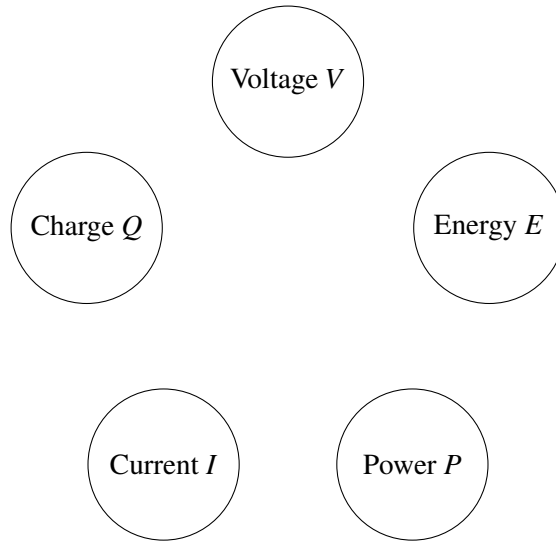


1. Fundamental Quantities in Electronics



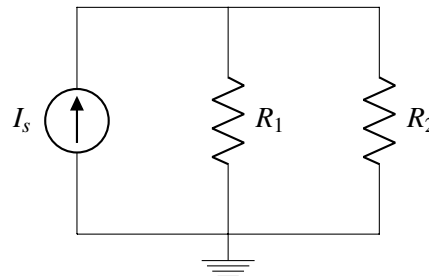
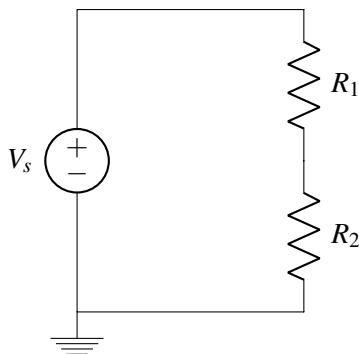
- (a) Write down three things that you associate with each bubble.
- (b) Draw lines between intrinsically connected quantities.
- (c) For each line that you drew, write an equation describing the relationship.

2. Current Divider

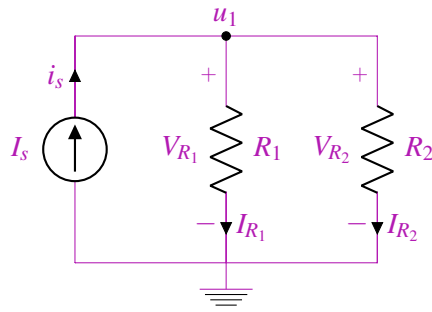
In lecture, we showed that for the circuit shown to the left, we can find the voltage across the resistor R_2 using a simple formula:

$$V_{R_2} = \frac{R_2}{R_1 + R_2} V_s$$

Try to derive a similar formula for the circuit shown below to the right, relating the current I_{R_2} through R_2 to the current through the current source I_s .



Answer:



We will use the circuit labeled as above. For this circuit, there are 3 unknown currents and 1 unknown potential. The IV relations give us the following equations:

$$\begin{aligned} u_1 - 0 - R_1 i_1 &= 0 \\ u_1 - 0 - R_2 i_2 &= 0 \\ i_s &= I_s \end{aligned}$$

KCL gives us one more equation: $i_s - i_1 - i_2 = 0$. From these equations, we set up the matrix equation shown below:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & -R_1 & 0 & 1 \\ 0 & 0 & -R_2 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_s \\ i_1 \\ i_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_s \end{bmatrix}$$

Now, we are looking for an equation for i_2 in terms of I_s , R_1 , and R_2 since these are elements with known values. We can row-reduce the matrix to get a row only in terms of I_s , R_1 , and R_2 .

$$\left[\begin{array}{cccc|c} 0 & -1 & -1 & 0 & -I_s \\ 0 & -R_1 & 0 & 1 & 0 \\ 0 & 0 & -R_2 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array} \right] \sim \left[\begin{array}{cccc|c} 0 & -1 & -1 & 0 & -I_s \\ 0 & -R_1 & 0 & 1 & 0 \\ 0 & R_1 & -R_2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array} \right] \sim \left[\begin{array}{cccc|c} 0 & -1 & -1 & 0 & -I_s \\ 0 & -R_1 & 0 & 1 & 0 \\ 0 & 0 & -R_2 - R_1 & 0 & -I_s R_1 \\ 1 & 0 & 0 & 0 & 0 \end{array} \right]$$

From row 3 of the matrix, we see $(-R_1 - R_2)i_2 = -I_s R_1$ or $i_2 = \frac{R_1}{R_1 + R_2} I_s$.

3. Bio-Molecule Detector

One application for electronics that has gained a lot of attention over the past several years is in so-called “bio-molecule” detection. The idea is to build a system that detects the presence of specific molecules and/or cells (e.g. specific viruses, proteins, etc.) in a biological sample; if this detection can be performed automatically and using relatively low-cost components, it can have a dramatic impact on a number of areas such as medical diagnosis, drug development, DNA sequencing, etc.

In this problem, we’ll look at how some of the techniques we learned about in the touchscreen module can be applied to realize a hypothetical bio-molecule detector. (Real bio-molecule detection systems involve quite a bit more complexity than what we’ll include here, but in many designs the same basic principles apply.)

