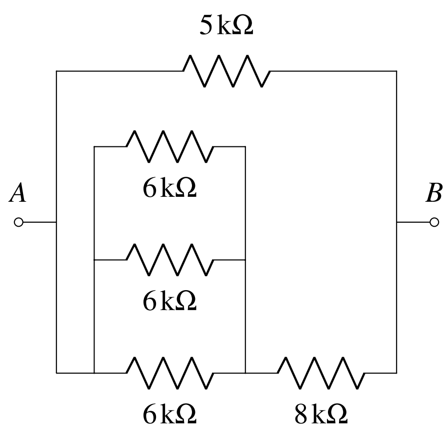
1. **Series and Parallel Combinations**

For the resistor network shown below, find an equivalent resistance between the terminals A and B using the resistor combination rules for series and parallel resistors.



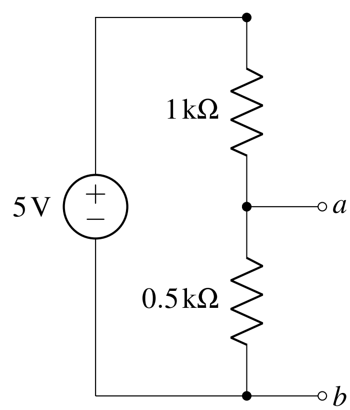
Answer:

$$5\,\text{k}\Omega \parallel ((6\,\text{k}\Omega \parallel 6\,\text{k}\Omega \parallel 6\,\text{k}\Omega) + 8\,\text{k}\Omega) = 5\,\text{k}\Omega \parallel (2\,\text{k}\Omega + 8\,\text{k}\Omega) = 5\,\text{k}\Omega \parallel 10\,\text{k}\Omega = 3.33\,\text{k}\Omega$$

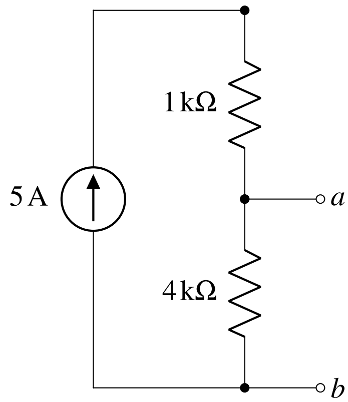
2. **Equivalence**

Find the Thévenin and Norton equivalents across terminals a and b for the circuits given below.

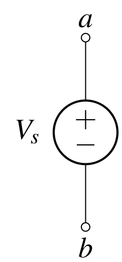
(a)



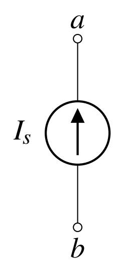
(b)



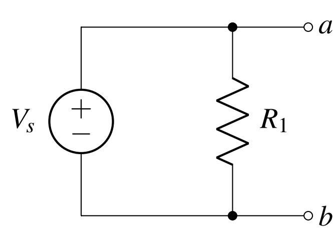
(c)



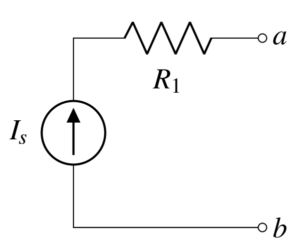
(d)



(e)

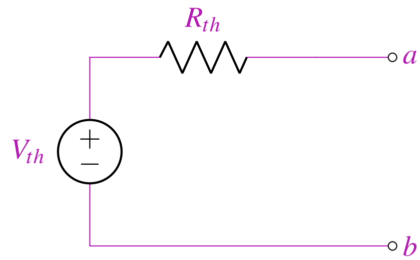


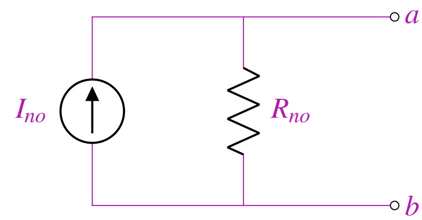
(f)



Answer:

The general Thévenin and Norton equivalents are shown below:





(a)

$$V_{th} = 1.67\,\text{V}, I_{no} = 5\,\text{mA}, R_{th} = R_{no} = 333\,\Omega$$

(b)

$$V_{th} = 20\,000\,\text{V}, I_{no} = 5\,\text{A}, R_{th} = R_{no} = 4000\,\Omega$$

(c) A Norton equivalent is not possible because $I_{sc} = I_{no}$ is arbitrary, depending on the load. The Thévenin equivalent is just a voltage source with voltage V_s, that is, $R_{th} = 0$.

