EECS 16A   Designing Information Devices and Systems I
Spring 2018
Final Exam

Exam Location: Recreational Sports Facility (RSF)

PRINT your student ID: _____

PRINT AND SIGN your name: _____, _____ _____
                                    (last name)   (first name)   (signature)

PRINT your discussion section and GSI(s) (the one you attend): _____

Name and SID of the person to your left: _____

Name and SID of the person to your right: _____

Name and SID of the person in front of you: _____

Name and SID of the person behind you: _____

1. **Which lab was your favorite lab in EE16A? (1 Point)**
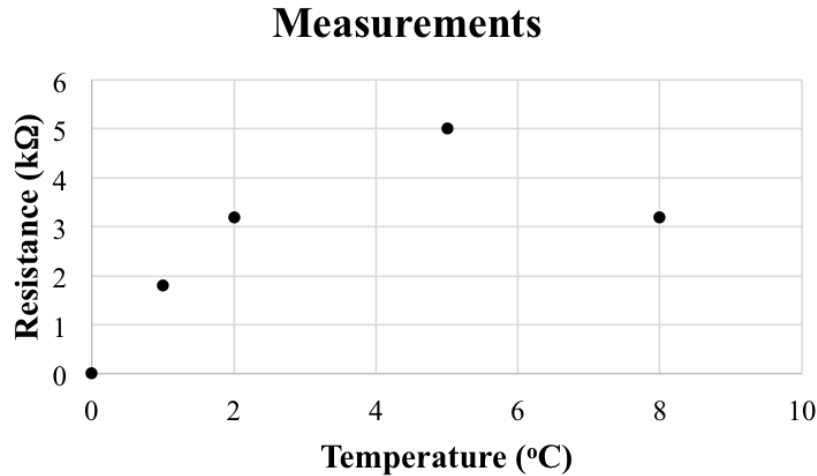
2. **What are your plans for the summer? (1 Point)**

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

3. Temperature-Dependent Resistor (10 points)

You scored a summer research position in Professor Stojanovic's lab, where you are developing a new type of resistor with a resistance that varies as a function of its temperature T, $R = f(T)$. Your task is to determine an approximation for $f(T)$.

You measure the resistance by applying a test voltage and measuring the current. Then you change the temperature and repeat this procedure. Your findings can be summarized by the following scatter plot:

Measurements

Temperature ($°C$)	0	1	2	5	8
Resistance ($k\Omega$)	0.0	1.8	3.2	5.0	3.2

You look through Professor Stojanovic's notes, and find that he believes the temperature function $f(T)$ is a polynomial that can be expressed as $R = aT^3 + bT^2 + cT + d$, where the T values are temperature and R values are resistance.

(a) (4 points) Set up a linear system of equations in matrix form that when solved will give the values of a, b, c, and d. *Do not solve the system of equations.*

(b) (4 points) Write the expression for the least-squares estimator for a, b, c, d. You can define matrices and/or vectors and use them in your solution. *You do not need to compute the values.*

(c) (2 points) You solve the above expression with your handy iPython notebook and get the following values:

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ -0.2 \\ 2 \\ 0 \end{bmatrix}$$

What is the equation of the best-fit polynomial, and what kind of polynomial is it?

○ **Constant**   ○ **Linear**   ○ **Quadratic**   ○ **Cubic**

4. Completely Normal Eigenvectors (20 points)

(a) (6 points) Consider matrix A that has eigenvectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

$$A = \begin{bmatrix} 2.5 & 0.5 & 1.5 \\ 0.5 & 2.5 & -0.5 \\ 0. & 0. & 4. \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Orthonormalize the eigenvectors using Gram-Schmidt to get vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$. Perform the orthonormalization in the order $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

(b) (5 points) Write the vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$ as a linear combination of the eigenvectors. Are any of $\vec{u}_1, \vec{u}_2, \vec{u}_3$ still eigenvectors of the matrix A? Justify your answer.

(c) (3 points) Let $U = \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \vec{u}_3 \end{bmatrix}$. Calculate $U^T \cdot U$.
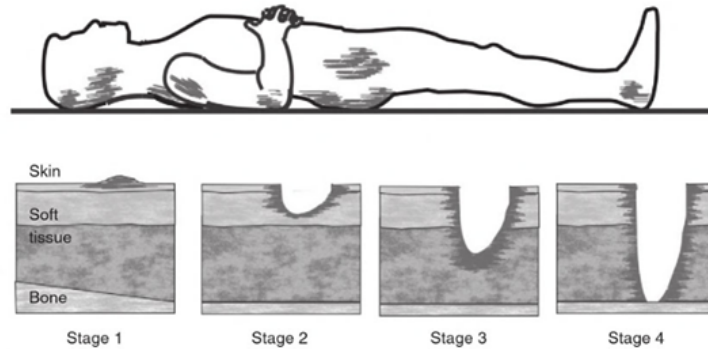
(d) (6 points) Prove that if an arbitrary matrix X has orthogonal eigenvectors then X is symmetric i.e. $X^T = X$. You may assume that X exists in $\mathbb{R}^{n \times n}$ and has n linearly independent eigenvectors.
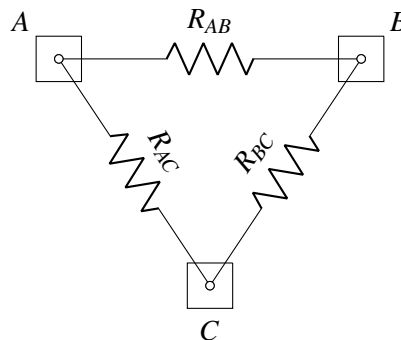
5. **Smart Bandage Saves Lives (40 points)**

In the U.S. alone, there are $60,000$ patient deaths per year from hospital-acquired pressure ulcers. The figure below illustrates the change in patient skin in various stages of ulcer development. Researchers in Professor Ana Arias' group in Cal's EECS department have decided to do something about it. They have developed a method to print electronics and batteries on flexible materials, and would like to get your help in designing a "smart bandage" that can detect skin deterioration and warn the hospital staff to turn the patient and treat the ulcer wounds.
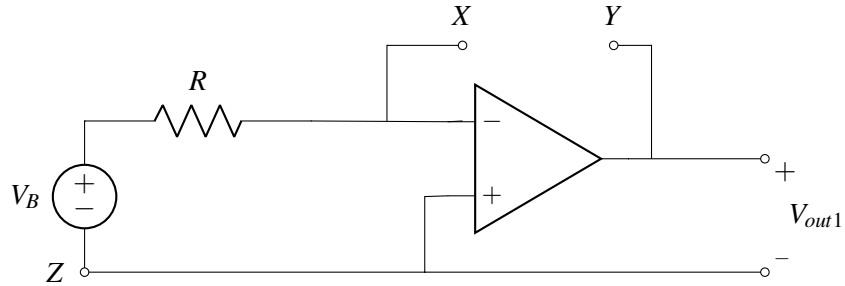

Various stages of the skin health above can be detected by measuring the skin resistance. The circuit below illustrates the skin resistance between three bandage electrodes.