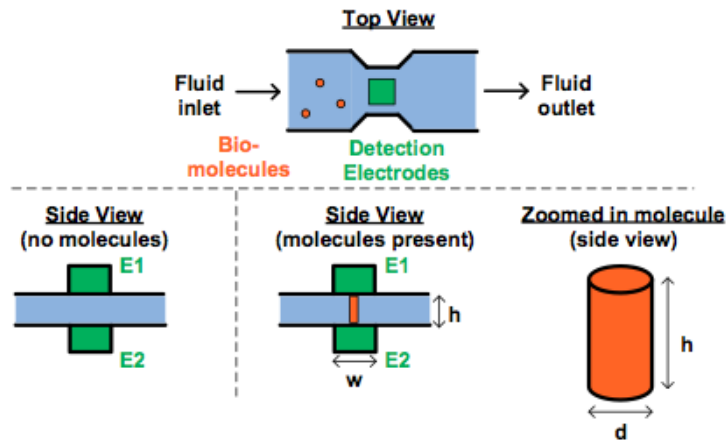


Figure 1: Bio-molecule detector.

As shown in Figure 1, the detector works by flowing a liquid that may or may not contain the biomolecules through a region in the device that has electrodes on the top and bottom of the liquid channel. The electrodes (E1/E2 in Figure 1) are chemically “functionalized” (using e.g. some appropriately designed antibodies), so that if the specific bio-molecule of interest is present in the fluid sample, one or more of the molecules will get physically trapped between the two electrodes (bottom right of Figure 1). After all of the fluid has been cleared out of the device (i.e., so that if there are bio-molecules present, there is only air in between the two electrodes E1/E2), we can then figure out whether or not one or more bio-molecules were trapped by measuring the resistance between the two electrodes.

- (a) Let’s first assume that we want to detect the presence of a bio-molecule by measuring resistance. If no bio-molecule is present, what should be the resistance between E1/E2? As shown in Figure 1, if each bio-molecule is a cylinder with diameter $d = 10\text{ nm}$, height $h = 100\text{ nm}$, and has a resistivity $\rho = 100\ \mu\Omega\text{ m}$, what would be the resistance between E1 and E2 if only a single bio-molecule has been trapped? Note that you can assume that the trapped molecule is exactly vertically oriented when it is trapped – i.e., the top and bottom faces of the molecule are both aligned with surfaces of the electrodes.

Answer:

If no bio-molecule is present between E1/E2, we are left with just air between the electrodes, so for all intents and purposes, the resistance between them should be *infinite*.

If there is a bio-molecule trapped between E1/E2, the resistance R should be:

$$R_{\text{molecule}} = \frac{\rho \cdot l}{A} = \frac{100 \times 10^{-6} \Omega \text{ m} \cdot 100 \times 10^{-9} \text{ m}}{\pi \cdot \left(\frac{1}{2} \cdot 10 \times 10^{-9} \text{ m}\right)^2} = 127.3 \text{ k}\Omega$$

- (b) Using the same numbers for d , h , and ρ as part (a), as a function of the number of trapped bio-molecules $N_{\text{molecules}}$, what is the resistance between E1 and E2? (Note that you can assume that $N_{\text{molecules}}$ is small enough that all of the molecules fit within the electrode area and that all of the molecules are still trapped in an exactly vertical orientation.)

Answer:

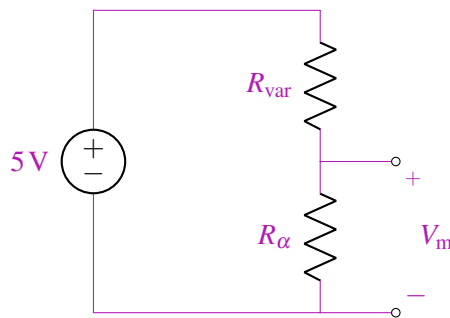
By having many bio-molecules in the detector, the cross-sectional area becomes the sum of the areas of all of the molecules. Therefore, the net resistance between E1/E2 is:

$$R = \frac{127.3 \text{ k}\Omega}{N_{\text{molecules}}}$$

- (c) Given your answers to parts (a) and (b), design a circuit that will output a voltage greater than 2.5 V if more than 5 molecules are trapped.

Answer:

Design idea: use the relationship of the resistors in a voltage divider and the respective drops across them.



We use a voltage divider with a 5 V source and resistors R_{α} and R_{var} in series. R_{var} is the resistance across the molecules, and R_{α} is used to compare against R_{var} . We measure across R_{α} . We want the measured voltage $V_m > 2.5 \text{ V}$ when we have more than 5 molecules, so we want:

$$R_{\alpha} > R_{\text{var}}(6)$$

$$R_{\alpha} > \frac{127.3 \text{ k}\Omega}{6}$$

When we have 5 or less molecules, we want $V_m < 2.5 \text{ V}$ Therefore,

$$R_{\alpha} < R_{\text{var}}(5)$$

$$R_{\alpha} < \frac{127.3 \text{ k}\Omega}{5}$$

We can then set R_{α} to $\frac{127.3 \text{ k}\Omega}{5.5} = 23.15 \text{ k}\Omega$.