

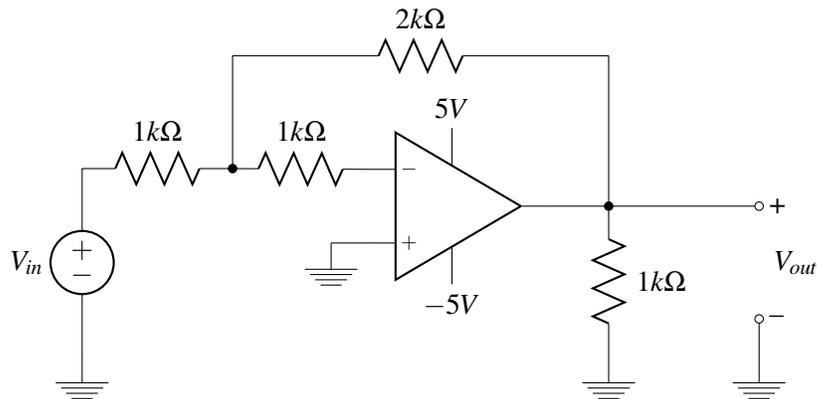
**This homework is due March 19, 2015 at 5PM.**

Note that unless explicitly stated otherwise, you can assume that all op-amps in this homework are ideal (i.e.  $R_{in} = \infty\Omega$ ,  $R_{out} = 0\Omega$ , and  $A = \infty$ ).

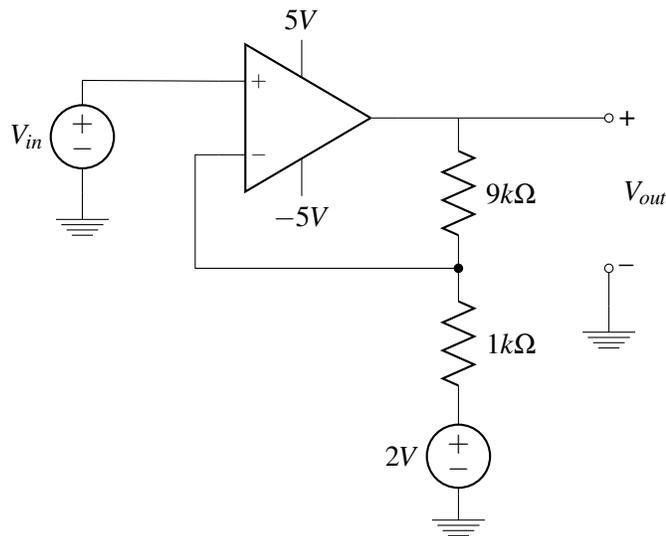
**1. Practice with Negative Feedback Amplifiers**

For each of the circuits shown below, plot  $V_{out}$  for  $V_{in}$  ranging from  $-2V$  to  $+2V$ . Note that understanding what these circuits do and how they work may help you with the design problems in the rest of the homework.

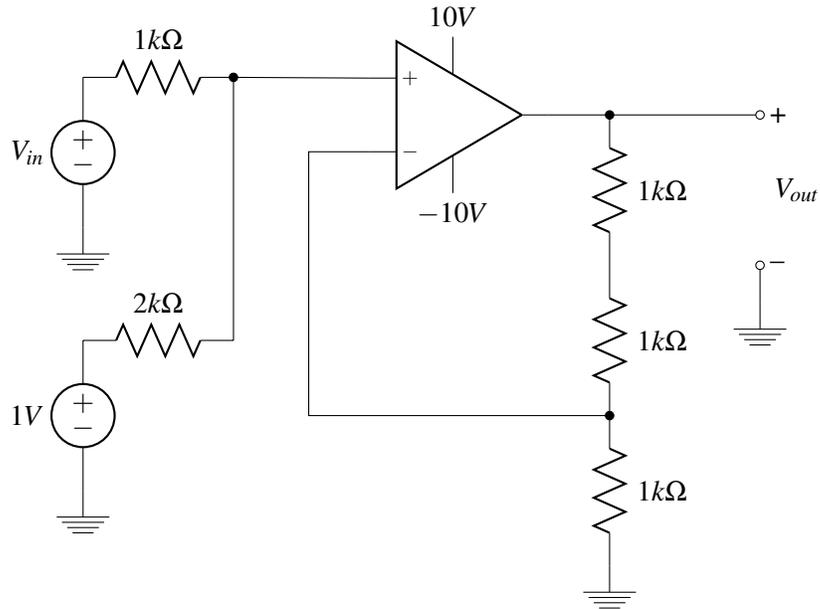
(a)