The resistance between the electrodes can vary between 50Ω and $150\ \Omega$, depending on the stage of skin health.

(a) (15 points) One of Prof. Arias' students has found the schematic of the following circuit but needs your help to figure out how to use it to turn the skin resistance measurement to voltage. V_B is the voltage of battery printed on the bandage. The battery also powers up the op-amp with $\pm V_B$.

i. (3 points) You would like V_{out1} to be a function of only R, V_B, and R_{AB} (the resistance between electrodes A and B).
How should you attach the bandage electrodes (A, B or C) to the labeled nodes (X, Y and Z) to achieve this? Each labeled node should be attached to one electrode.

X: _____   Y: _____   Z: _____

ii. (6 points) Find an expression for V_{out1} as a function of circuit components above.

iii. (6 points) Pick the value of R and calculate the maximum current through the skin, such that current through the skin between any two electrodes is less than 100 μA, and V_{out1} fills the range from -1.5 mV to -0.5 mV, depending on the value of skin resistance between the electrodes, as mentioned above. Assume that $V_B = 5$ V.
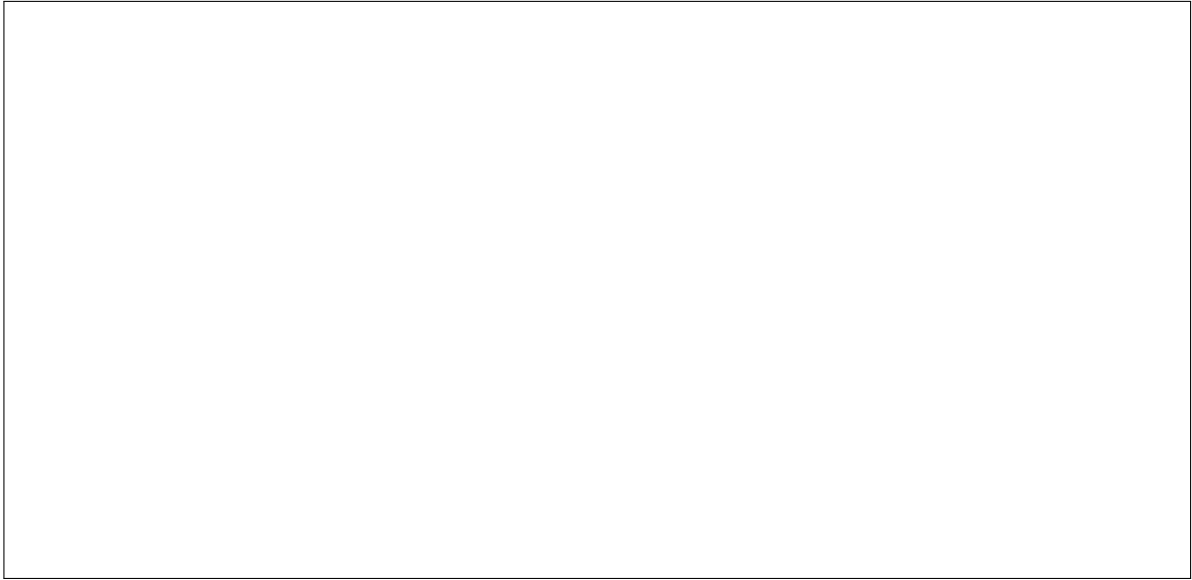
$R =$ _____ $I_{\text{skin, max}} =$ _____

(b) (10 points) Since V_{out1} is a small voltage in the range of -1.5mV to -0.5mV, we need to design an amplifying stage that will amplify V_{out1} into V_{out2} in the range of 500mV to 1.5V. For this you can use

- **one** op amp
- **two** resistors

You do not need to specify power supplies on the op amp.

    i. (5 points) Draw your circuit below, clearly labeling the circuit components and circuit nodes. Derive an expression for V_{out2} as a function of V_{out1} and circuit component values.

