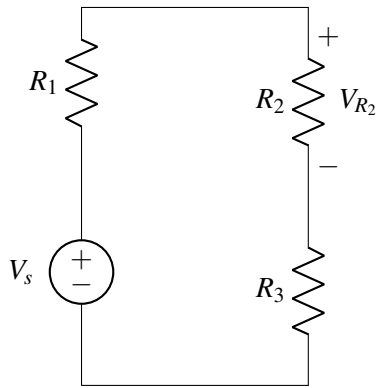


Figure 7.1

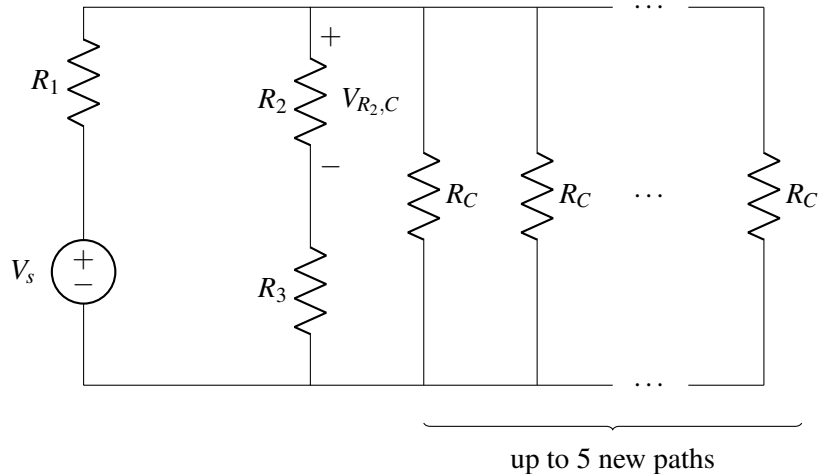


Figure 7.2

Which choice below **most accurately** describes the relationship between V_{R_2} , the voltage drop across R_2 in the original circuit of Figure 7.1, and $V_{R_2,C}$, the voltage drop across R_2 in Figure 7.2 when exactly five parallel paths are added?

- $V_{R_2} > V_{R_2,C}$
 $V_{R_2} < V_{R_2,C}$
 $V_{R_2} = V_{R_2,C}$
 $V_{R_2} \geq V_{R_2,C}$
 $V_{R_2} \leq V_{R_2,C}$

Solution: When we connect additional finite-resistance resistors in parallel to the two resistors representing the membrane layers, the effective resistance across the entire membrane decreases. As such, the proportion of the total voltage drop that occurs across the membrane resistors (R_2 and R_3) decreases when the additional resistors are added in parallel. Since the source voltage is unchanged between the two circuits, the absolute voltage drop across the membrane resistors decreases, which also decreases the voltage drop across the outer layer resistor R_2 , relative to the circuit with no paths added. Therefore, $V_{R_2} > V_{R_2,C}$. The first choice is correct.

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8. Getaway Car (8 Points)

You would like to design a sensor that detects if a driver is touching the steering wheel, as part of an autonomous driving system. Anastasia offers to help you with the warning system design.

(a) (3 points) You first need to design a new parallel plate capacitor. You are given the following requirements and information:

- The capacitor must have a total capacitance of $400\epsilon_0 \cdot 10^{-2} \cdot \text{m}$ (recall the units of ϵ_0 are F/m).
- The cross-sectional area of the plates is 10^{-3} m^2 .
- The dielectric (i.e. the material between the capacitor plates), has a permittivity 80 times that of vacuum (i.e. $80\epsilon_0$).

Finalize your design by **determining the distance between the plates of the capacitor in meters.**
Show your work.

Solution: Recall that:

$$C = \frac{\epsilon_0 A}{d}$$

Plugging in known values:

$$400\epsilon_0 \cdot 10^{-2} \cdot \text{m} = \frac{80\epsilon_0 * 10^{-3} \text{m}^2}{d}$$

$$d = \frac{80\epsilon_0 * 10^{-3}}{400\epsilon_0 \cdot 10^{-2}}$$

Simplifying leads us to a thickness of 2 cm, or 0.02 m.

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- (b) (5 points) You are given the following waveform of voltage across your capacitor, V_C , in the touch and no-touch cases.

