EECS 16A Designing Information Devices and Systems I
Spring 2015

Discussion 3W

Vector Spaces

Let $(\mathbb{V},\mathbb{F})$ be a vector space. To qualify as a vector space, the set $\mathbb{V}$ and the operations of addition and multiplication must adhere to a number axioms. Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be arbitrary vectors in $\mathbb{V}$, and a and b scalars in $\mathbb{F}$.

- Associativity of addition: $\mathbf{u}+(\mathbf{v}+\mathbf{w}) = (\mathbf{u}+\mathbf{v})+\mathbf{w}$
- Commutativity of addition: $\mathbf{u}+\mathbf{v} = \mathbf{v}+\mathbf{u}$
- Existence of additive identity: There exists an element $\mathbf{0} \in \mathbb{V}$, called the zero vector, such that $\mathbf{v}+0 = \mathbf{v}$, $\forall \mathbf{v} \in \mathbb{V}$.
- Existence of additive inverse: $\forall \mathbf{v} \in \mathbb{V}$, there exists an element $-\mathbf{v} \in \mathbb{V}$, such that $\mathbf{v}+(-\mathbf{v}) = 0$.
- Compatibility of scalar multiplication with field multiplication: $a(b\mathbf{v}) = (ab)\mathbf{v}$
- Existence of scalar identity: $1\mathbf{v} = \mathbf{v}$, where 1 denotes the multiplicative identity in $\mathbb{F}$.
- Distibution of addition and multiplication:
  - $a(\mathbf{u}+\mathbf{v}) = a\mathbf{u}+a\mathbf{v}$
  - $(a+b)\mathbf{v} = a\mathbf{v}+b\mathbf{v}$

1. Zeros and additive inverses

What is the additive identity of vector space of (3×3) matrices with entries in $\mathbb{R}$? What is the additive inverse of the matrix $\begin{bmatrix} 1 & -2 & 3 \\ 2 & -9 & 0 \\ -1 & 1 & 1 \end{bmatrix}$?

2. Vector Spaces

(a) Consider the set of polynomials of degree n, i.e. $\mathscr{S} = \{p(x) | p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0, a_i \in \mathbb{R}\}$. Is this $(\mathscr{S}, \mathbb{R})$ a vector space?

(b) Consider the set of polynomials of degree n, i.e. $\mathscr{T} = \{p(x) | p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0, a_i \in \mathbb{R}^+\}$. Is this $(\mathscr{T}, \mathbb{R}^+)$ a vector space?

(c) Under the usual matrix operations is the set of matrices defined by $\left\{ \begin{pmatrix} a & 1 \\ b & c \end{pmatrix} \,\middle|\, a, b, c \in \mathbb{R} \right\}$ a vector space?

3. Transformations

You would like to unwrap a matrix

$$A_{\text{matrix}} = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix}$$

to get the vector, $A_{\text{vector}} = \begin{bmatrix} A_{1,1} & A_{1,2} & A_{2,1} & A_{2,2} \end{bmatrix}^T$. Can you construct a matrix (or a set of matrices) that can achieve this?