    ii. (5 points) To prevent skin damage, you need to pick the resistor values such that the two resistors each dissipate less than $1\mu W$ of power. Modify your resistor values if necessary to ensure that the patient is not burned. Show your calculations.
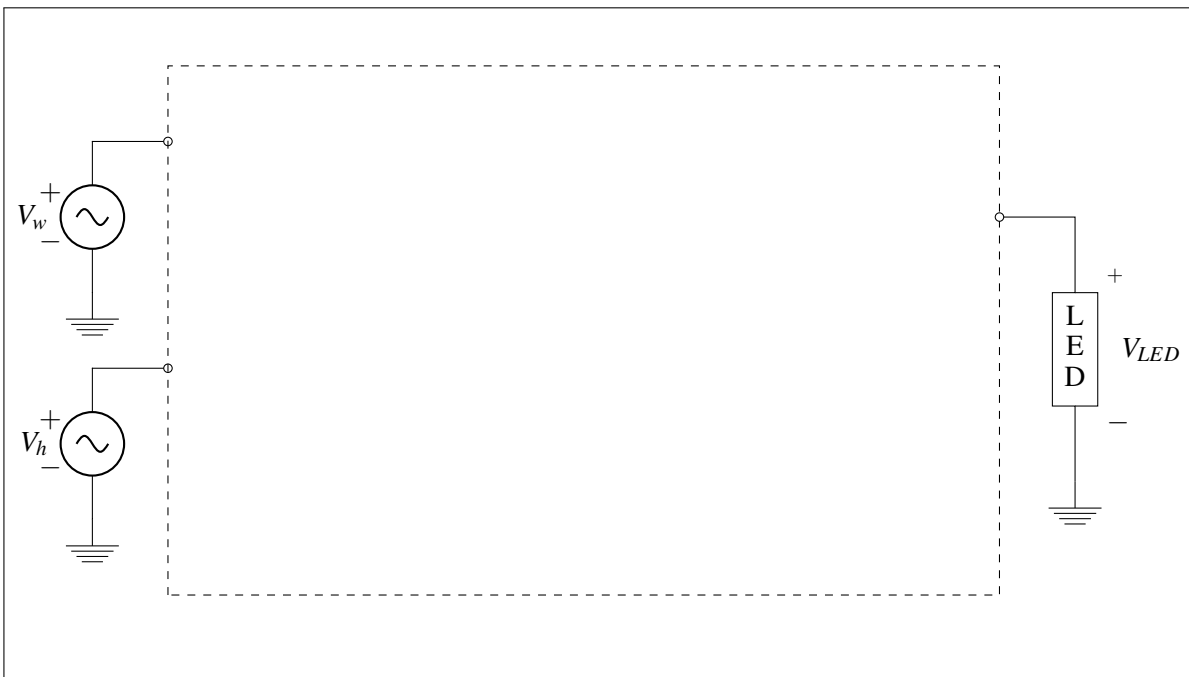
(c) (15 points) Finally, we'd like to make the bandage "smart". Students in Prof. Arias' group have figured out how to print red light emitting diodes (LEDs) on the bandage, too, and would like to turn it on when the monitored skin resistance is larger than that of reference "healthy" skin. In the figure below, V_w represents the output signal from the wound sensor that we designed in Part B, and V_h represents the output signal from the reference sensor placed over the healthy skin.

Design a circuit that provides positive voltage V_{LED} to the LED diode whenever $V_w > V_h$. We'd like to make the LED shine brighter (larger V_{LED}) when the difference between the two voltages is larger. The LED does not shine when $V_{LED} < 0$.

For this part, you can use

- **one** op amp
- **four identical** resistors

i. (7 points) Draw your circuit in the box in the schematic provided below. You do not need to specify power supplies on the op amp.

ii. (8 points) Find the expression for V_{LED} as a function of V_w and V_h.

6. Track Timer (40 points)

Jewanna Befast is an up and coming track star that has asked you to design a smart stopwatch to tell her exactly how fast she runs around the track. As an enthusiastic 16A student, you immediately agree.

Your idea is to have Jewanna start the stopwatch by stepping on a pressure sensor, time her as she runs along the track, and stop the timer when she steps on the pressure sensor again.

(a) (10 points) In order to start and stop your timer, you decide to use a resistive pressure sensor and edge-triggered switch. An edge-triggered switch has the following properties:

- If the switch is initially open, it will close when it sees a voltage change from 0.5 V to 2 V.
- If the switch is initially closed, it will open when it sees a voltage change from 0.5 V to 2 V.
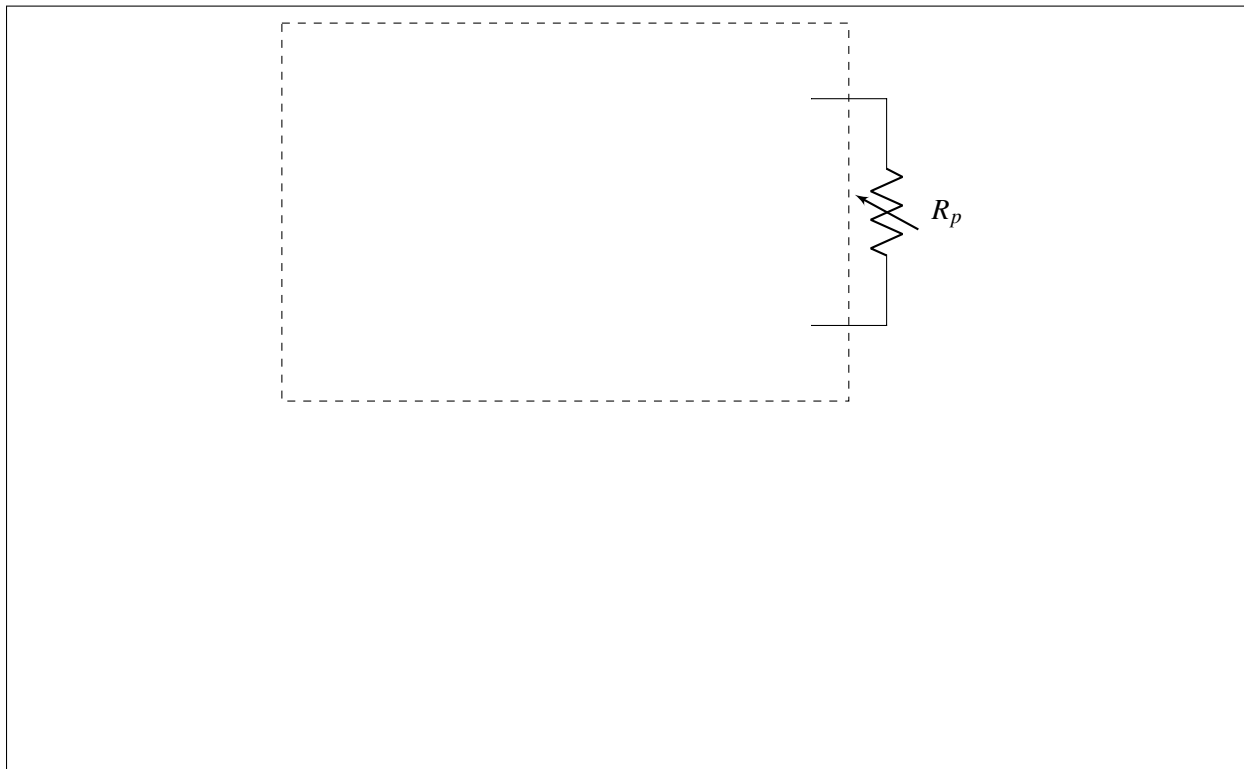
You want to design a circuit to control the switch. When Jewanna steps off the sensor, you want to output a change in voltage from 0.5 V to 2 V to open/close the switch, which will correspondingly start/stop the stopwatch.

