
EE16A: Lecture 8
Introduction to Eigenvalues and
Eigenvectors

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Outline

- Computing eigenvalues and eigenvectors
- Examples:
 - Reflection
 - Stretching, squashing
 - Rotation
 - Vertical position of a quadrotor
- NOTE: the attached notes are a VERY rough outline, we'll fill them in during lecture

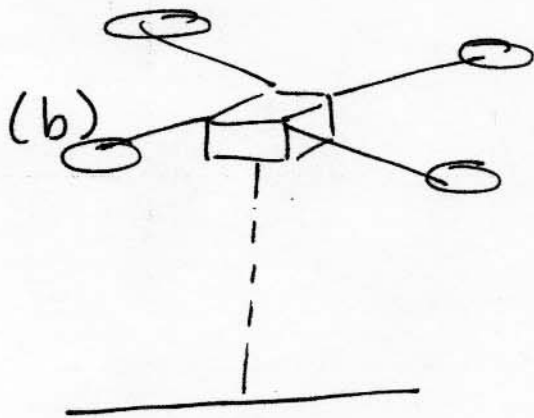
Eigenvalues and eigenvectors of a matrix

Ax : matrix-vector multiplication

$$x(k+1) = Ax(k)$$

examples:

(a) PageRank.



$$x(k) = \begin{bmatrix} \text{vertical position} \\ \text{vertical velocity} \end{bmatrix}$$

(c) performing linear operations on images, like rotate, stretch, shrink, flip...

Computing eigenvalues and eigenvectors:

$$Av = \lambda v$$

$$\Rightarrow Av - \lambda v = 0$$

$$\Rightarrow Av - \lambda I v = 0$$

$$\Rightarrow (A - \lambda I)v = 0$$

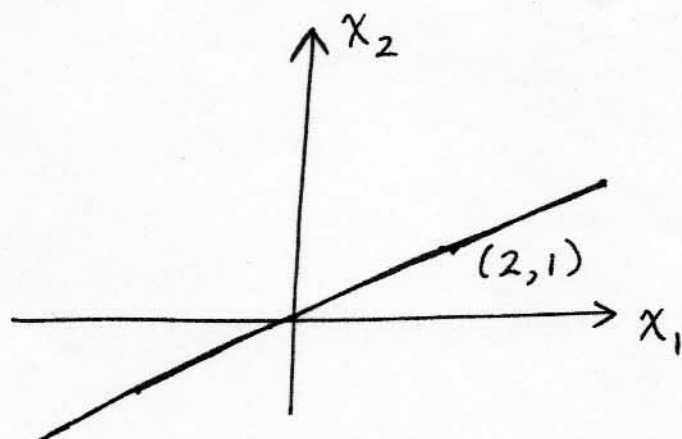
$$\Rightarrow (\lambda I - A)v = 0$$

$$\Rightarrow \det(\lambda I - A) = 0$$

example: $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

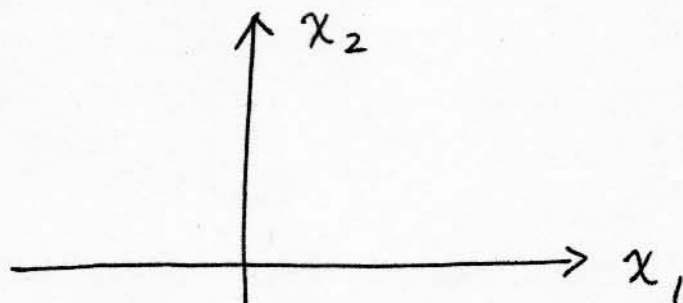
find the eigenvalues } of A.
eigenvectors }

example: reflection across a line



What is A ?

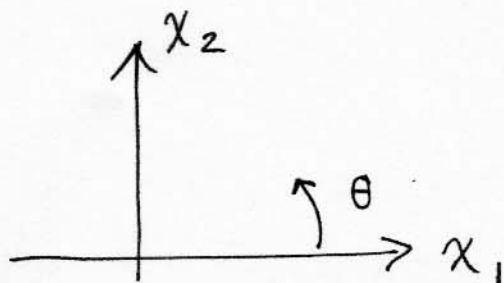
example: stretching, squashing.



x_1 coordinate to be stretched by 2
 x_2 coordinate to be squashed by 2.

What is A ?

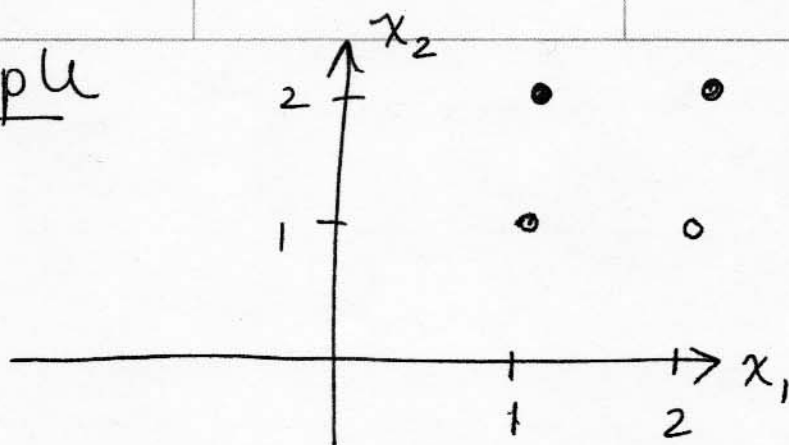
example:



rotate by θ .

What is A ?

example



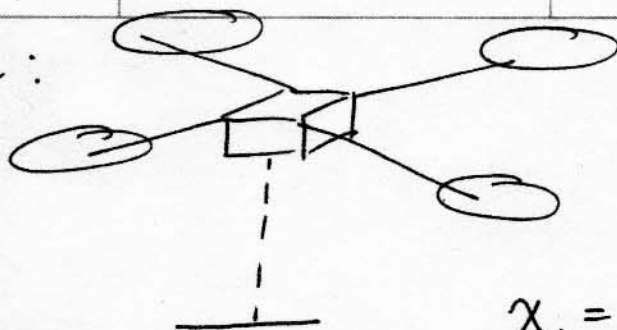
$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{cases} \text{pos.} \\ \text{pixel} \\ \text{value.} \end{cases}$

Consider a rotation by 90° , which doesn't affect pixel value:

$$A = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What are A 's eigenvalues and eigenvectors?

example:



$x_1 = \text{vertical pos}^n$
 $x_2 = \text{vertical vel.}$

(a) x_2 constant: $A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$