EECS 16A Designing Information Devices and Systems I
Spring 2018 Discussion 7A

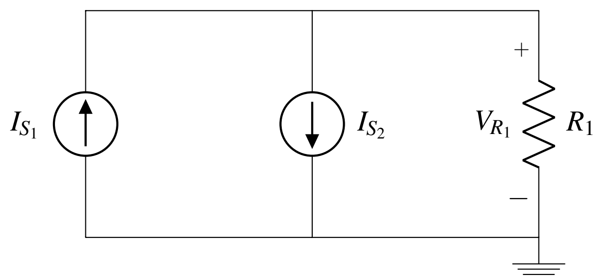
1. Super-power

For the following circuits:

i. Use the superposition theorem to solve for the voltages across the resistors.

ii. For parts (a) and (b) only, find the power dissipated/generated by all components. Is power conserved?

(a)



Answer:

i. While we could apply the algorithm we have learned in class, let's see if there's a way to find the answer quicker than before. We're looking for the voltage across the resistor, which could be found quickly using Ohm's law if we knew the current. If we were to apply KCL at the node at the top of the circuit, one source is coming in, the other source is leaving, and the current through the resistor is leaving. From KCL, we then know $i_{R_1} = I_{S_1} - I_{S_2}$. Applying Ohm's Law we find:

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

We could also solve this using superposition. Turning on I_{S_1} gives $V_R = I_{S_1}R_1$. Turning on I_{S_2} gives $V_R = -I_{S_2}R_2$. Finally, the total V_R is the sum of the individual V_R's or

$$V_{R_1} = (I_{S_1} - I_{S_2})R_1$$

ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = (I_{S_1} - I_{S_2})^2 R_1$$

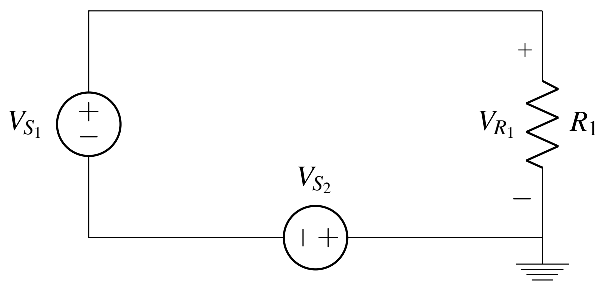
$$P_{I_{S_1}} = -I_{S_1}V_{R_1} = -(I_{S_1} - I_{S_2})I_{S_1}R_1$$

$$P_{I_{S_2}} = I_{S_2}V_{R_1} = (I_{S_1} - I_{S_2})I_{S_2}R_1$$

$$P_{R_1} + P_{I_{S_1}} + P_{I_{S_2}} = (I_{S_1} - I_{S_2})^2 R_1 - (I_{S_1} - I_{S_2})I_{S_1}R_1 + (I_{S_1} - I_{S_2})I_{S_2}R_1 = 0$$

Power is conserved.

(b)



Answer:

i. Once again, we could apply the circuit analysis algorithm or find the answer directly. Notice the circuit only has one loop, so we can use KVL to find the voltage across the resistor.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

We could also solve with superposition. Turning on V_{S_1} gives $V_{R_1} = V_{S_1}$. Turning on V_{S_2} gives $V_R = -V_{S_2}$. The overall voltage is then the sum.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

ii.

$$P_{R_1} = \frac{V_{R_1}^2}{R_1} = \frac{(V_{S_1} - V_{S_2})^2}{R_1}$$

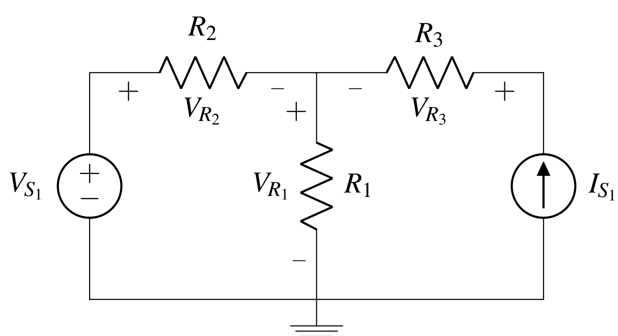
$$P_{V_{S_1}} = -I_{R_1} V_{S_1} = -\frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1}$$

$$P_{V_{S_2}} = I_{R_1} V_{S_2} = \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1}$$

$$P_{R_1} + P_{V_{S_1}} + P_{V_{S_2}} = \frac{(V_{S_1} - V_{S_2})^2}{R_1} - \frac{V_{S_1}(V_{S_1} - V_{S_2})}{R_1} + \frac{V_{S_2}(V_{S_1} - V_{S_2})}{R_1} = 0$$

Power is conserved.