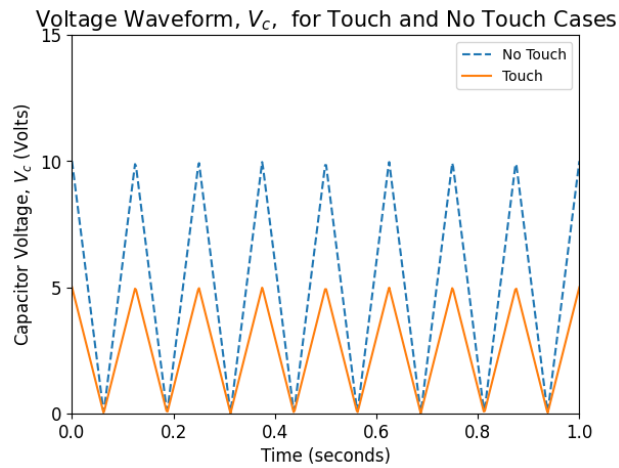
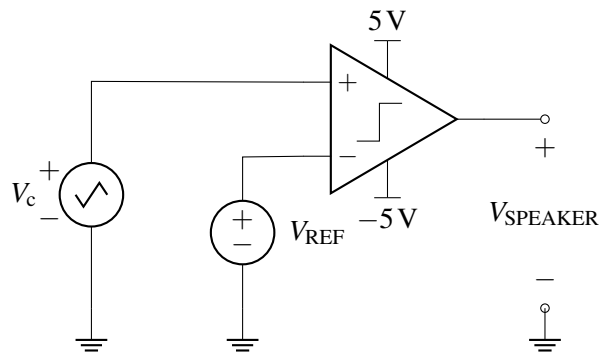


Figure 8.1: Voltage Waveform

You would like to use this voltage, V_C , and utilize it to beep a speaker (i.e. turn on and off repeatedly) when there is no touch. You discover this circuit schematic, but the value of V_{REF} is erased. The speaker turns on when the voltage across it, $V_{SPEAKER}$, is +5 Volts.



Select an appropriate value, with units, for V_{REF} such that the speaker beeps on and off when there is no touch, and is quiet when there is a touch. Justify your answer.

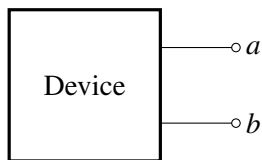
Solution: Notice that the speaker turns on whenever the voltage goes above V_{REF} . Since the speaker

should only turn on when we are not touching the steering wheel, V_{REF} should be selected to be any value strictly between the touch peak and no-touch peak. In this case, V_c is *always* below V_{REF} . This means that the comparator will *always* output -5V, and the speaker will always be off. On the other hand, in the no touch case, V_c spends some of its time above V_{REF} (in which case the comparator outputs 5V and the speaker is powered), and the rest of it below V_{REF} , in which case the comparator outputs -5V and the speaker is off. This switching behavior results in the beeping. To be fully precise, we cannot pick a value of 10 volts (the no-touch peak), nor can we pick 5V (the touch peak).

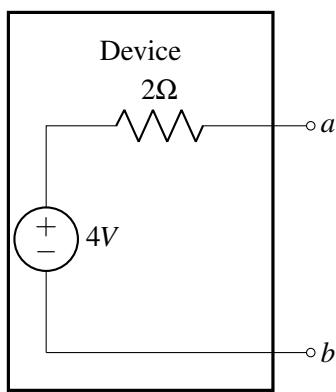
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9. Sparks Fly (22 points)

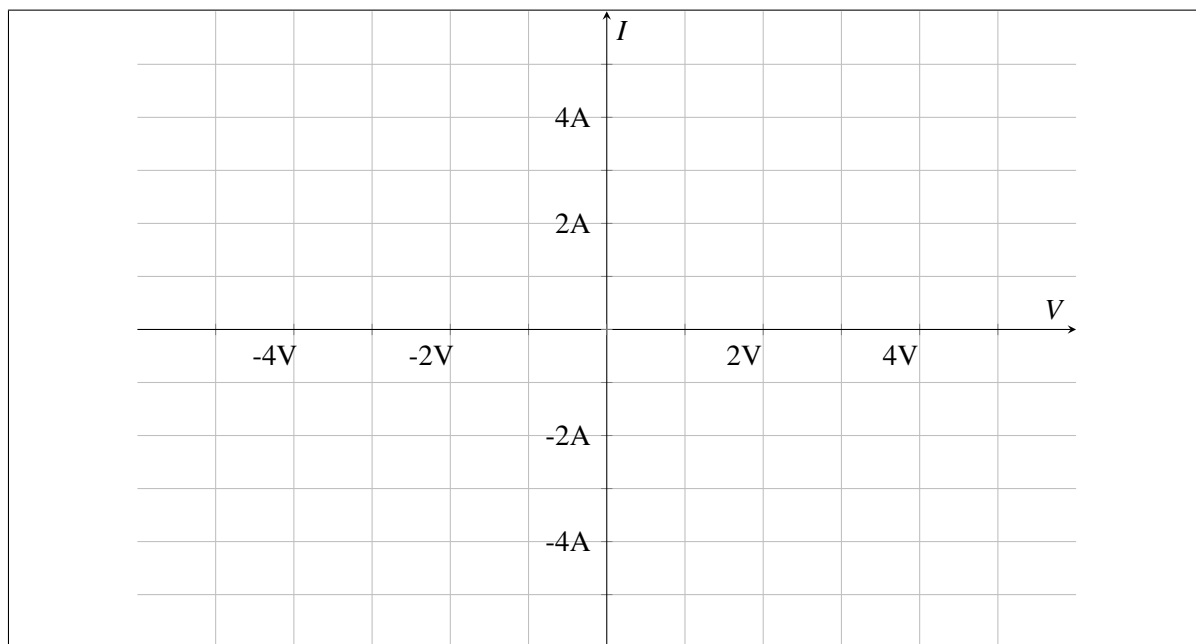
- (a) (4 points) Ryan is trying to use old circuits and devices from a junk pile to light up a lightbulb. He finds the following device in the junk pile.



