EECS 16A Designing Information Devices and Systems I
Spring 2015 Note 20

Lecture notes by Nathan Zhang (04/07/2015)

Multiplexing

When sending sending messages across the airwaves, we use a a combination of a "carrier wave" and our message in order to get the transmission. To do so, we multiply a message $x(t)$ by our carrier $cos(\omega t)$ to get our output $q(t) = x(t)cos(\omega t)$.

Normally, many signals are sent on many carriers to form the received signal, that is, $q(t) = \Sigma x_i(t)cos(\omega_i t)$

Taking a simple case, $N = 2$ with no phase shift, we have $q(t) = cos(\omega_1 t)x_1(t) + cos(\omega_2 t)x_2(t) = \frac{x_1}{2} + 2x_1 cos(2\omega_1 t) + \frac{x_2}{2}cos((\omega_1 + \omega_2)t) + \frac{x_2}{2}cos((\omega_1 - \omega_2)t)$. Using ω_1 and ω_2, we can find a fundamental frequency ω_0 such that $\omega_1 = k_1\omega_0$ and $\omega_2 = k_2\omega_0$, for $k_1, k_2 \in \{0, 1, 2, 3...\}$.

Using the same technique we have several times before. In order to get the messages back out, we take averages to filter out the "noise" that we have. By definition of a continuous average, we have $\bar{q} = \frac{1}{b-a}\int_a^b q(t)dt$. This, however doesn't cancel out the extraneous cos terms (note, we only want x_1 for this example). Conveniently, $\int_a^b cos(t)dt = 0$ when $a - b = 2k\pi$; that is, the cos terms will evaporate if we choose a, b such that $b - a$ covers an integer number of periods. Since $T = \frac{2\pi}{\omega}$, we can find a period length $T_0 = \frac{2\pi}{\omega_0}$ that we can then integrate over. Doing so yields $\bar{q} = \frac{x_1}{2} + \frac{x_1}{2}\frac{1}{T_0}\int_0^{T_0} cos(2\omega_1 t)....dt = \frac{x_1}{2}$ since all terms inside the integral contain some cos term.

Complex variables on the unit circle

Here, we introduce the concept of a phasor $q(t) = e^{it}$. Signals can be represented as the sum of complex exponentials $x(t) = \Sigma x_k e^{i\omega_k t}$.

$q(t)$ is always on the unit circle

Let $q(t)$ be a phasor $q(t) = e^{it}$. It is immediately obvious that $q(0) = 1$. Taking the derivative $\frac{dq}{dt}$, we have $\frac{dq}{dt} = ie^{it} = iq(t)$. If we let $a(t) = Re(q(t)), b(t) = Im(q(t))$, we have $\frac{dq}{dt} = \frac{da(t)}{dt} = i\frac{db(t)}{dt}$. However, we also found earlier that $\frac{dq}{dt} = iq(t) = -b(t) + ia(t)$, so we have $\frac{da(t)}{dt} = -b(t); \frac{db(t)}{dt} = a(t)$. Combining these two together, we arrive at $\frac{da}{dt}a = -\frac{db}{dt}b$, or $\frac{da}{dt}a + \frac{db}{dt}b = \frac{1}{2}\frac{d}{dt}[a^2 + b^2] = 0$. This means that $a^2 + b^2 = C$ for some constant C, and therefore $q(t)$ lies on a circle. Since $q(0) = 1$, we know that $a^2 + b^2 = 1$, so $q(t)$ lies on the unit circle. QED