(c)



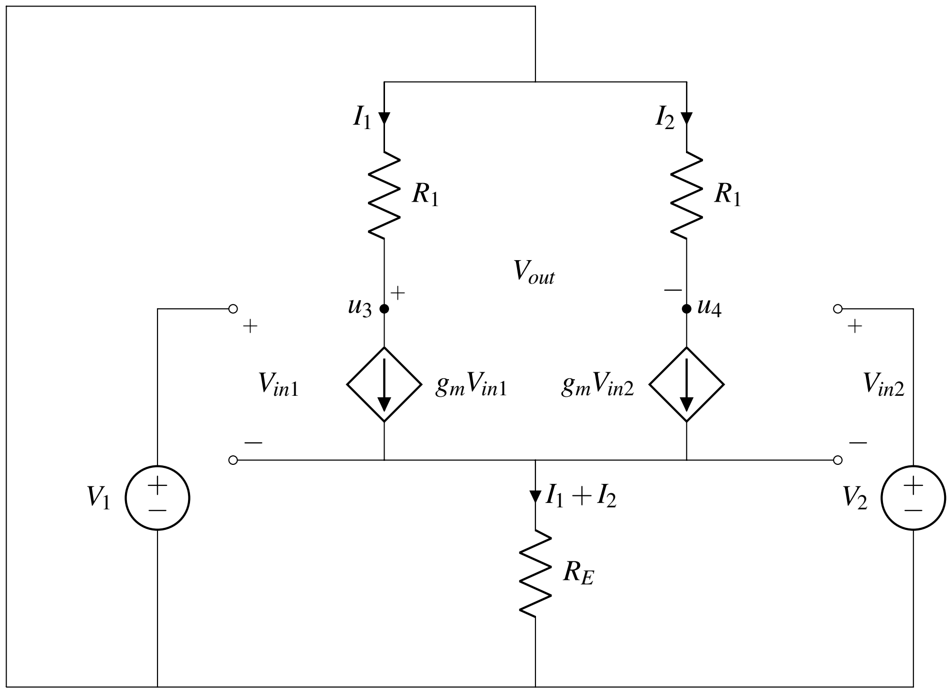
Answer:

$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_{S_1} + \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$

$$V_{R_2} = V_{S_1} - V_{R_1} = \frac{R_2}{R_1 + R_2} V_{S_1} - \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$

$$V_{R_3} = I_{S_1} R_3$$

2. Superposition



(a) Calculate V_{out} with only V_1 on. Repeat this with only V_2 on.

Answer:

First we calculate V_{out} when V_1 is on. We will call this $V_{out,1}$.

$$V_{in1} = V_1 - (I_1 + I_2)(R_E)$$
$$I_1 = g_m V_{in1} = g_m(V_1 - (I_1 + I_2)(R_E))$$
$$u_3 = 0 - R_1 I_1 = -R_1 g_m(V_1 - (I_1 + I_2)(R_E))$$
$$V_{in2} = -(I_1 + I_2)(R_E)$$
$$I_2 = g_m V_{in2} = g_m(-(I_1 + I_2)(R_E))$$
$$u_4 = 0 - R_1 I_2 = -R_1 g_m(-(I_1 + I_2)(R_E))$$
$$V_{out,1} = u_3 - u_4 = -g_m R_1(V_1)$$

Similarly, we can find V_{out} when V_2 is on. Let us call this $V_{out,2}$. We see that the circuit in this situation is a mirror image of the one we just solved for. Therefore, we can conclude the expression for $u_3 - u_4$ in the new circuit is the same as $-(u_3 - u_4)$, where we substitute V_2 for V_1. We thus have:

$$V_{out,2} = g_m R_1(V_2)$$

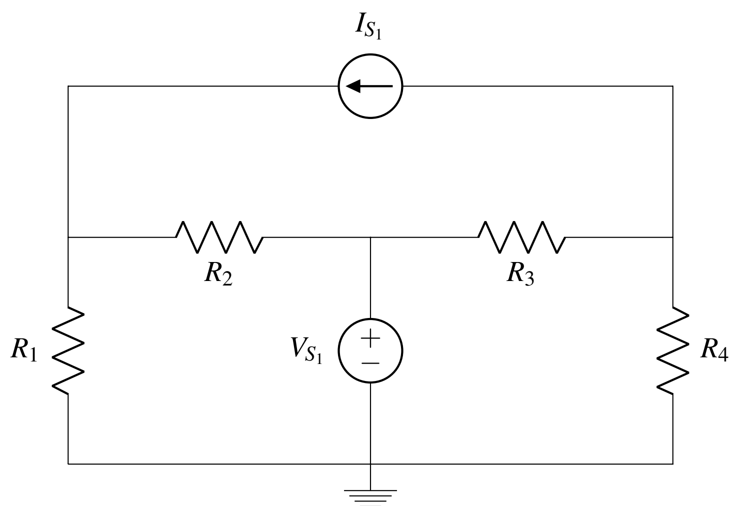
(b) Let's now turn on both V_1 and V_2. What is the output V_{out}? What does this circuit do to arbitrary input voltages?

Answer:

From superpostion, we can calculate $V_{out} = -g_m R_1 (V_1 - V_2)$. For two arbitrary inputs, this circuit *amplifies* the *difference*. We will learn more about amplification in the coming weeks. This particular circuit actually forms one of the most essential blocks inside op-amps, a device we will explore in further detail in the coming weeks.

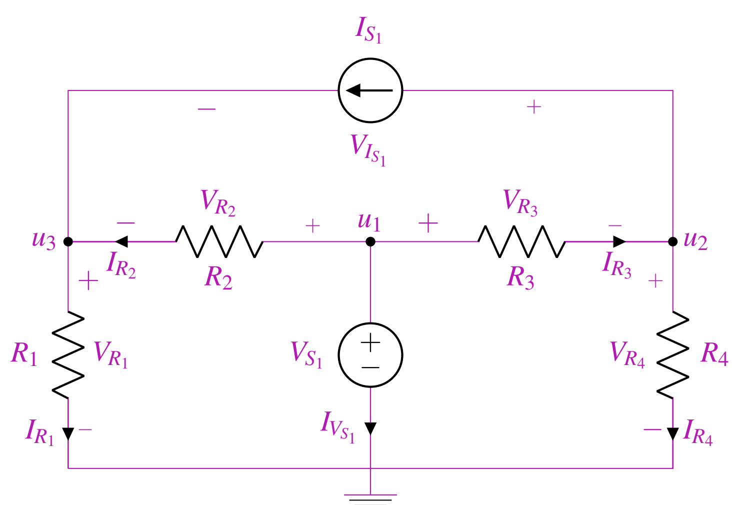
3. Circuit Analysis

Solve for the voltages across and the currents flowing through each component.

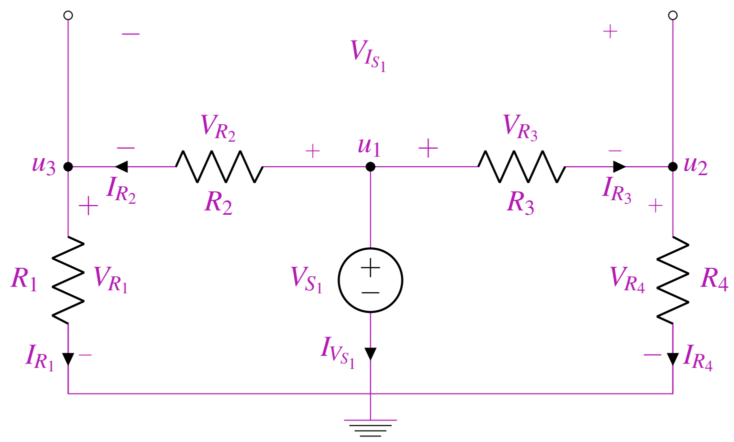


Answer:

We first label all potentials, currents, and voltages.



We can now use superposition. Let's first consider only the voltage source.



$$u_1 = V_{S_1}$$

Using the voltage divider equation:

$$u_3 = V_{R_1} = \frac{R_1}{R_1 + R_2} V_{S_1}$$

$$u_2 = V_{R_4} = \frac{R_4}{R_3 + R_4} V_{S_1}$$

$$V_{R_2} = u_1 - u_3 = \frac{R_2}{R_1 + R_2} V_{S_1}$$

$$V_{R_3} = u_1 - u_2 = \frac{R_3}{R_3 + R_4} V_{S_1}$$

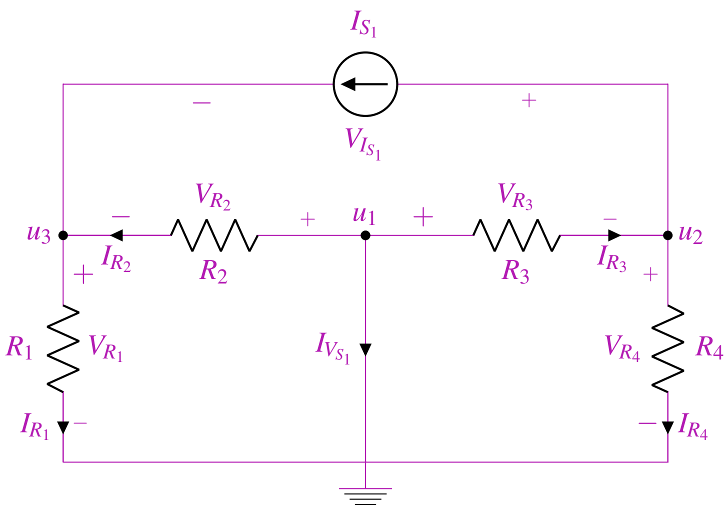
$$I_{R_1} = I_{R_2} = \frac{1}{R_1 + R_2} V_{S_1}$$

$$I_{R_3} = I_{R_4} = \frac{1}{R_3 + R_4} V_{S_1}$$

$$V_{I_{S_1}} = u_2 - u_3 = \left(\frac{R_4}{R_3 + R_4} - \frac{R_1}{R_1 + R_2} \right) V_{S_1}$$

$$I_{V_{S_1}} = -I_{R_2} - I_{R_3} = -\left(\frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} \right) V_{S_1}$$

Now let's consider only the current source.



$$u_1 = 0$$

Using the current divider equation:

$$I_{R_1} = \frac{R_2}{R_1 + R_2} I_{S_1}$$

$$I_{R_2} = -\frac{R_1}{R_1 + R_2} I_{S_1}$$

$$I_{R_3} = \frac{R_4}{R_3 + R_4} I_{S_1}$$

$$I_{R_4} = -\frac{R_3}{R_3 + R_4} I_{S_1}$$

$$u_3 = V_{R_1} = \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$

$$V_{R_2} = -\frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$

$$V_{R_3} = \frac{R_3 R_4}{R_3 + R_4} I_{S_1}$$

$$u_2 = V_{R_4} = -\frac{R_3 R_4}{R_3 + R_4} I_{S_1}$$

$$V_{I_{S_1}} = u_2 - u_3 = -\left(\frac{R_3 R_4}{R_3 + R_4} + \frac{R_1 R_2}{R_1 + R_2} \right) I_{S_1}$$

$$I_{V_{S_1}} = -I_{R_2} - I_{R_3} = \left(\frac{R_1}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) I_{S_1}$$

To find the voltages across and the currents flowing through each component, we would then sum up the partial currents and partial voltages for each component.

$$u_1 = V_{S_1}$$

$$u_3 = V_{R_1} = \frac{R_1}{R_1+R_2}V_{S_1} + \frac{R_1R_2}{R_1+R_2}I_{S_1}$$

$$V_{R_2} = \frac{R_2}{R_1+R_2}V_{S_1} - \frac{R_1R_2}{R_1+R_2}I_{S_1}$$

$$V_{R_3} = \frac{R_3}{R_3+R_4}V_{S_1} + \frac{R_3R_4}{R_3+R_4}I_{S_1}$$

$$u_2 = V_{R_4} = \frac{R_4}{R_3+R_4}V_{S_1} - \frac{R_3R_4}{R_3+R_4}I_{S_1}$$

$$I_{R_1} = \frac{1}{R_1+R_2}V_{S_1} + \frac{R_2}{R_1+R_2}I_{S_1}$$

$$I_{R_2} = \frac{1}{R_1+R_2}V_{S_1} - \frac{R_1}{R_1+R_2}I_{S_1}$$

$$I_{R_3} = \frac{1}{R_3+R_4}V_{S_1} + \frac{R_4}{R_3+R_4}I_{S_1}$$

$$I_{R_4} = \frac{1}{R_3+R_4}V_{S_1} - \frac{R_3}{R_3+R_4}I_{S_1}$$

$$V_{I_{S_1}} = \left(\frac{R_4}{R_3+R_4} - \frac{R_1}{R_1+R_2}\right)V_{S_1} - \left(\frac{R_3R_4}{R_3+R_4} + \frac{R_1R_2}{R_1+R_2}\right)I_{S_1}$$

$$I_{V_{S_1}} = -\left(\frac{1}{R_1+R_2} + \frac{1}{R_3+R_4}\right)V_{S_1} + \left(\frac{R_1}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right)I_{S_1}$$