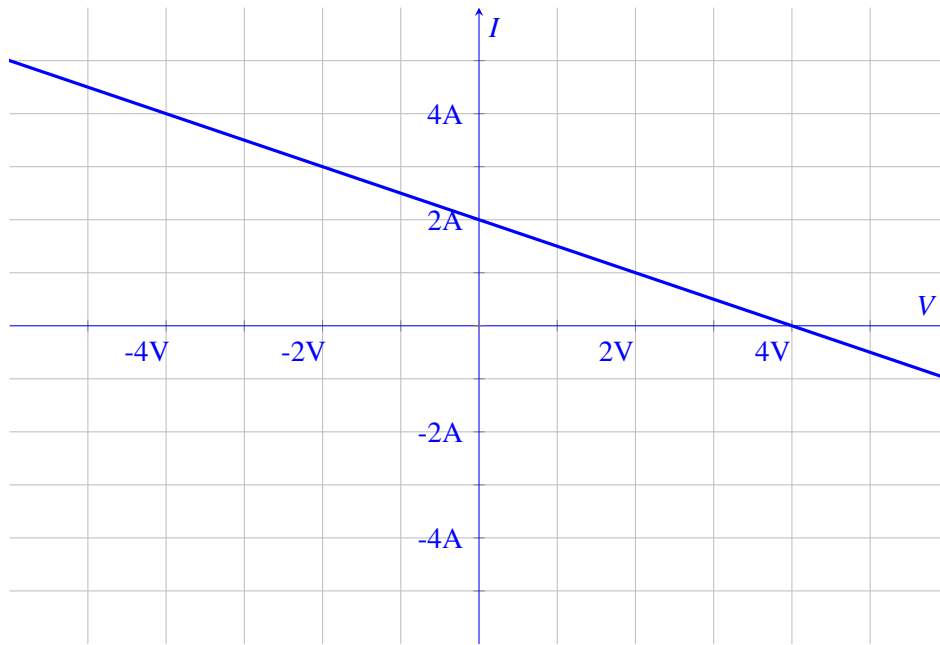
The device has the following Thévenin equivalent circuit:



Plot the I-V curve of this device on the grid lines below.



Solution:



We can determine two points on the IV curve as follows:

- When we short terminals a and b , a $4V/2\Omega = 2A$ current occurs through the wire with a $0V$ drop. So $(0V, 2A)$ is a point on the IV curve.
- When we open terminals a and b , a $4V$ drop occurs across the device with a $0A$ current. So $(4V, 0A)$ is a point on the IV curve.

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- (b) (6 points) Suppose Ryan instead finds the circuit shown in Figure 9.1 in the junk pile. To simplify the analysis of this circuit, Ryan finds out that the circuit has a Thévenin equivalent circuit from the perspective of nodes a and b , shown in Figure 9.2:

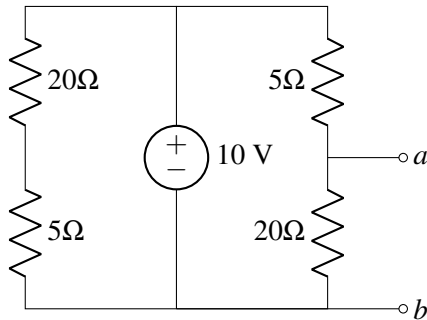


Figure 9.1

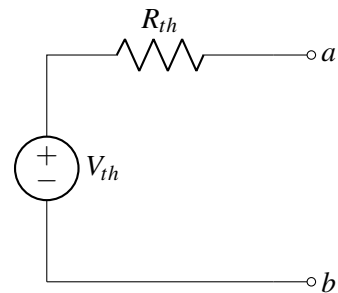


Figure 9.2

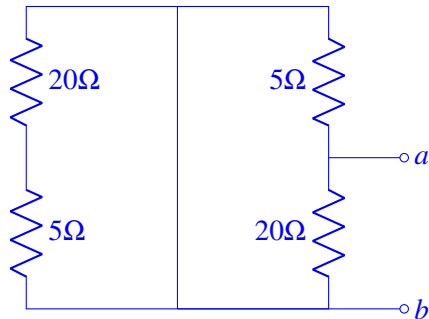
Find V_{th} and R_{th} . Show your work.

Solution: First, let us find $V_{th} = V_{ab}$. Looking at the right side of the circuit, this is just a voltage

divider. Therefore we have that:

$$V_{ab} = 10V \cdot \frac{20\Omega}{20\Omega + 5\Omega} = 10V \cdot \frac{4}{5} = 8V$$

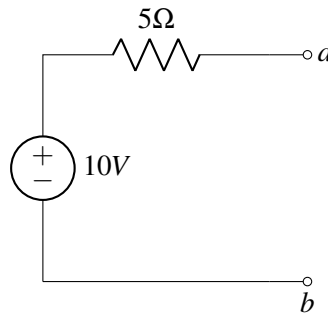
Next, to find R_{eq} , we short out the voltage source, which gives us the following circuit:



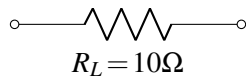
Note that the resistors on the left are shorted from the perspective of nodes a and b . The resistors on the right are in parallel from the perspective of a and b . Therefore $R_{eq} = 5\Omega || 20\Omega = 4\Omega$. Thus we have that $R_{th} = 4\Omega$, $V_{th} = 8V$.

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(c) (7 points) Suppose that Ryan finds a different circuit that has the following Thévenin equivalent circuit:



Ryan wants to light up a lightbulb with load $R_L = 10\Omega$. However, the light bulb is very sensitive and needs **exactly** $5V$ across it in order to function. After searching around, he finds a variable resistor whose resistance, R_{extra} , can be adjusted to any resistance.



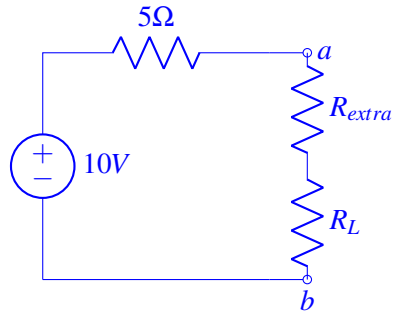
Design a circuit using R_L , R_{extra} , and the circuit above so that the voltage across R_L is $5V$. Specify what the value of R_{extra} is if you decide to use it. **Draw the circuit you design in the space below. Justify your answer.**

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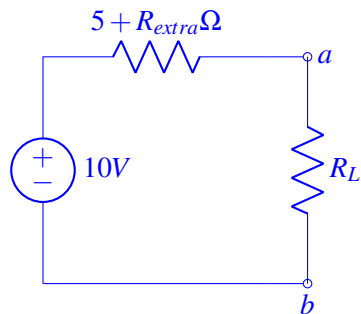
Solution: If we directly connect the resistor R_L across nodes a and b , we have a voltage divider that gives us $V_{R_L} = 10V \cdot \left(\frac{10\Omega}{5\Omega+10\Omega}\right) = \frac{20}{3}V$, which is not what we need. Therefore we need to use R_{extra} in some way. There are two main solutions to this. Connecting the resistors in parallel or connecting them in series.

Series:

If connect them in series, we get the following circuit:



Then combining the top 2 resistors, we get:

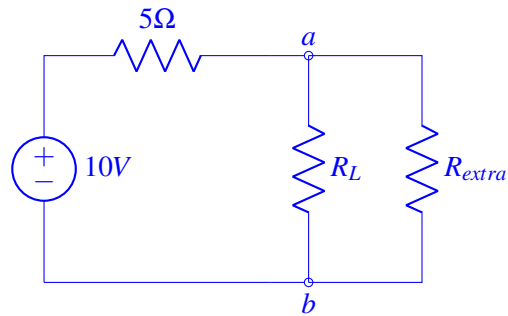


Therefore this is a voltage divider with $V_{R_L} = 10V \cdot \left(\frac{10}{5+10+R_{extra}}\right)$. Since we want $V_{R_L} = 5V$, we get that:

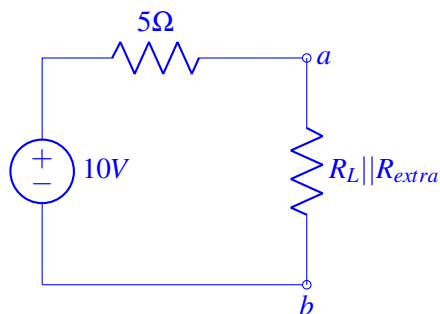
$$\begin{aligned} V_{R_L} &= 10V \cdot \left(\frac{10}{5+10+R_{extra}}\right) \\ 5 &= \frac{100}{15+R_{extra}} \\ 75+5R_{extra} &= 100 \\ R_{extra} &= 5\Omega \end{aligned}$$

Parallel:

If connect them in parallel, we get the following circuit:



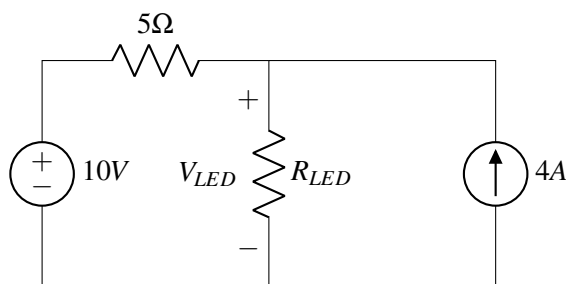
Combining them, we get:



Therefore this is a voltage divider with $V_{ab} = 10V \cdot \left(\frac{R_L || R_{extra}}{5 + R_L || R_{extra}}\right)$. Since $V_{ab} = V_{R_L}$ and we want $V_{R_L} = 5V$, we get that:

$$\begin{aligned}
 5V &= 10V \cdot \frac{R_L || R_{extra}}{5 + R_L || R_{extra}} \\
 25 + \frac{50R_{extra}}{10 + R_{extra}} &= \frac{100R_{extra}}{10 + R_{extra}} \\
 25 &= \frac{50R_{extra}}{10 + R_{extra}} \\
 250 + 25R_{extra} &= 50R_{extra} \\
 R_{extra} &= 10\Omega
 \end{aligned}$$

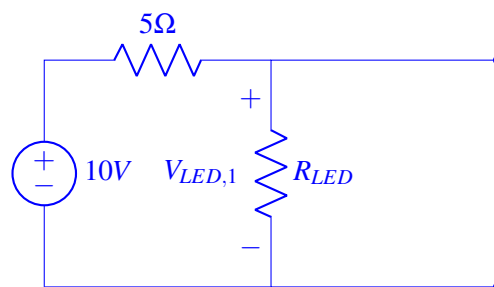
(d) (5 points) Ryan now connects up two circuits he finds to get the following circuit:



If $R_{LED} = 5\Omega$, what is V_{LED} , the voltage across R_{LED} ? Show your work.



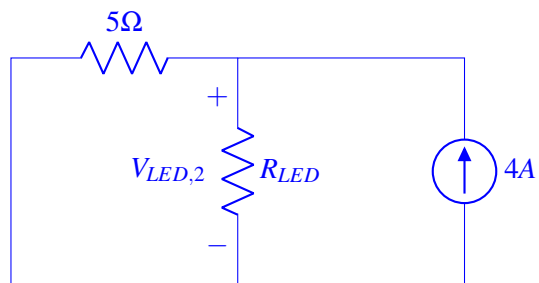
Solution: First, we replace the current source with an open circuit to get:



This is just a voltage divider, so we get:

$$V_{LED,1} = 10 \cdot \frac{R_{LED}}{5 + R_{LED}} = 10 \cdot \frac{1}{2} = 5V$$

Now we short out the voltage source to get:



Combining the parallel resistors, we get total resistance $5 || R_{LED} = \frac{5}{2}\Omega$. Therefore the voltage of $V_{LED,2} = \frac{5}{2}\Omega \cdot 4A = 10V$.

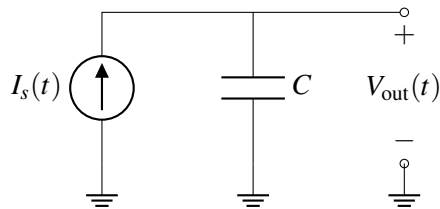
Therefore $V_{LED} = 10V + 5V = 15V$

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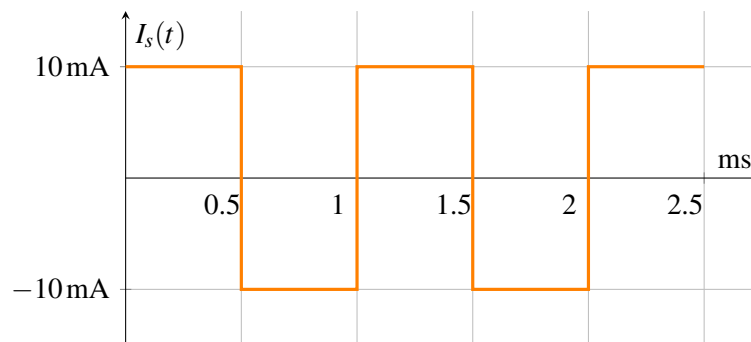
10. London Boy (16 points)

