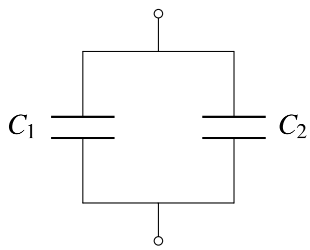
# EECS 16A Designing Information Devices and Systems I
# Spring 2018 Discussion 8A

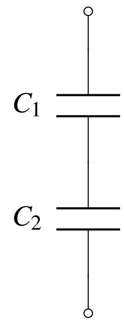
**1. Series And Parallel Capacitors**

Derive $C_{eq}$ for the following circuits.

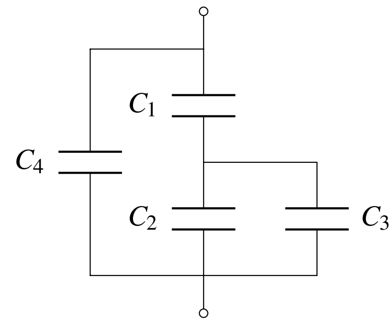
(a)




(b)




(c)




**Answer:**

(a)

$$C_{eq} = C_1 + C_2$$

We can derive this using the methods shown in lecture and in the notes. Alternatively, note that there is a more physical approach, where we consider the charges.

The total charge stored on the capacitors is $q$ with $q_1$ on the first capacitor and $q_2$ on the second.

Then $C_{eq} = \frac{q}{V} = \frac{q_1+q_2}{V} = \frac{q_1}{V} + \frac{q_2}{V} = C_1 + C_2$.

(b)

$$C_{eq} = C_1 \parallel C_2 = \frac{C_1 C_2}{C_1 + C_2}$$

Again, we can derive this using the methods shown in lecture and in the notes, but we will cover the more physical approach.

The voltage across the two capacitors $V_1$ and $V_2$ must sum to the voltage between the two terminals $V$. Since charge is conserved, the charge $q$ is the same on the two capacitors. (If there is $+q$ charge on the top plate of the top capacitor, there is $-q$ charge on the bottom plate. To conserve charge, there is $+q$ charge on the top plate of the bottom capacitor and thus $-q$ charge on the bottom plate.)

Then $\frac{1}{C_{eq}} = \frac{V}{q} = \frac{V_1+V_2}{q} = \frac{V_1}{q} + \frac{V_2}{q} = \frac{1}{C_1} + \frac{1}{C_2}$.

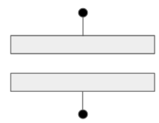
(c)

$$C_{eq} = (C_4 + (C_1 \parallel (C_2 + C_3))) = \frac{C_4(C_1 + C_2 + C_3) + C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

**2. Capacitance Equivalence**

For the structures shown below, assume that the plates have a depth $L$ into the page and a width $W$ and are always a distance $d$ apart.

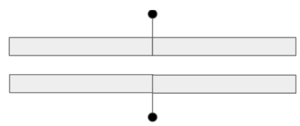
(a) What is the capactance of the structure shown below?



**Answer:**

The capacitance of two parallel plate conductors is given by $C = \varepsilon \frac{A}{d}$. The cross-sectional area $A$ is $WL$, so the capacitance is $C = \varepsilon \frac{WL}{d}$.

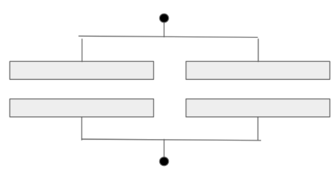
(b) Suppose that we take two such structures and put them next to each other as shown below. What is the capacitance of this new structure?



**Answer:**

Here, we have just doubled the width of the capacitor plates. The new capacitance is $C = \varepsilon \frac{2WL}{d}$. Notice that this is just double the capacitance from the first part.

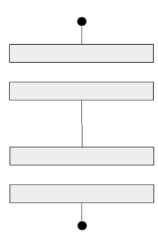
(c) Now suppose that rather than connecting the together as shown above, we connect them with an ideal wire as shown below. What is the capacitance of this structure?



