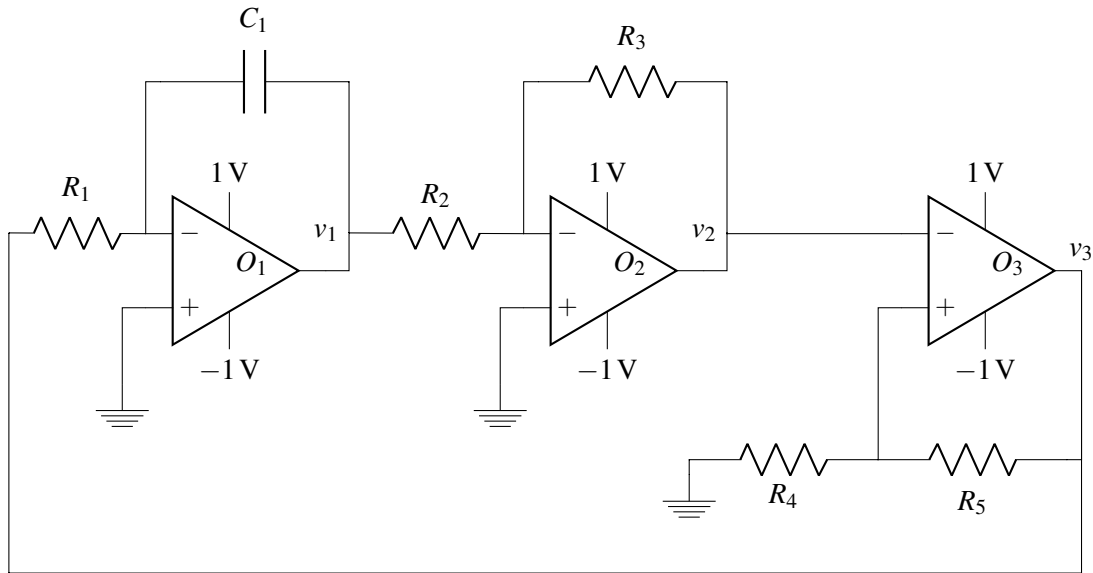


1. Timer Circuit

In this problem, we will walk through the timer circuit, shown below, similar to the one seen in lecture. The circuit is shown below. All resistors have a resistance of  $1\text{ k}\Omega$  and  $C_1 = 1\text{ }\mu\text{F}$ .



- (a) Find the current through the capacitor  $C_1$  in terms of the voltage  $V_3$  and the resistor  $R_1$ .

**Answer:**

For an op-amp, no current flows into the input terminals. Therefore, all the current through  $R_1$  must flow through  $C_1$ . Applying the Golden Rules, we know that  $v_+ = v_- = 0\text{ V}$ .

$$i_{R_1} = i_{C_1} = \frac{v_3}{R_1}$$

- (b) Suppose that at time  $t = 0$ ,  $C_1$  is uncharged. Find the voltage  $v_1$  in terms of  $t$ ,  $v_3$ , and  $R_1$ . What is the maximum  $|v_1|$  could be?

**Answer:**

Recall the voltage across a capacitor is related to the charge on the capacitor, that is  $Q = CV$ . Current is related to charge with the equation  $I = \frac{dQ}{dt}$ .

$$v_{C_1} = \frac{Q}{C_1} = \frac{1}{C_1} It = \frac{v_3}{R_1 C_1} t = \frac{v_3}{1\text{ ms}} t$$

Note that a  $\Omega\text{F}$  is a second. Because the current is flowing into the capacitor, as the voltage across the capacitor increases, the output voltage decreases.

$$v_1 = -v_{C_1} = -\frac{v_3}{1\text{ ms}} t$$

The maximum or minimum for  $v_1$  is the top or bottom supply rail, so either  $+1\text{ V}$  or  $-1\text{ V}$ . Therefore, the maximum  $|v_1| = 1\text{ V}$ .

- (c) How is  $v_2$  related to  $v_1$ ? What is the voltage  $v_2$ ?