Taylor Swift has hired you to design her personal electric scooter using your new circuits knowledge from EECS16A. You must build a circuit to produce a pulse-width modulation (PWM) signal used to control the motor's speed. *Note: you do not need to know how PWM signals work for this problem.*

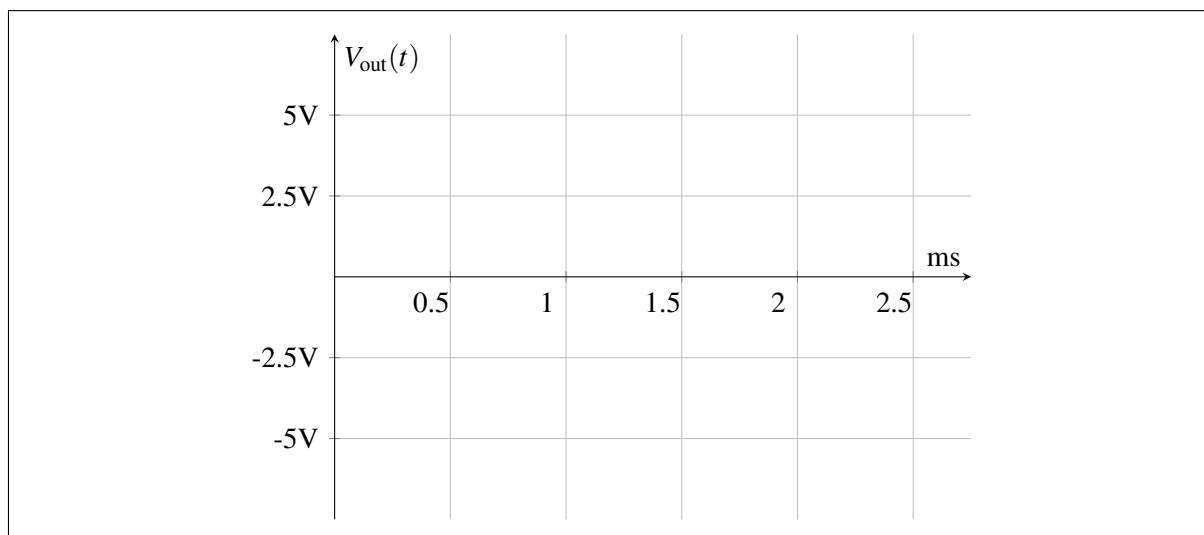
- (a) (5 points) You are given a circuit that contains a time-varying current source $I_s(t)$ connected to a capacitor, with capacitance C , as shown below:



You are told that $C = 1 \mu\text{F}$ and $I_s(t)$ outputs a square wave shown below:

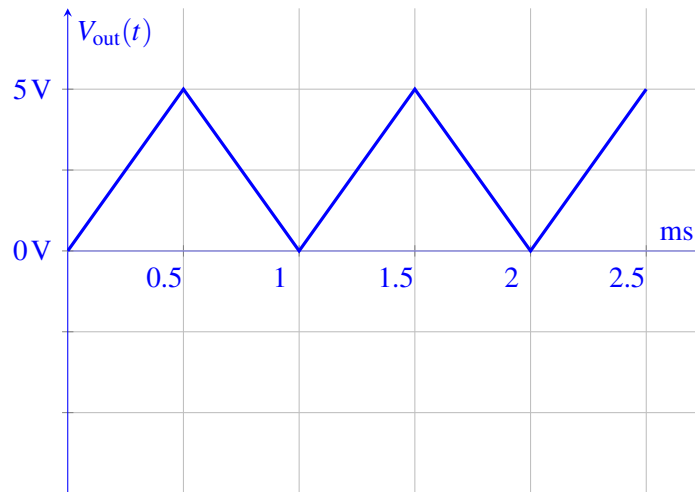


Assuming $V_{\text{out}}(0) = 0\text{V}$, plot $V_{\text{out}}(t)$ from $t = 0\text{ms}$ to $t = 2.5\text{ms}$ in the space provided below. Clearly label the minimum and maximum values.



