EECS 16A Designing Information Devices and Systems I
Spring 2015 Note 22

Lecture notes by Edward Wang (04/16/2015).

Amplitude modulation

Babak is whistling. We can define f_B, **Babak's frequency**, as follows:

$$f_B = 3.3 \text{ kHz}$$

Then, we can define the wave representing Babak's whistling as follows:

$$x(t) = B_0 \cos(2\pi(3.3 * 10^3)t)$$

Why not just transmit this wave $x(t)$ directly? Well, the frequency of Babak's voice is in a (relatively) low frequency range called the **baseband**. Unfortunately, the atmosphere is very unforgiving towards baseband frequencies.

Also, there is another problem:

$$\underbrace{c}_{3*10^8} = f_B \underbrace{\lambda_B}_{\text{Wavelength}}$$

Generally, the antenna length needs to be $\leq \frac{\lambda_B}{4}$. This means that the antenna length must be about 100 km! Clearly this will not be feasible.

Solution: map to higher frequencies for shorter wavelengths.

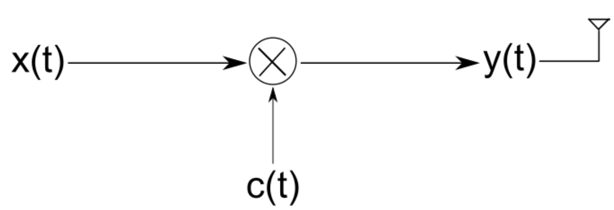

Figure 1: Transmission apparatus.

- $x(t)$ is the information bearing signal.
- $c(t)$ is the carrier signal, at a very high frequency.

Consider once again the previous transmission setup, where:

- $f_B << f_c$
- $x(t) = e^{i2\pi f_B t}$
- $c(t) = e^{i2\pi f_c t}$
- And so $y(t) = x(t)c(t) = e^{i2\pi(f_B+f_c)t}$.

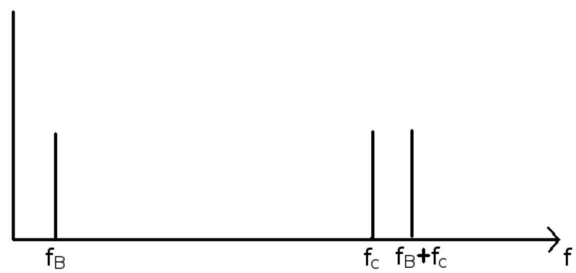

Figure 2: Spectrum of x, c, y.

Notice that multiplication by $c(t)$ has **shifted** the spectrum of $x(t)$ over to the right by f_c Hz. It has **not** stretched the frequency!

In real life, signals (such as a speaking voice) occupy more than just a single pure frequency. Instead, they typically occupy a certain bandwidth, which determines how much frequency real estate is occupied by this signal.

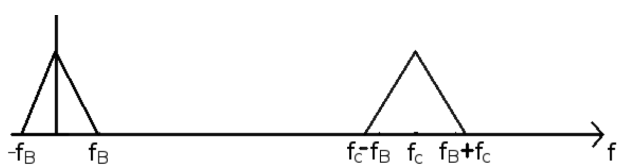

Figure 3: Premium real estate in the frequency domain.

What about the relationship between f_c and f_B?

- Theoretically: $f_c - f_B > 0 \Rightarrow f_c > f_B$.
- Actually: We want f_c to **PWN** f_B!
  (If you did not understand that: call 1-800-GET-LIFE)

Question 1: Why do we have to modulate to get higher frequencies? Could we not directly start with those higher frequencies?

Answer 1: The human voice (fortunately or unfortunately) cannot produce frequencies which are that high. Also, the voice box produces acoustic (longitudinal) waves rather than electromagnetic waves.

Question 2: Why do high frequencies dissipate as they travel through the atmosphere?

Answer 2: Atmospheric physics. The atmosphere acts like a band-pass/high-pass filter. Also, acoustic waves do not move very far due to physical movement of air molecules, but electromagnetic waves move much faster.

Question 3: Does the speed v of the wave matter?