Suppose you have read the specs of your pressure sensor and calculated that any time Jewanna steps on the sensor, the resistance is $1\,k\Omega$, and any time she is not on it, the resistance is $10\,k\Omega$. We model the pressure sensor as a variable resistor with resistance R_p, shown below.
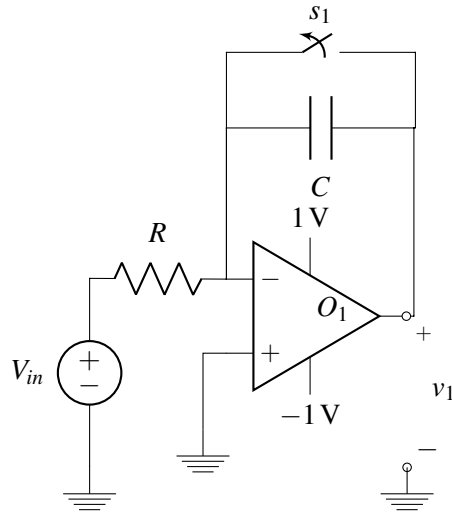
Design a circuit that:

- outputs 0.5 V when Jewanna is on the pressure sensor
- outputs 2 V when Jewanna is not on the sensor

You are only allowed to use resistors and voltage sources. Clearly label where your output voltage, V_{out}, is, and label the values of all resistors and voltage sources.
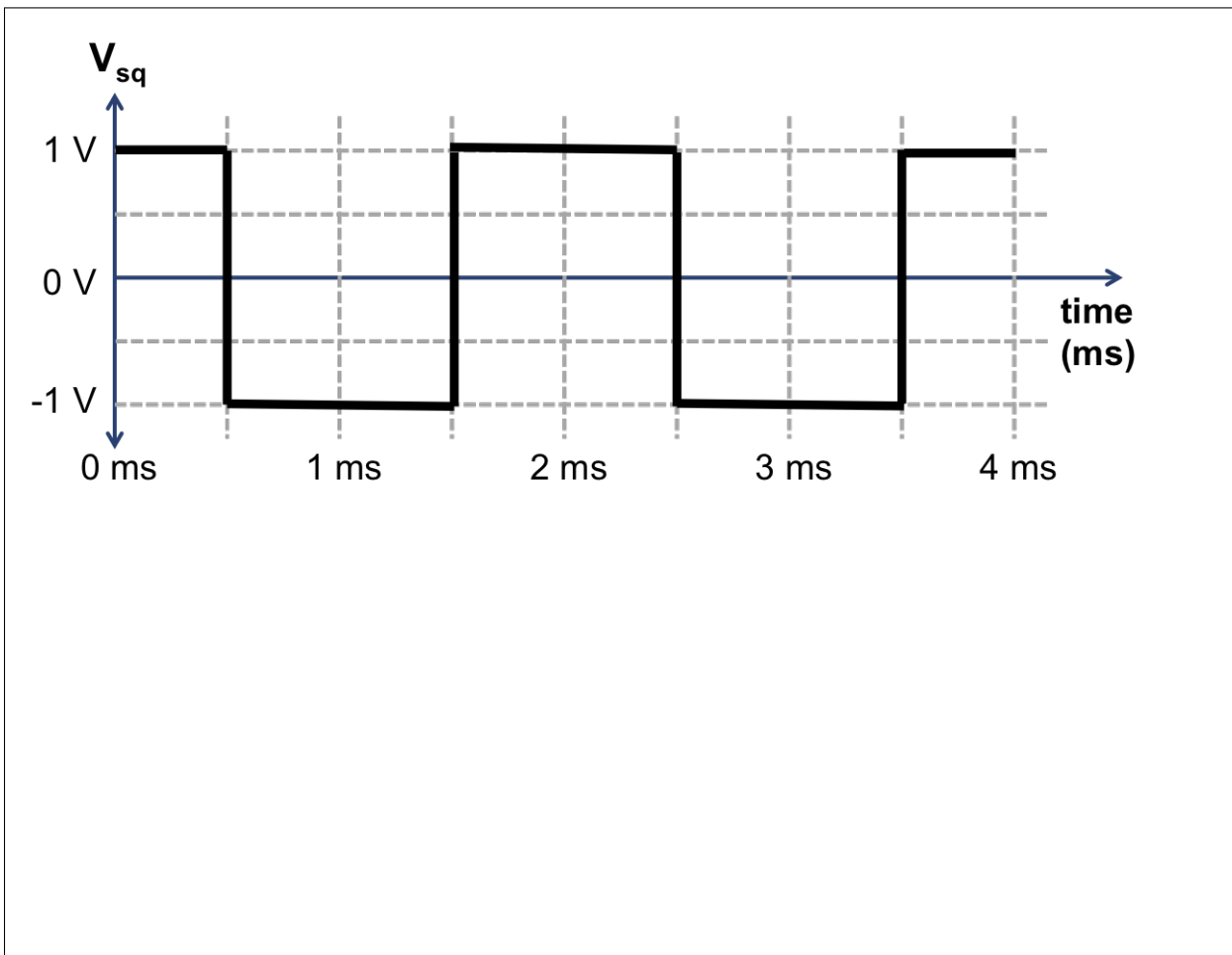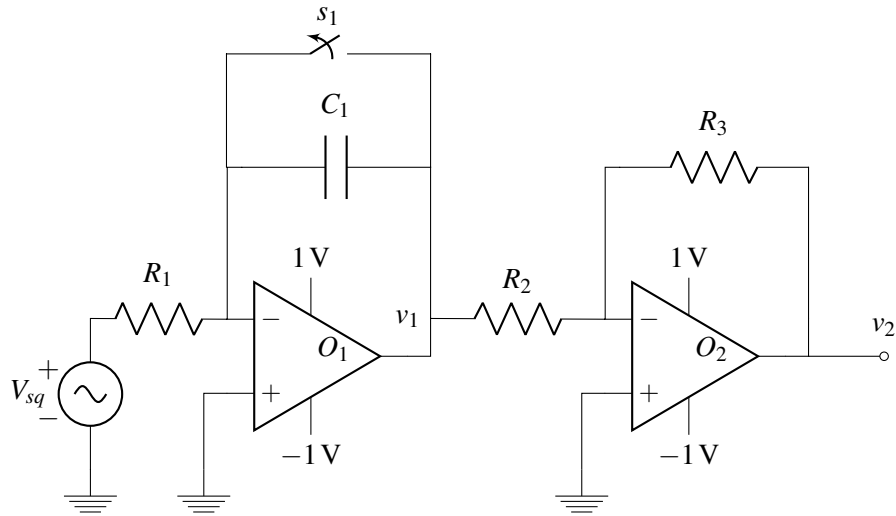
(b) (10 points) You would like to measure how much time it takes for Jewanna to run the track. You start with the RC circuit below where s_1 is the edge-triggered switch from part A. Assume the switch is closed until time $t = 0$, after which it opens because Jewanna starts running.