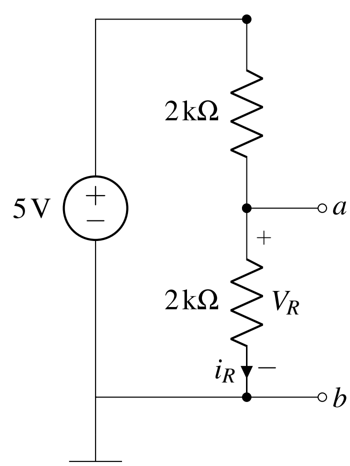
(d) A Thévenin equivalent is not possible because $V_{oc} = V_{th}$ is arbitrary, depending on the load. The Norton equivalent is just a current source with current I_s, that is, $R_{no} = \infty$.

(e) A Norton equivalent it not possible because $I_{sc} = I_{no}$ is arbitrary, depending on the load. The Thévenin equivalent is just a voltage source with voltage V_s, that is, $R_{th} = 0$. Notice that adding a parallel resistor does not change the Thévenin equivalent.

(f) A Thévenin equivalent is not possible because $V_{oc} = V_{th}$ is arbitrary, depending on the load. The Norton equivalent is just a current source with current I_s, that is, $R_{no} = \infty$. Notice that adding a series resistor does not change the Norton equivalent.

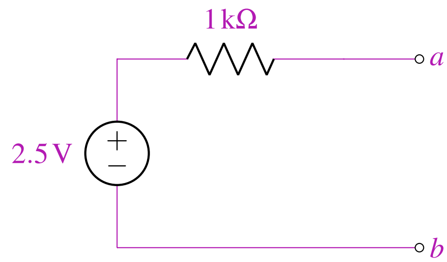
3. Why Bother With Thévenin Anyway?

(a) Find a Thévenin eqiuvalent for the circuit shown below.

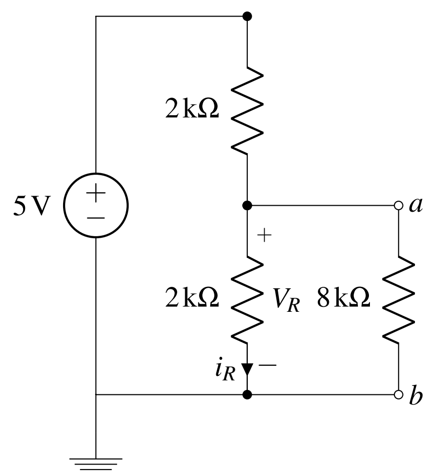


Answer:

$$V_{th} = \frac{2\,\text{k}\Omega}{2\,\text{k}\Omega + 2\,\text{k}\Omega} \cdot 5\,\text{V} = 2.5\,\text{V}$$
$$R_{th} = 2\,\text{k}\Omega \parallel 2\,\text{k}\Omega = 1\,\text{k}\Omega$$

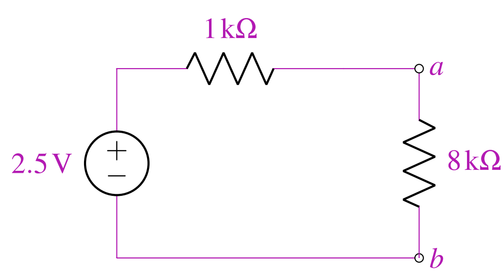


(b) What happens to the output voltage V_{ab} if we attach a load of $8\,\text{k}\Omega$ to the output as depicted in the circuit below. Use your Thévenin equivalent from part (a).



Answer:

We just attach the $8\,\text{k}\Omega$ resistor to our Thévenin equivalent circuit and calculate the voltage across it.