Answer 3: Not in this case. It has more to do with the fact that physically moving air molecules requires energy.

What about cosine modulation - modulating with a real signal instead of an imaginary one? Consider the setup in figure 1, configured as follows:

- $x(t) =$ a signal which occupies bands $-f_B$ to f_B
- $c(t) = \cos(2\pi f_c t)$

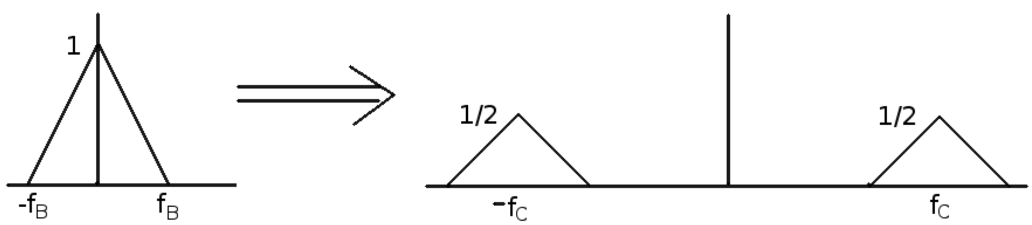

Figure 4: The result of cosine modulation.

Question 1: What is with the negative frequency business?

Answer 1: A negative frequency is a phasor which goes clockwise. To form a cosine wave, two phasors in sync, one going clockwise and the other going counterclockwise, are required to make the cosine wave real. The two phasors' imaginary components always add to zero, leaving only a real cosine wave.

Question 2: Why the $\frac{1}{2}$ factor?

Answer 2: It comes from the $\frac{1}{2}$ factor from expanding cosine using the inverse Euler formula.

Meanwhile, at the receiver headquarters...

Consider the following setup for receiving, assuming that the signal $y(t)$ received is the same signal which was sent at the source. $d(t) = \cos(2\pi f_c t)$.

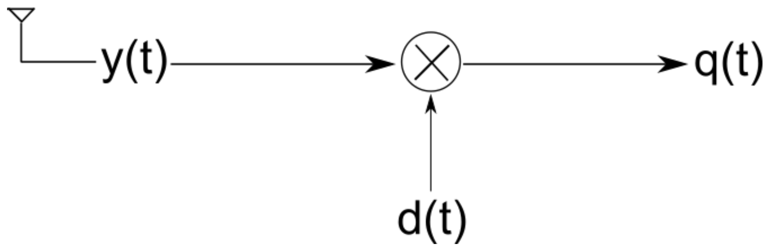

Figure 5: Receiving the signal.

$$
\begin{aligned}
q(t) &= y(t)\cos(2\pi f_c t) \\
&= x(t)\cos(2\pi f_c t)\cos(2\pi f_c t) \\
&= x(t)\cos^2(2\pi f_c t) \\
&= \frac{x(t)}{2} + \frac{x(t)}{2}\cos(2\pi 2 f_c t)
\end{aligned}
$$

Previously: $x(t) = X_0$ (constant, average to recover)

Now: $x(t)$ is not a constant, **but** $f_B << f_c$. Notice that $x(t)$ varies much slower than the carrier wave $c(t)$.