Find an expression of v_1 with respect to V_{in}, R, C and time t. Assume that the capacitor is discharged at time $t = 0$ and that V_{in} is a constant voltage. You must show all work to receive credit.

s_1

C
$1\,V$

R

O_1
$+$

V_{in}
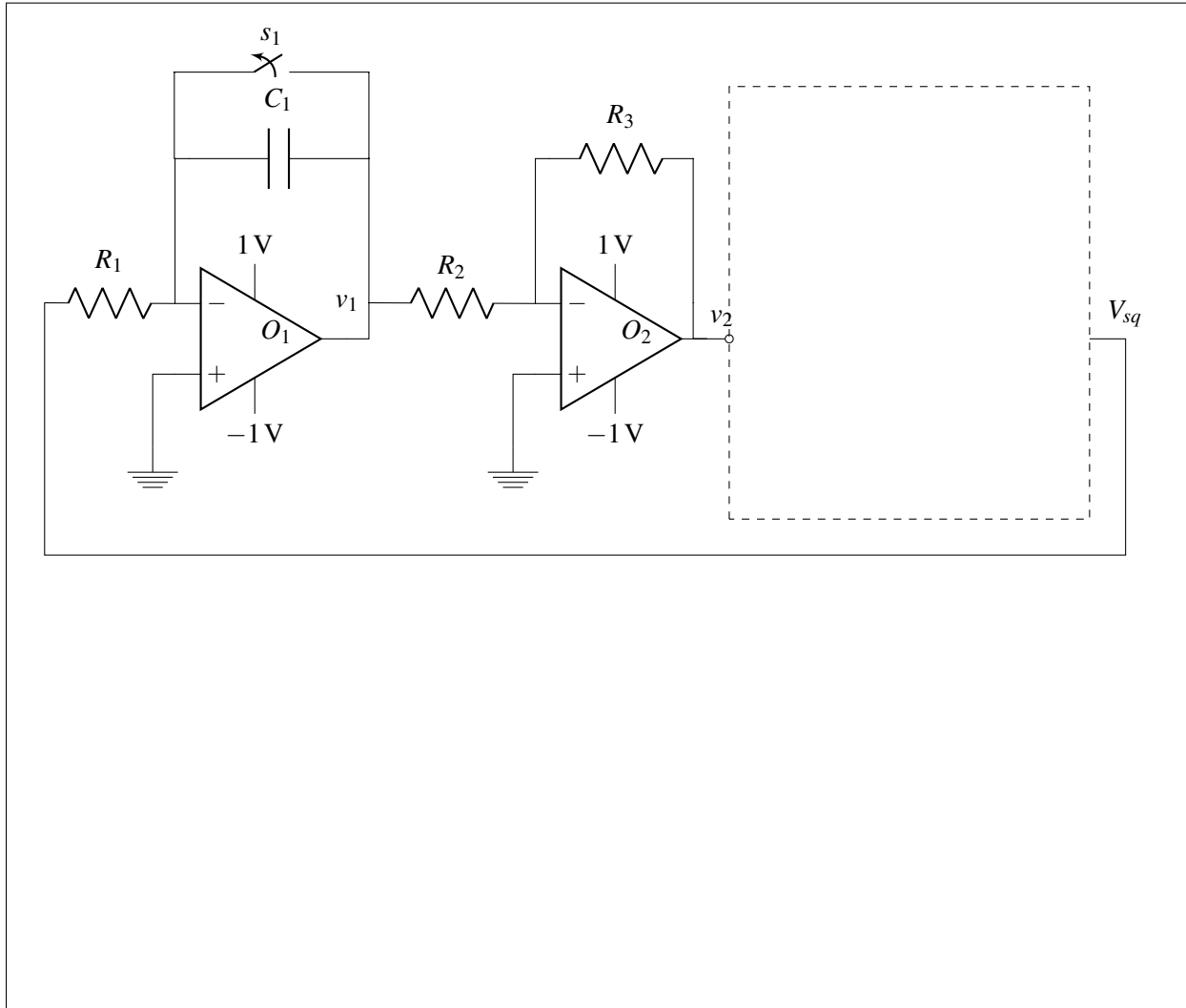$+$
$-$

$-1\,V$

v_1

$-$

(c) (8 points) Now consider the following circuit. The voltage source V_{sq} outputs a square wave, which is plotted below. On the same plot, draw the output voltage v_2 as a function of time. Assume that $C_1 = 1\mu F$ and is initially discharged, and all resistors have resistance of $1k\Omega$. Show your work in the box below.

(d) (10 points) Design a circuit that will convert v_2 into the square wave, V_{sq} shown in the previous part. You can use the following components only:

- **one** op amp (assume $\pm 1V$ power supplies)
- **two identical** resistors

Draw your circuit in the dotted box and show any work below.

(e) (2 points) Suppose you have a device that counts the number of pulses of your square wave (ie. you get one count per period of the square wave). How would you use this information to get the total time it takes for Jewanna to run around the track?

PRINT your student ID: _____

7. Cactus Care (30 points)

On Midterm 2 you designed a light sensor to check that there is sufficient light in your room for your cactus
to be happy and healthy. But you want to monitor the light levels over the course of the day, when you aren't
around. You design a transmitter that sends the following periodic code of length $N = 5$:

$$\vec{c} = \begin{bmatrix} 1 & -3 & 2 & 1 & 2 \end{bmatrix}^T$$

You encode information about the light by multiplying the code with the light intensity (y). With your cell
phone, you receive a shifted version of the code (since it had to travel an unknown distance), multiplied by
the light intensity.