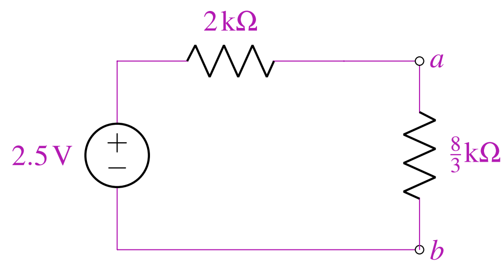


$$V_R = \frac{8\,\text{k}\Omega}{1\,\text{k}\Omega + 8\,\text{k}\Omega} \cdot 2.5\,\text{V} = 2.22\,\text{V}$$

(c) What if the load is $\frac{8}{3}\text{k}\Omega$? What if the load is $80\,\text{k}\Omega$?

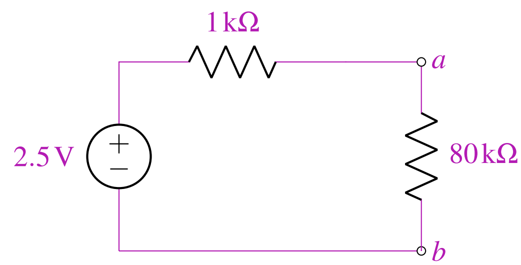
Answer:

$R = \frac{8}{3}\text{k}\Omega$:



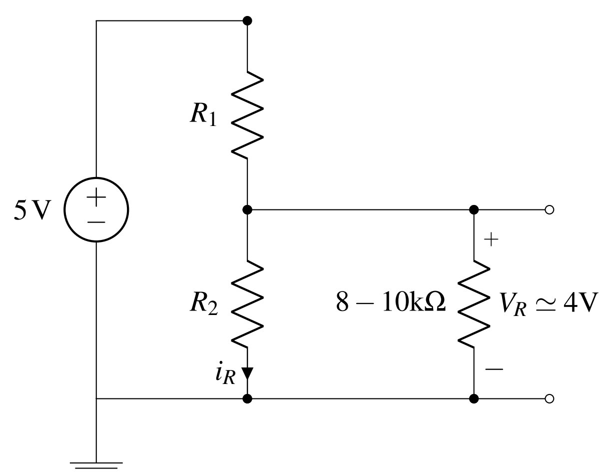
$$V_R = \frac{\frac{8}{3}\,\text{k}\Omega}{1\,\text{k}\Omega + \frac{8}{3}\,\text{k}\Omega} \cdot 2.5\,\text{V} = 1.82\,\text{V}$$

$R = 80\,\text{k}\Omega$:



$$V_R = \frac{80\,\text{k}\Omega}{1\,\text{k}\Omega + 80\,\text{k}\Omega} \cdot 2.5\,\text{V} = 2.46\,\text{V}$$

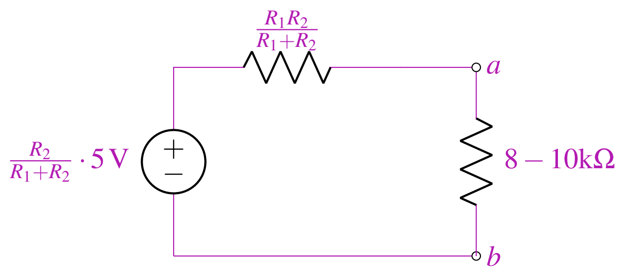
(d) Say that we want to support loads in the range of $8\,\text{k}\Omega$ to $10\,\text{k}\Omega$. We would like to maintain $4\,\text{V}$ across these loads. How can we approximately achieve this by setting R_1 and R_2 in the following circuit?