**Answer:**

$O_2$  is an inverting amplifier. The output voltage  $v_2$  is equal to  $-v_1$ .

$$v_2 = \frac{v_3}{1\text{ ms}}t$$

$O_3$  is not connected in negative feedback. However, we can analyze its behavior by considering it to be a comparator. Let's independently analyze the circuit in the two possible outputs of the comparator, when  $v_3 = 1\text{ V}$  and when  $v_3 = -1\text{ V}$ .

- (d) Assume that the output of the comparator  $v_3$  has railed to the top rail. With this value of  $v_3$ , what is  $v_2$  as a function of time? What is the voltage at the positive input of  $O_3$ ? At what time will the two inputs of the comparator be equal?

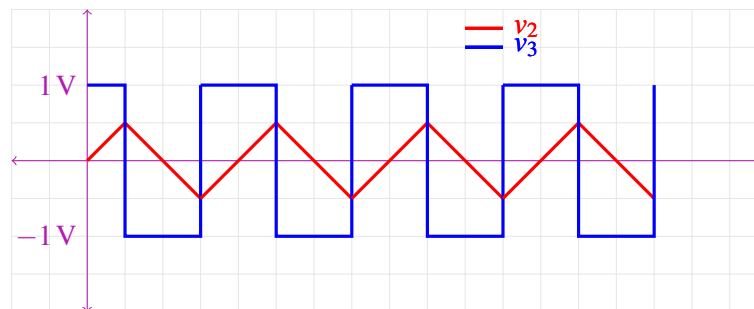
**Answer:**

With  $v_3$  at the top rail,  $v_2$  is  $\frac{t}{1\text{ ms}}\text{ V}$ . The voltage at the positive input of the opamp is  $0.5\text{ V}$  because of  $R_5$  and  $R_4$ . Therefore, when  $t = 0.5\text{ ms}$ ,  $v_2 = 0.5\text{ V}$ .

- (e) Now assume that the reverse occurs, that is, the output of the comparator has railed to the bottom rail. Repeat part (d) with this value of  $v_3$ .
- (f) What is  $v_3$  as a function of time? Draw a graph of  $v_3$  and  $v_2$ . Since the graph is periodic, find its period and frequency.

**Answer:**

Notice that in each of the above cases, once  $v_2$  was equal to  $v_+$ , the output of the comparator would flip. This leads to a periodic function, where  $v_3$  is either  $+1\text{ V}$  or  $-1\text{ V}$ . The period of this function is  $T = 2\text{ ms}$ . Notice that in each of the above cases we analyzed, we always assumed that the capacitor was initially uncharged. However, when  $v_3$  switches, the capacitor will already have some charge built up on it, so it must first be drained. This is why the period is twice what we expect.



- (g) Suppose that we changed the value of  $C_1$  to be  $2\mu\text{F}$ ? What is the new period? Suppose that we change  $R_5$  to be  $2\text{ k}\Omega$ . What is the new period? What if we change  $R_5$  to be  $0\Omega$ ? Will this circuit still operate?

**Answer:**

Notice above we got the constant  $1\text{ ms}$  by multiplying  $R_1$  and  $C_1$  together. If we double  $C_1$ , the effective period would double because it would take longer to charge  $C_1$  to the same voltage with the same current.

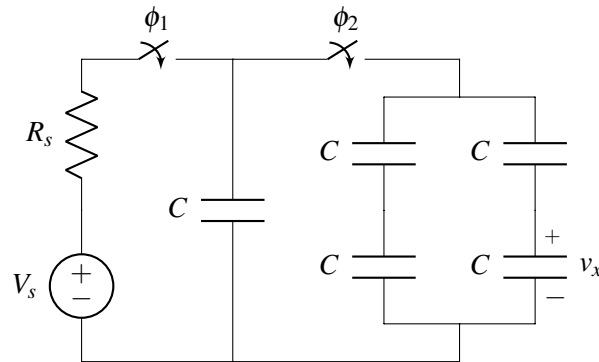
Changing  $R_5$  affects the "flip" threshold because  $v_+$  is at a different voltage. Increasing  $R_5$  decreases the voltage at  $v_+$ , so we would expect the flip voltage to decrease. In fact, the new period is  $\frac{4}{3}\text{ ms}$ .