(a) (4 points) Write a matrix A such that
$$A\vec{y} = \vec{r}$$
where $\vec{r}$ is the received signal (length 5) and $\vec{y}$ is a vector of all zeros except one entry which contains
the light intensity y. (*Hint: The position of y in the vector $\vec{y}$ will depend on the unknown shift in the
signal.*)

(b) (6 points) This semester your learned several techniques for solving linear systems of equations. For
each of the following techniques, could you use it to solve the matrix equation from Part A? Justify
your answer in 1-2 sentences. Assume there is no noise.

Gaussian Elimination   ◯ yes   ◯ no
Explain:

Least Squares   ◯ yes   ◯ no
Explain:

Orthogonal Matching Pursuit   ◯ yes   ◯ no
Explain:

You're not sure that your room is really the right place for your cactus, so you set up another light detector in the lab to see if it's better. Each of the two light detectors has a transmitter with a different periodic code c_1, c_2.

$$\vec{c_1} = \begin{bmatrix} 1 & -3 & 2 & 1 & 2 \end{bmatrix}^T$$
$$\vec{c_2} = \begin{bmatrix} 3 & 1 & 2 & -2 & -1 \end{bmatrix}^T$$

As before, the codes are multiplied by the light intensities at each location, y_1 and y_2, and your cell phone receives the sum of shifted codes, each weighted by the light at that location.

(c) (5 points) Write a new matrix A such that

$$A\vec{y} = \vec{r}$$

where $\vec{r}$ is the received signal (length 5) and $\vec{y}$ is a vector of all zeros except two entries which contain y_1 and y_2.

Hint: The positions of y_1 and y_2 in the vector $\vec{y}$ will depend on the unknown shifts in c_1 and c_2, respectively.

(d) (6 points) For each of the following techniques, could you use it to solve the matrix equation from Part D, with two different light sensors? Justify your answer in 1-2 sentences. Assume there is no noise.

Gaussian Elimination   ◯ yes   ◯ no
Explain:

Least Squares   ◯ yes   ◯ no
Explain:

Orthogonal Matching Pursuit   ◯ yes   ◯ no
Explain:

(e) (3 points) In order to judge if your codes are "good", you want to calculate the autocorrelations and cross-correlation of your codes. Professor Waller helps you calculate the following:

$$\text{autocorrelation of } \vec{c_1} = \begin{bmatrix} 19 & -3 & ?? & -2 & -3 \end{bmatrix}^T$$

$$\text{autocorrelation of } \vec{c_2} = \begin{bmatrix} 19 & 0 & -5 & -5 & 0 \end{bmatrix}^T$$

$$\text{cross-correlaton of } \vec{c_1} \text{ with } \vec{c_2} = \begin{bmatrix} 0 & -10 & 12 & 11 & -4 \end{bmatrix}^T$$

Finish the set by calculating the unknown term in the autocorrelation of c_1.

(f) (6 points) Consider the following set of codes (c_3 and c_4).

$$\vec{c_3} = \begin{bmatrix} 1 & -2 & -3 & 2 & 1 \end{bmatrix}^T \qquad\qquad \vec{c_4} = \begin{bmatrix} 1 & 1 & 2 & -2 & -3 \end{bmatrix}^T$$

$$\text{autocorr. of } \vec{c_3} = \begin{bmatrix} 19 & 1 & -10 & -10 & 1 \end{bmatrix}^T$$

$$\text{autocorr. of } \vec{c_4} = \begin{bmatrix} 19 & 2 & -11 & -11 & 2 \end{bmatrix}^T$$

$$\text{cross-correlation of } \vec{c_3} \text{ with } \vec{c_4} = \begin{bmatrix} -14 & -16 & 5 & 18 & -2 \end{bmatrix}^T$$

If you use OMP to solve for the light intensities, which set of codes (c_1,c_2 OR c_3,c_4) is more robust to noise in the received signal? Justify your answer. For the set of codes that is worse, what mistake will is most likely to happen during the OMP algorithm in the presence of noise?

⟳ c_1, c_2 are more robust   ⟳ c_3, c_4 are more robust
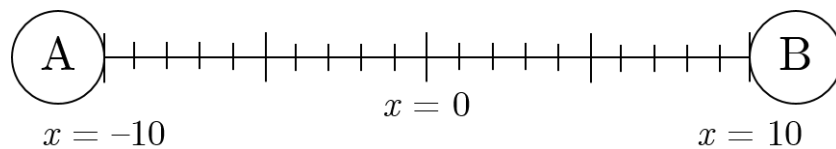
8. APS Lab by Hand (38 points)

In the APS labs, you wrote code for cross-correlation functions that separated raw recorded signals from known beacon signals. These beacon signals were very large–upwards of 10,000 samples long. In this problem, you will walk through a similar process as the APS lab except your beacon signals are much smaller and much much slower.

For the subsequent parts, the beacon signals are comprised of binary numbers. Each element can be treated as a unit of time, t, in seconds with the signals traveling at a velocity $v = 1$ meter per second (m/s) away from beacons located some distance d away from our microphone in meters. Friendly reminder: $v = d/t$.

First, we will look at a 1D system where we are only trying to find our x-coordinate based on signals from two beacons: Beacon A and Beacon B. The signals coming from each beacon, which are **periodic** and have a length of $N = 7$, are represented below:
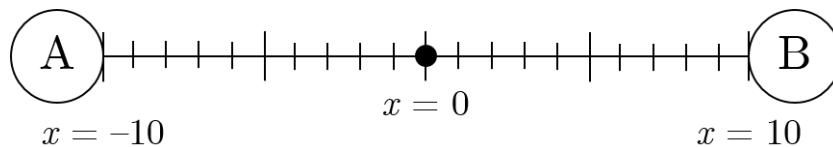
$$\vec{A} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\vec{B} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Our 1D system is only the x-axis, with Beacon A and Beacon B at opposite ends, as shown below:

For all parts of this problem, assume that Beacon A and Beacon B both start sending their signals at the same time, but we don't know exactly what time they started transmitting. Therefore, we can only know *relative* information about our location compared to A and B.