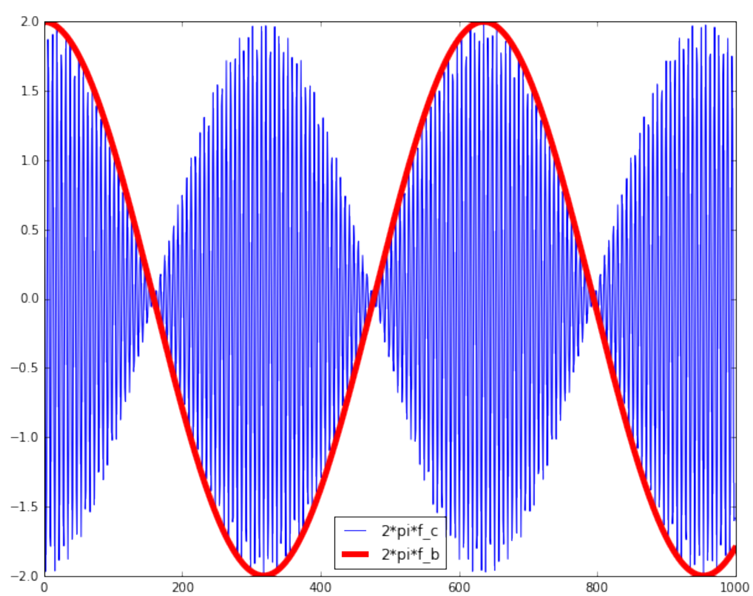

Figure 6: $\cos(2\pi f_B t)\cos(2\pi f_c t)$

Claim: averaging gives a darn good approximation to $x(t)$.

$$q(t) = \underbrace{\frac{\cos(2\pi f_B t)}{2}}_{\text{around 0 (baseband)}} + \underbrace{\cos(2\pi f_B t)\cos(2\pi 2 f_c t)}_{\text{around } 2f_c}$$

Average over $\frac{1}{T_c}$:

$$\begin{aligned} &= \frac{1}{T_c}\int_t^{t+T_c} q(\tau)\,\mathrm{d}\tau \\ &= \frac{\cos(2\pi f_B t)}{2}\frac{T_c}{T_c} + 0 \\ &= \frac{x(t)}{2} \end{aligned}$$

This works because the slowly-varying cosine wave stays about the same while the fast-varying cosine wave completes a period.

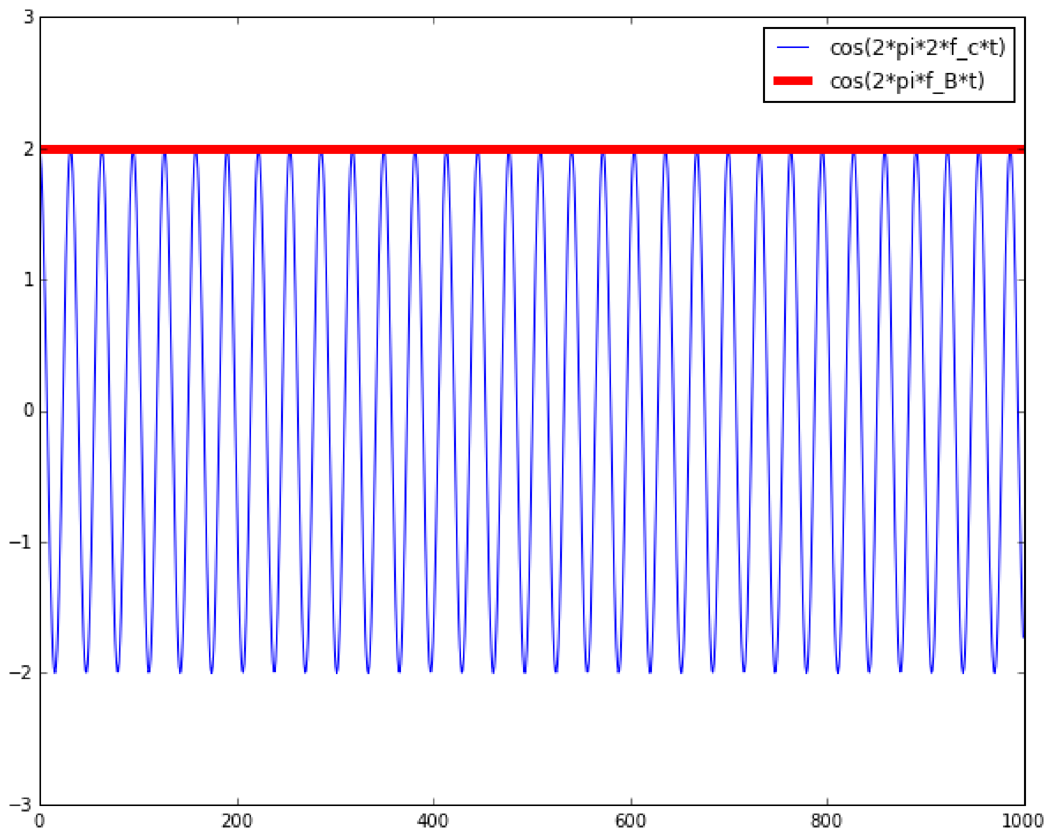

Figure 7: Zoomed in view. The envelope barely moves while the fast-varying cosine completes many periods.