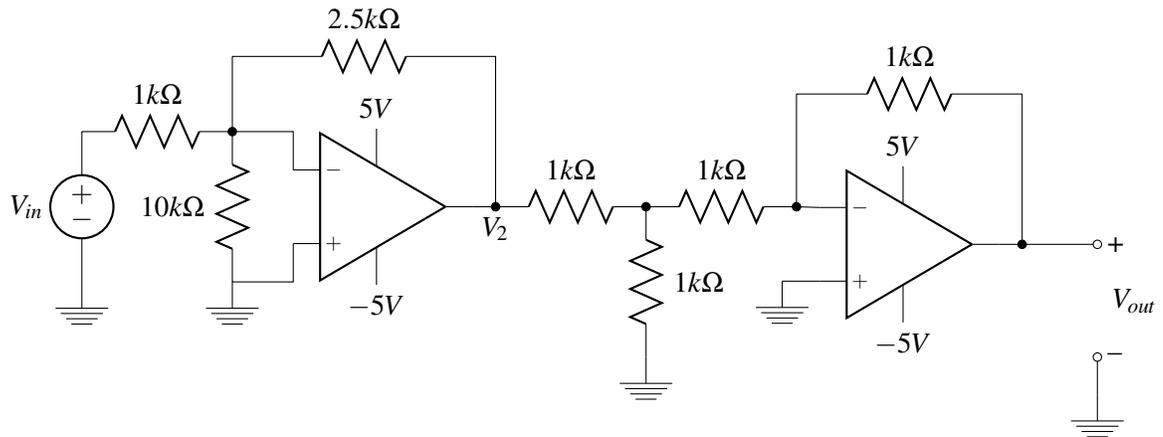
(b) Once you have solved for the behavior of this specific circuit, you should consider what type of function this circuit might be able to implement. In particular, what if the  $2V$  voltage source was not fixed in value, but was actually another input to the circuit?



- (c) Much like in part b), after solving for this specific circuit, you should think about what type of function this circuit might be able to implement. In particular, what if the 1V voltage source was not fixed in value, but was actually another input to the circuit?



- (d)



## 2. IoT4eva Revisited

After guiding them to make an intelligent selection for their super-capacitors, IoT4eva was so happy with your performance that you got a promotion! The good news is that you're getting paid more, but the "bad" news is that you have more responsibilities too. In particular, you are now responsible not only for selecting the super-capacitors used to power the device, but also for building the rest of the circuitry associated with the power supply.

In practice, many real circuits (especially sensors that are trying to detect very small signals) don't like to operate with supply voltages that vary substantially over time. Remembering that the voltage on our super capacitors drops linearly as we pull current out of them, this means that if we want to use these super capacitors for our device, we need to build another circuit. This circuit is powered by the super-capacitor and produces a constant voltage at its output, where this voltage will then be used to supply power to rest

of the device. These circuits are often referred to as "voltage regulators", and in this problem we'll explore how to build the simplest form of such a voltage regulator.

- (a) The first problem we may have had to solve to realize such a voltage regulator is to figure out how to build a reference that would allow us to set the voltage at the output of our regulator to a known absolute value. Fortunately someone else in the company has already built one of those and made it available to you - you can model this circuit as a voltage source whose value is  $0.8V$  with a source resistance of  $1k\Omega$ . (The internals of this voltage reference circuit aren't important for this problem, but as you should see shortly, this circuit by itself is not appropriate for supplying power to the rest of the device.)

Now that we have a reference we can focus on the core of the voltage regulator itself. Using this reference circuit, an op-amp, and resistors, design a circuit that is powered by the super-capacitor voltage  $V_{sc}$  (which for now you can assume is always high enough for the circuit to work) and that would produce a constant  $1.2V$  supply voltage for the rest of the device. Note that you can model the load from the rest of the device as a  $10mA$  current source; please be sure to choose specific values for any resistors you use in your circuit as well.