(a) (4 points) What is a **raw signal** ($\overrightarrow{raw}$) the microphone could record at the position $x = 0$? The 1D system with the microphone position at $x = 0$ ($d_A = 10$ m, $d_B = 10$ m) is shown below:
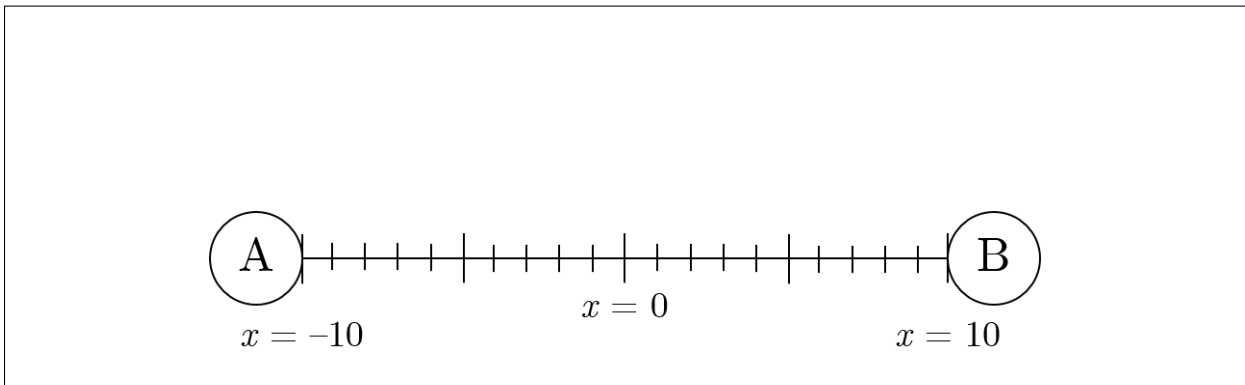
(b) (8 points) Suppose you have a raw signal recorded by the microphone as follows:

$$\overrightarrow{raw} = \begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

What is the smallest time delay between when the signals arrive? That is, what is their smallest time difference of arrival (TDOA)? Reminder, the signals are traveling at 1 m/s over a distance in meters, and each element in a signal array corresponds to some time t related by $v = d/t$.

(c) (2 points) Based on your answer from Part B, where is the microphone located in our 1D system? Please mark the location on the axis below.

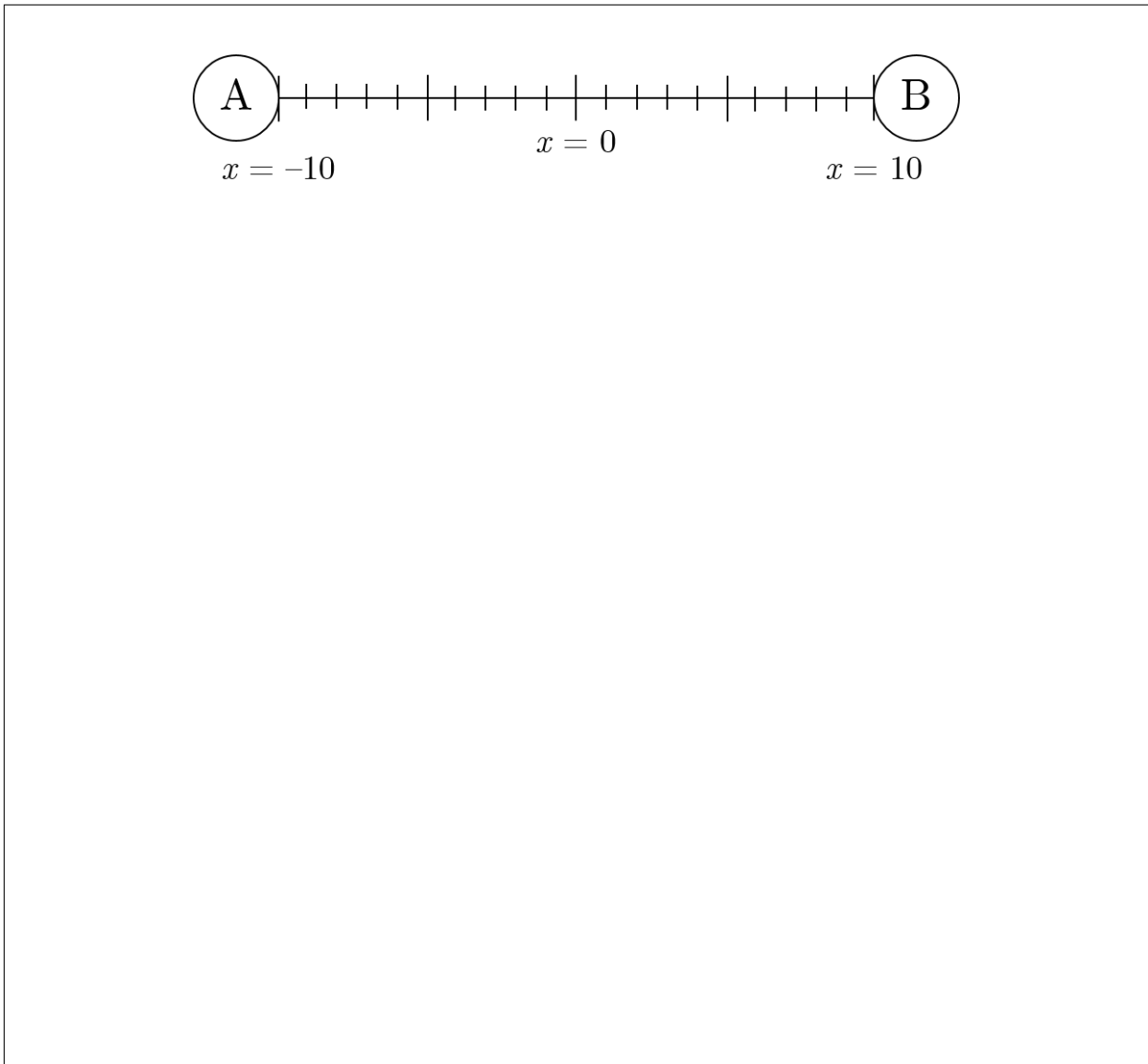*Hint: Don't forget, if you move closer to Beacon A, you're moving farther away from Beacon B.*

(d) (10 points) In part (b), we asked for the smallest time delay, which is implied to be less than the length of our signal, $N = 7$, since our signals are periodic. (If you got a delay > 7 time steps in part (b), you may want to double check your work!) Now, suppose we consider all time delays in which the microphone is located at an **integer number** on the x-axis. Given the same raw signal below:

$$\vec{raw} = \begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

please identify all **integer** solutions of where the microphone is in the 1D system, x_{mic}. Show any relevant work, and **please mark any found solutions on the axis**. Reminder, the signals are traveling at 1 m/s over a distance in meters, and each element in a signal array corresponds to some time t related by $v = d/t$.