The circuit would not operate if  $R_5 = 0\Omega$ . The inverting input needs to be able to go above and below the non-inverting input, which is not possible if the non-inverting input is constant at the rail.

**Optional Review Problems:**

**2. Charge-Sharing (Fall 2016 MT2)**

Initially, all capacitors are uncharged and have the same capacitance  $C$ . For  $t < 0$ , the switch  $\phi_1$  is *closed*,  $\phi_2$  is *open*, and the circuit has achieved steady state. Later, at  $t = 0$ ,  $\phi_1$  is *open* and  $\phi_2$  is *closed*; then the system is allowed to reach steady state.



What is the voltage  $v_x$  at  $t \gg 0$ ?

**Answer:**

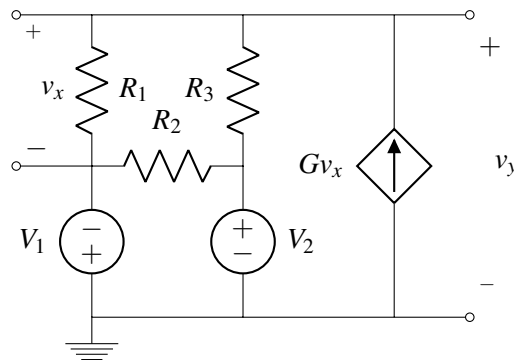
$$v_x = \frac{V_S}{4}$$

Note that  $C_{eq}$  of 2 capacitors of capacitance  $C$  in series is  $\frac{C}{2}$  and in parallel is  $2C$ . Based on the symmetry of the setup, the voltage splits evenly across the capacitor on the left and the capacitor setup on the right. This implies that half of the initial voltage  $V_S$  exists on both sides. On the capacitor network on the right, symmetry indicates a secondary split across each capacitor in series.

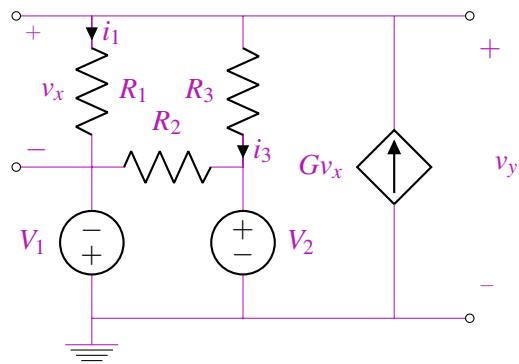
**3. Take Node of the Voltage Sources (Fall 2016 MT2)**

Use nodal analysis to solve for the voltages  $v_x$  and  $v_y$ . Use the following values for numerical calculations. **Note the polarity on the voltage sources.**

$$\begin{aligned} V_1 &= 5\text{ V} & R_1 &= 10\ \Omega \\ V_2 &= 5\text{ V} & R_2 &= 50\ \Omega \\ G &= \frac{1}{4}\text{ S} & R_3 &= 40\ \Omega \end{aligned}$$



**Answer:**



Applying KCL at the node  $v_y$ :

$$\begin{aligned}i_1 + i_3 - Gv_x &= 0 \text{ A} \\ \frac{v_y + 5 \text{ V}}{10 \Omega} + \frac{v_y - 5 \text{ V}}{40 \Omega} - \left(\frac{1}{4} \text{ S}\right)(v_y + 5 \text{ V}) &= 0 \text{ A} \\ 4(v_y + 5 \text{ V}) + (v_y - 5 \text{ V}) - 10(v_y + 5 \text{ V}) &= 0 \text{ V} \\ 4v_y + 20 \text{ V} + v_y - 5 \text{ V} - 10v_y - 50 \text{ V} &= 0 \text{ V} \\ -5v_y - 35 \text{ V} &= 0 \text{ V} \\ v_y &= -7 \text{ V} \\ v_x = v_y + 5 \text{ V} &= -7 \text{ V} + 5 \text{ V} = -2 \text{ V}\end{aligned}$$

Finally, we arrive at:

$$\begin{aligned}v_x &= -2 \text{ V} \\ v_y &= -7 \text{ V}\end{aligned}$$