- (b) Now that we've built the voltage regulator and we know that we want its output voltage to stay fixed at  $1.2V$ , what is the minimum voltage we need on our super capacitors  $V_{sc,min}$  to ensure that the regulator can indeed produce a fixed  $1.2V$  output?
- (c) One of the most important things to evaluate about a voltage regulator is its efficiency - i.e., the power dissipated by the load circuits (in this case, the rest of the IoT4eva device) divided by the total amount of power delivered by the power supply. Continuing to model the rest of the IoT4eva device as a  $10mA$  current source, how much power is dissipated by the  $10mA$  current source? As a function of  $V_{sc}$  (and assuming  $V_{sc}$  is higher than the minimum you found in part b), how much power is actually delivered by the super-capacitor? What is therefore the efficiency of your voltage regulator circuit?

Note that you can assume that the op-amp does not dissipate any power except for what is required to supply the current to its output. (Hint: The op-amp itself can't generate any power, so you should think about where this current would have to originate from.) It is also worth noting that the voltage reference circuit that was given to you would actually dissipate some power from the super-capacitor as well, but you can ignore that for this problem.

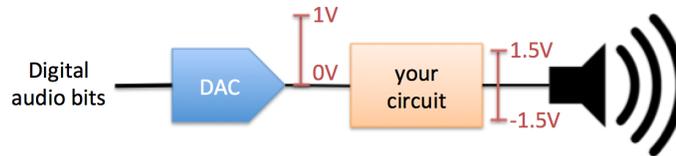
- (d) Still using only op-amps and resistors, is there anything you can do to improve the efficiency of your voltage regulator design?

### 3. Noise Cancelling Headphones

Almost everyone has probably used "noise cancelling" headphones - in some cases without even knowing it. The basic goal of a noise cancelling headphone is for the user to hear only the desired audio signal and not any other sounds that may have been produced by external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources, and then feeds a signal in to your speakers that cancels (subtracts out) that externally-generated sound.

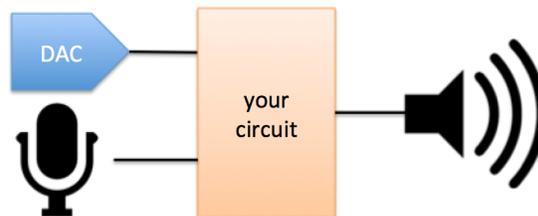
- (a) Let's start by looking at the most basic part of the headphones, which is driving the speaker itself with the audio stream we would like to hear. In our system, the source of the audio comes from a digital-to-analog converter or DAC (a component that converts the digital bits we use to represent our audio stream in the tablet/phone/computer/etc. to analog voltages) that can modeled as a voltage source with a  $50\Omega$  source impedance, and with min/max values of  $0V$  and  $1V$ . The speaker can be modeled as

an  $8\Omega$  resistor, but in order to produce loud enough sounds and not damage the speaker (driving the speaker with non-zero average voltage can damage the transducer within the speaker), it needs to be driven from  $-1.5V$  to  $1.5V$  (relative to the ground connected to the DAC, which is the same ground used throughout the system).



Assuming you are given two voltage sources with values  $-1.5V$  and  $1.5V$  as well as an op-amp and any resistors you would like, design a circuit that could drive the speaker while meeting the specifications above. Hint: You may want to think about how to build a circuit that centers the effective voltage coming out of the DAC at  $0V$  (instead of the  $0.5V$  center point directly produced by the DAC voltage source).

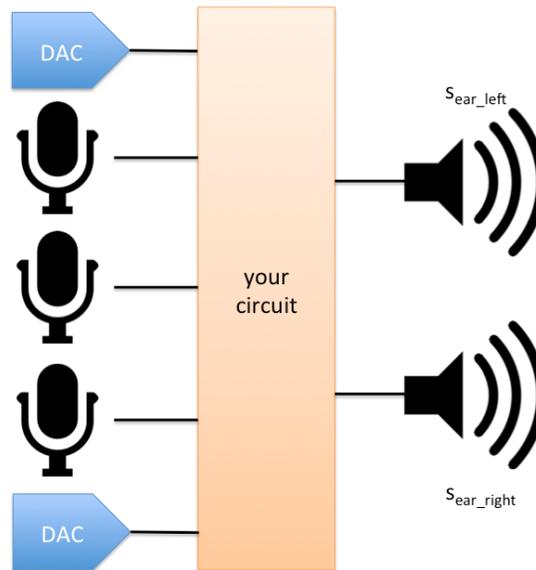
- (b) Now that we know how to drive the speaker, let's look at implementing the noise cancellation. Conceptually, there are two ways we could do this. The first is that if we digitize (using an analog-to-digital converter or ADC) the signal from the microphone, then in software we could subtract it (after appropriate scaling) from the digital version of our desired audio stream before feeding it in to the DAC. This however requires us to have access to the original digital representation of the audio and be able to reprogram the device feeding the DAC, which we may or may not be able to do. In this problem we'll therefore focus on implementing the cancellation with the other method, which is to directly take the (analog) voltage produced by the microphone and subtract it out from the voltage we feed to the speaker.