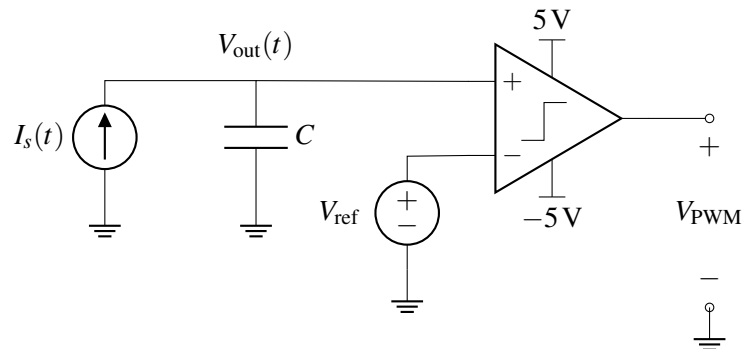
Solution:

$$Q = CV$$
$$I_s = \frac{dq}{dt} = C \frac{dV}{dt}$$
$$\frac{dV_{\text{out}}}{dt} = \frac{I_s}{C} = \frac{\pm 10 \text{ mA}}{1 \mu\text{F}} = \pm 10 \text{ V ms}^{-1}$$
$$V_{\text{out,max}} = 0 + \frac{dV_{\text{out}}}{dt} \Delta t = 10 \text{ V ms}^{-1} (0.5 \text{ ms}) = 5 \text{ V}$$

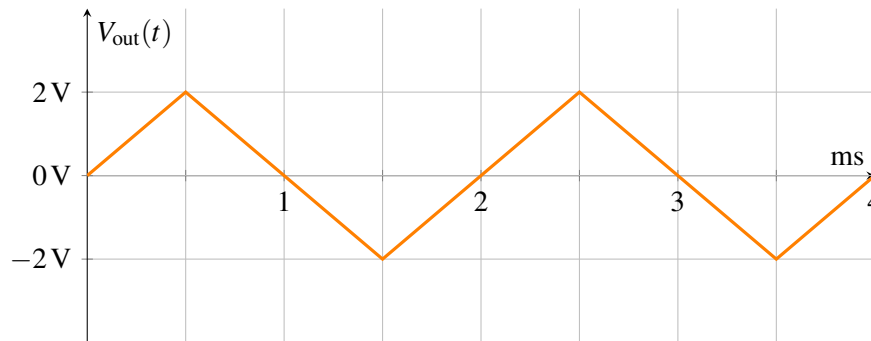


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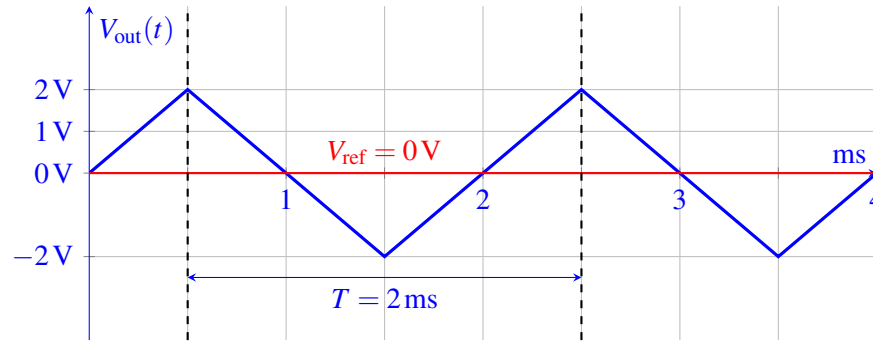
- (b) (4 points) In order to create the desired PWM signal $V_{\text{PWM}}(t)$, you connect $V_{\text{out}}(t)$ to the comparator circuit below.



For this part, assume $V_{\text{out}}(t)$ is the following triangle wave:



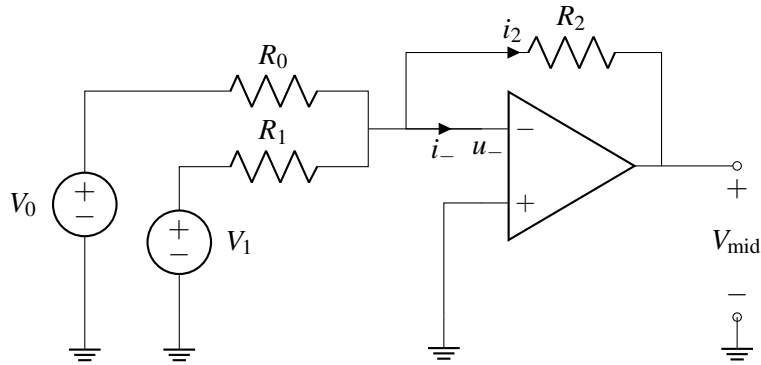
$V_{\text{PWM}}(t)$ will also be a periodic signal that switches between -5 V and $+5\text{ V}$. **Find the value of V_{ref} such that $V_{\text{PWM}}(t) = +5\text{ V}$ for 50% of the time (also called a 50% duty cycle). Justify your answer.**