Low-pass filters?

Averaging is a type of **low-pass** filter which smooths out high frequencies.

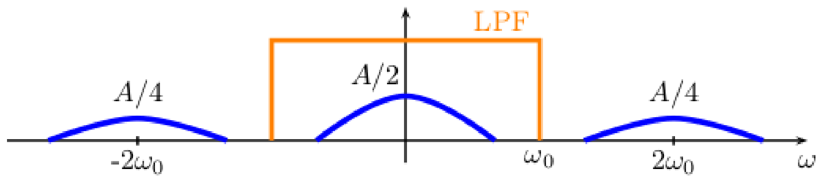

Figure 8: A wild low-pass filter appeared!

Question 1: How are the fast-varying carrier waves with frequency f_c *actually* generated?

Answer 1: There are physical crystals which oscillate and produce waves. But it is not exactly guaranteed that the crystal in the transmitter will oscillate in phase with the receiver. In real life there is a a phase shift as well as a frequency shift as the wave travels.

Assumptions so far

- Received $y(t)$ is the same as the transmitted $y(t)$.
- Receiver and transmitter are in phase and at the exact same frequency.

What if that is not the case?

Consider again the transmitter setup in figure 1 as well as the receiver setup in figure 5.

As before, $c(t) = \cos(2\pi f_c t)$ but this time, $d(t) = \cos(2\pi(f_c + \Delta f)t)$ to account for frequency drift (think of the Doppler effect or red-shifting of distant stars). Of course, $\Delta f << f_B << f_c$.

$$\begin{aligned} q(t) &= x(t)\cos(2\pi f_c t)\cos(2\pi(f_c + \Delta f)t) \\ &= \frac{x(t)}{2}(\underbrace{\cos(2\pi(2f_c + \Delta f)t)}_{\text{High frequency, about } 2f_c} + \underbrace{\cos(2\pi\Delta f)}_{\text{Baseband term}}) \end{aligned}$$

Pass $q(t)$ thrgouh a low-pass filter (such as an average of appropriate window) to obtain $r(t)$ as below:

$$r(t) = \frac{\cos(2\pi\Delta f t)}{2} x(t)$$

Plugging in some test case values: if f_c is a radio frequency of about 1 MHz, then Δf might be 1 Hz (typical drift is ≈ 50 Hz). As a result, we get $r(t) = \cos(2\pi t)x(t)$. Where did the $\frac{1}{2}$ factor go? No, it was not forgotten, but rather omitted for clarity, since it is just a scalar constant, which can always be removed by adjusting the gain of the filter to 2 to eliminate it.

What does this sound like? It is the effect of someone turning up and down the volume knob on the radio twice a second. What to do?

So far our $r(t)$ was obtained by multiplication with $\cos(2\pi(f_c + \Delta f)t)$ followed by a low-pass filter. We can perform the same procedure as outlined in this section to obtain an analogous $r_2(t) = \sin(2\pi t)x(t)$ by multiplying the received signal $y(t)$ by the sine instead of the cosine.

Quick digression: notice that in this scheme we have assumed that $x(t) \geq 0$. But if that is not true, we can still without loss of generality assume that $-\alpha \leq x(t) \leq \alpha$. So we can add α to make $\hat{x}(t) = x(t) + \alpha$ which is indeed always ≥ 0.

Putting it all together

Given the following:

$$\begin{aligned} r(t) &= \cos(2\pi\Delta f\ t)x(t) \\ r_2(t) &= \sin(2\pi\Delta f\ t)x(t) \end{aligned}$$

It should be very straightforward to reconstruct the signal $x(t)$, given $r(t)$ and $r_2(t)$. (Hint: consider the quantity $(r(t))^2 + (r_2(t))^2$!)