Let's assume that the microphone can be modeled as voltage source with  $10k\Omega$  source resistance. The loudest sounds we will ever pick up with the microphone make the voltage source (before the source resistance and any voltage drop associated with it) that swings between  $0V$  and  $1V$  (relative to the same DAC ground). Note however that because the materials in the headphones attenuate some of the sound waves coming from outside of it, this loudest signal picked up by the microphone should correspond to a voltage of only  $-125mV$  to  $+125mV$  driven on to the speaker.

Expand the circuit from a) to take the signal from the microphone and subtract it out from the signal that will be driven on to the speaker. You can use op-amps and resistors to do this, but no new voltage sources (except for the model of the microphone of course). Note however that since our speaker driver now needs to handle both the cancellation and the desired audio signal, you can assume that the supply voltages fed to the op-amp have sufficiently large magnitude to ensure that they never clip (reach the power rails). In other words, you can continue to assume that you have  $+/- 1.5V$  voltage sources available to use in the rest of your circuit, but that the op-amps are supplied by a separate set of voltage sources (e.g.,  $+/- 2.5V$ , although you may not need that high of a voltage to make everything work).

- (c) So far we've had just one speaker and one microphone, but almost all headphones today have two speakers (one for each ear - i.e., stereo sound). Adding an extra speaker that can be driven by a separate audio stream typically makes things sound better (more real) to us, and for similar reasons, if we can use that information in the right way, having more than one microphone to pick up ambient sounds from multiple different locations can help us do a better job of cancellation.



Let's now assume that our system has 3 microphones and 2 speakers, and that the source of our audio is stereo - i.e., we have two different audio streams  $s_{left}$  and  $s_{right}$  (produced by two different DACs) that represent the ideal sounds we would like the user to hear in their left and right ear. Let's call the three audio signals picked up by the microphones  $s_{mic1}$ ,  $s_{mic2}$ , and  $s_{mic3}$ , and let's assume that without any active noise cancellation, some fraction of the signal picked up by each microphone would be heard by the user in each of their ears. For example,  $a_{1left}$  would represent the fraction of the signal of the signal picked up by microphone 1 that will be heard in the user's left ear,  $a_{2right}$  would represent the fraction of the signal picked up by microphone 2 that will be in the user's right ear, etc.

Still assuming no noise cancellation and assuming that the DAC/driver circuitry is ideal in producing  $s_{left}$  and  $s_{right}$ , write a matrix-vector equation you could use to calculate the audio signals  $s_{ear\_left}$  and  $s_{ear\_right}$  heard by each of the users ears.

- (d) If we define the matrix that related the signals picked up by each of the microphones to the signals heard by each ear as  $A$ , what matrix  $B$  should the active noise cancellation circuitry be aiming to implement in order to ensure that the user doesn't hear any of the sounds picked up by the microphones?
- (e) Using resistors and op-amps and assuming that the microphones can be modeled as voltage sources with a series resistance of  $1k\Omega$  and whose value  $v_{micn}$  is proportional to  $s_{micn}$  (i.e.,  $v_{micn} = k \cdot s_{micn}$ ), design and sketch a circuit that would implement the cancellation matrix  $B$ . Since the circuit is electrical and deals with voltages (not sound waves), you should assume that this circuit has three voltage inputs  $v_{mic1}$ ,  $v_{mic2}$ , and  $v_{mic3}$  (corresponding to the signals picked up by the microphones) and two voltage outputs  $v_{cancel\_left}$  and  $v_{cancel\_right}$  (corresponding to the voltages that will be subtracted from the desired audio streams in order to cancel the externally-produced sounds). Note that in order to simplify the problem, you can assume that all of the  $v_{mic}$  voltages are centered at  $0V$  (relative to the DAC ground).

- (f) **BONUS:** Building upon your solutions to parts b), c), and e), and otherwise making the same assumptions about the relative voltage ranges of  $v_{mic1}$ ,  $v_{mic2}$ , and  $v_{mic3}$  and available supply voltages as we made in part b), sketch the complete circuit you would use to create the stereo audio on the two speakers while cancelling the noise picked up by the three microphones.