Solution:

Notice due to the inherent symmetry, the waveform spends half its time greater than zero, and half its time less than zero. Thus, a reference voltage of zero volts yields a 50% duty cycle—i.e. the voltage is above the reference voltage 1/2 the time.

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Your scooter's microcontroller outputs two digital control voltages V_0, V_1 to control your scooter's speed. You come across the op-amp circuit below and think it may be helpful but you are unsure of exactly how it operates. You may assume that $V_{DD} = -V_{SS}$ for this op-amp, and that it is ideal and in negative feedback.



- (c) (2 points) **What is the voltage at the negative terminal of the op-amp, u_- ?**

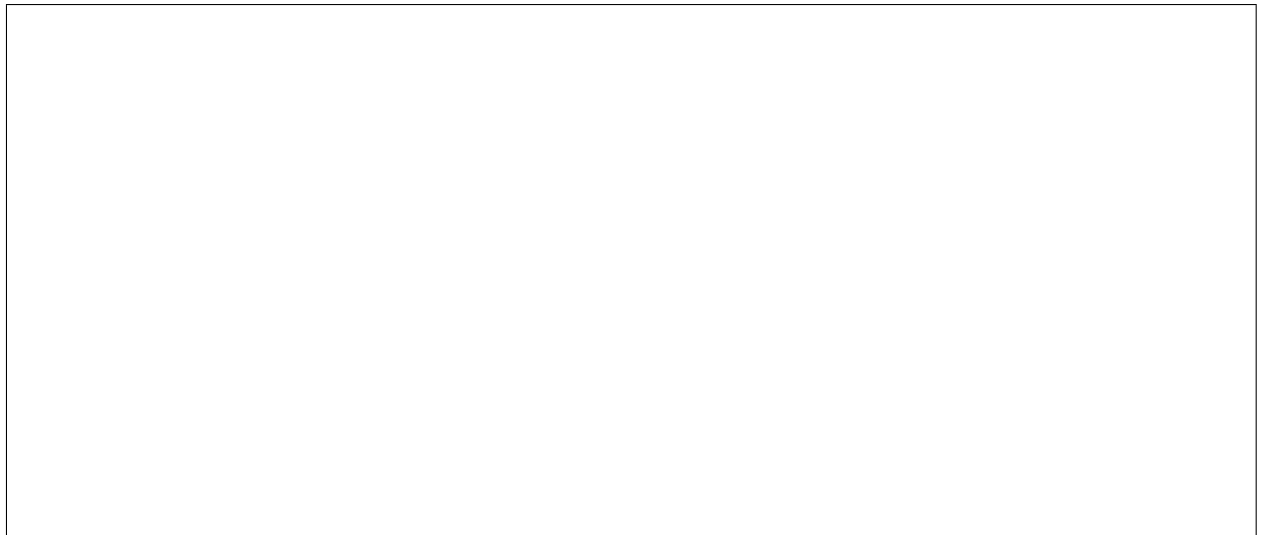
Solution: 0 Volts (from GR2, $u_+ = u_-$ in negative feedback, and the positive terminal is held at ground.)

- (d) (2 points) **What is the current into the negative terminal of the op-amp, i_- ?**

Solution: 0 Amps (from GR1, input current always zero)

- (e) (3 points) **What is the current i_2 through the upper resistor (R_2) in terms of V_0, V_1, R_0, R_1 , and R_2 ?** Note that you may not need to use all of these quantities in your expression. Show your work and justify your answer.

Hint: Use superposition and KCL at the u_- node.



Solution: According to the hint provided, we can write the following KCL equation:

$$i_2 = i_{R_0} + i_{R_1} + i_-$$

However, from part (d), we know that i_- is 0A. Therefore, our KCL equation simplifies to:

$$i_2 = i_{R_0} + i_{R_1}$$

We also know that the voltage at u_- is 0V (c). In other words, the current across each of R_0 and R_1 follows directly from Ohm's Law as:

$$i_{R_0} = V_0/R_0$$

$$i_{R_1} = V_1/R_1$$

Bringing it all together yields:

$$i_2 = \frac{V_0}{R_0} + \frac{V_1}{R_1}$$