Hint: Again, don't forget, if you move closer to Beacon A, you're moving farther away from Beacon B.

(e) (10 points) Uh-oh! It turns out we forgot to look up and are actually in 2D. My bad. Given the same raw signal from part (b):

$$\vec{raw} = \begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

assuming the signals are offset by the smallest time difference (as in part (b)), please write an expression for **all possible solutions** of where the microphone is in the **2D system** in terms of x_{mic}, y_{mic}. Show any relevant work. Some useful terms might be the coordinates of the beacons ($x_A = -10$, $y_A = 0$) and ($x_B = 10$, $y_B = 0$) and the distance of the microphone from the beacons d_A and d_B.

(*Hint 1: the same raw signal means the same delay in signal arrival.*)
(*Hint 2: think about APS 2, specifically.*)
(*Hint 3: as a final sanity check, check if your solution matches the 1D solution when $y_{mic} = 0$.*)
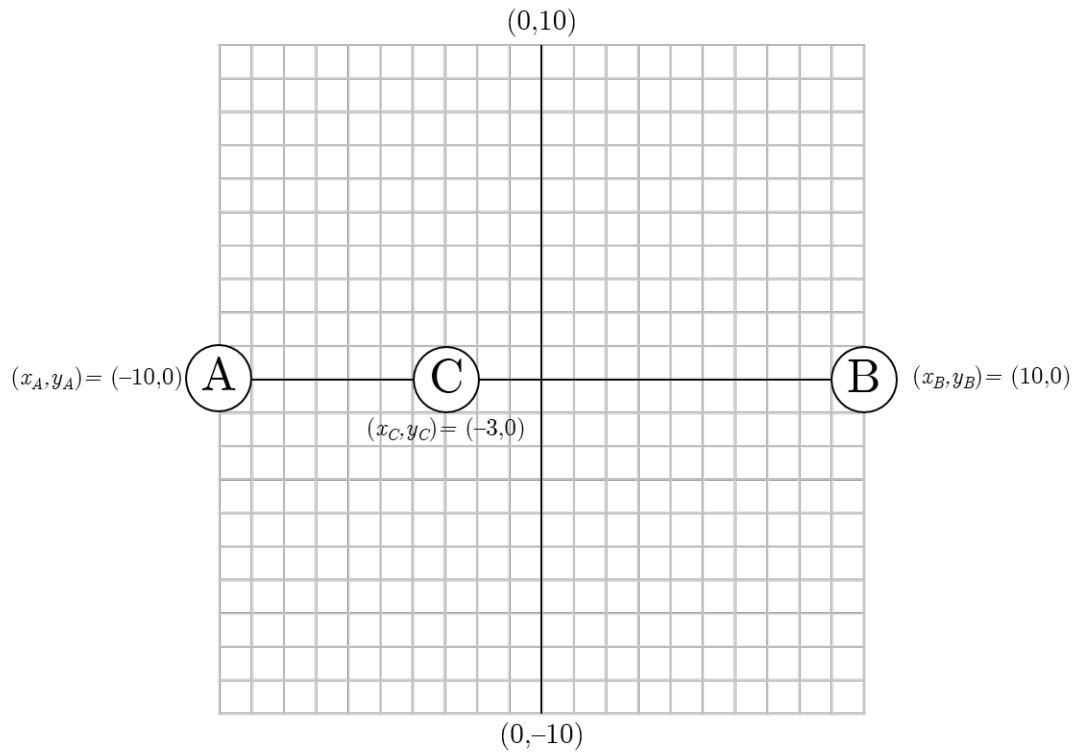
(f) (4 points) To narrow down where we are in the 2D space, Thomas the Lab TA decides to add a third beacon, Beacon C, with its own beacon signal. All of the system's beacon signals and their respective locations are shown below:

$$\vec{A} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\vec{B} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$
$$\vec{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(0,10)

$(x_A, y_A) = (-10,0)$ (A) ——— (C) ——— (B) $(x_B, y_B) = (10,0)$

$(x_C, y_C) = (-3,0)$

(0,−10)

Do you see any issues with the new setup? If so, please note them and explain why they are issues.

Extra page for scratchwork.
Work on this page will NOT be graded.

Doodle page!
Draw us something if you want or give us suggestions, compliments, or complaints.
You can also use this page to report anything suspicious that you might have noticed.

Read the following instructions before the exam.

You have 180 minutes for the exam. There will be 8 problems of varying numbers of points. The problems are of varying difficulty, so pace yourself accordingly and avoid spending too much time on any one question until you have gotten all of the other points you can. Problems may not be in order of difficulty, so make sure you read all of the problems.

There are 180 points possible on this exam. Partial credit will be given for substantial progress on each problem.

Distribution of the points:

Problem	1	2	3	4	5	6	7	8	Total
Points	1	1	10	20	40	40	30	38	180

The exam is printed double-sided. Do not forget the problems on the back sides of the pages!

There are 26 pages on the exam, so there should be 13 sheets of paper in the exam. Notify a proctor immediately if a page is missing. **Do not tear out or remove any of the pages. Do not remove the exam from the exam room.**

Write your student ID on each page before time is called. If a page is found without a student ID, we are not responsible for identifying the student who wrote that page.

You may consult THREE handwritten 8.5" × 11" note sheets (front and back). No phones, calculators, tablets, computers, other electronic devices, or scratch paper are allowed. **No collaboration is allowed, and do not attempt to cheat in any way. Cheating will not be tolerated.**

Please write your answers legibly in the spaces provided on the exam; we will not grade outside a problem's designated space. In general, show all of your work in order to receive full credit.

If you need to use the restrooms during the exam, bring your student ID card, your phone, and your exam to a proctor. You can collect them once you return from the restrooms.

Our advice to you: if you can't solve the problem, state and solve a simpler one that captures at least some of its essence. You might get some partial credit, and more importantly, you will perhaps find yourself on a path to the solution.

Good luck!