Answer:

$$V_{th} = \frac{R_2}{R_1 + R_2} \cdot 5\,\text{V}$$

$$R_{th} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$



$$V_R = \frac{R}{R+\frac{R_1R_2}{R_1+R_2}} \cdot \frac{R_2}{R_1+R_2} \cdot 5\,\text{V} \simeq 4\,\text{V}$$

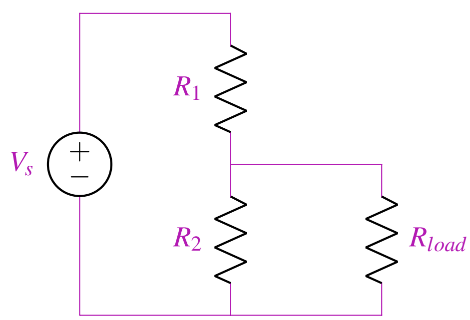
$$\frac{RR_2}{R(R_1+R_2)+R_1R_2} = \frac{4}{5}$$

If we set $R_1, R_2 \ll R$, then $\frac{RR_2}{R(R_1+R_2)+R_1R_2} \approx \frac{RR_2}{R(R_1+R_2)} = \frac{R_2}{R_1+R_2}$. Therefore, we can just choose two small resistors $R_1, R_2 \ll 8\,\text{k}\Omega$, such that $R_2 = 4R_1$.

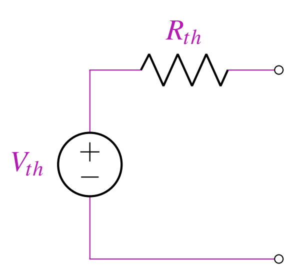
(e) For part (b), how much power does each element dissipate? Calculate the power using your Thévenin equivalent and using the original circuit. Are the values the same?

Answer:

We will ignore the power dissipated by R_{th} initially and just explore V_s vs. V_{th} and R_{load} in either case. This could be done for the specific example above, but it's more useful to go through this exercise generally. Thus, we will use the circuit shown below:



Recall that the Thévenin equivalent for the circuit above looks as follows:



where $R_{th} = \frac{R_1R_2}{R_1+R_2}$ and $V_{th} = \frac{R_2}{R_1+R_2}V_s$.

Because we are going to end up writing a few expressions multiple times, we are going to define a new variable:

$$\beta = R_1R_2 + R_{load}R_1 + R_{load}R_2$$

Let's start with our equivalent circuit. In the equivalent circuit, the current through the load resistor and equivalently every other element in the circuit is:

$$I = \frac{V_{ab}}{R_{load}} = \frac{V_{th}}{R_{load} + R_{th}}$$

With this current, we find the power dissapated across the source and the load resistor.

$$P_{V_{th}} = -IV = -\frac{V_{th}^2}{R_{load} + R_{th}} = -\frac{V_{th}^2(R_1 + R_2)}{\beta} = -\frac{V_s^2 R_2^2}{\beta(R_1 + R_2)}$$

$$P_{R_{load}} = I^2 R = \frac{V_{th}^2}{(R_{load} + R_{th})^2} \cdot R_{load} = \frac{V_{th}^2(R_1 + R_2)^2}{\beta^2} \cdot R_{load} = \frac{V_s^2 R_2^2}{\beta^2} \cdot R_{load}$$

Let's try to find the answer from the original circuit. We will begin by calculating the current through the source.

$$I_s = \frac{V_s}{R_{eq}} = \frac{V_s}{R_1 + R_2 \parallel R_{load}} = \frac{V_s(R_1 + R_2)}{\beta}$$

Now, we can calulate the power through the source.

$$P_{V_s} = -I_s V_s = -\frac{V_s^2(R_2 + R_{load})}{\beta}$$

The power dissipated by the source in the original circuit is not the same as the power dissipated in the new circuit. What about the load resistor? We will first calculate the voltage across the load resistor.

$$V_{load} = \frac{R_2 \parallel R_{load}}{R_1 + R_2 \parallel R_{load}} \cdot V_s = \frac{\frac{R_2 R_{load}}{R_2 + R_{load}}}{R_1 + \frac{R_2 R_{load}}{R_2 + R_{load}}} \cdot V_s = \frac{R_2 R_{load}}{\beta} \cdot V_s$$

$$P_{load} = \frac{V_{load}^2}{R_{load}} = \frac{V_s^2 R_2^2}{\beta^2} R_{load}$$

The power through the load is the same! Thévenin equivalents can be used to calculate the power through elements that are not part of the circuit that was transformed.