**Answer:**

Intuitively, nothing has changed here since we have just added an ideal wire between two capacitors. Thus, the answer remains $C = \varepsilon \frac{2WL}{d}$.

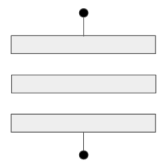
(d) Suppose that we now take two capacitors and connect them as shown below. What is the capacitance of the structure?



**Answer:**

We know that capacitors placed in series follow the parallel rule. Thus, the overall capacitance is half the individual capacitance.

(e) What is the capacitance of the structure shown below?

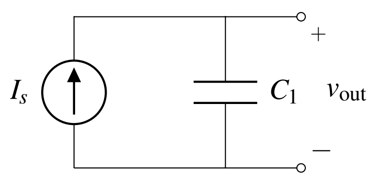


**Answer:**
Notice here that we are ignoring the material in the middle. Thus, from a modeling perspective, we can think of this as the original capacitor with the distance between the plates doubled.

## 3. Current Sources And Capacitors

For the circuits given below, give an expression for $v_{\text{out}}(t)$ in terms of $I_s$, $C_1$, $C_2$, and $t$. Assume that all capacitors are initially uncharged.

(a)



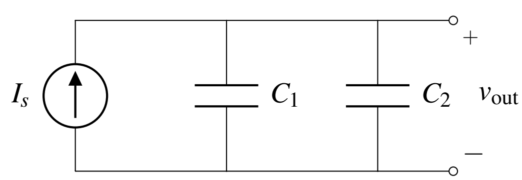

**Answer:**

$$I_s = C_1 \frac{dv_{\text{out}}(t)}{dt}$$

$$v_{\text{out}}(t) = \int \frac{I_s}{C_1} dt = \frac{I_s t}{C_1} + V_0$$

Since the capacitor is initially uncharged, $V_0 = 0$, so $v_{\text{out}}(t) = \frac{I_s t}{C_1}$.

(b)

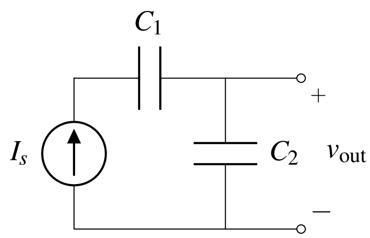



**Answer:**
We can combine the two capacitors into an equivalent capacitor with capacitance $C_1 + C_2$. Again, $V_0 = 0$ because all capacitors are initially uncharged.

$$I_s = (C_1 + C_2) \frac{dv_{\text{out}}(t)}{dt}$$

$$v_{\text{out}}(t) = \frac{I_s t}{C_1 + C_2} + V_0 = \frac{I_s t}{C_1 + C_2}$$

(c)



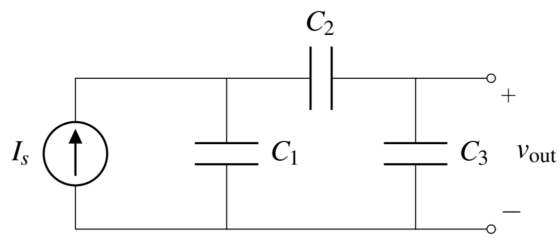

**Answer:**

By KCL, the current $I_s$ flowing through $C_1$ must be the current flowing through $C_2$. $V_0 = 0$ because all capacitors are initially uncharged.

$$I_s = C_2 \frac{dv_{\text{out}}(t)}{dt}$$

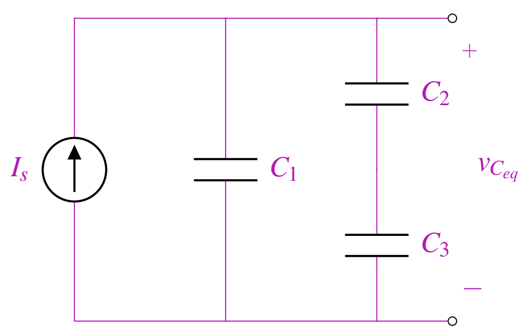
$$v_{\text{out}}(t) = \int \frac{I_s}{C_2} dt = \frac{I_s t}{C_2} + V_0 = \frac{I_s t}{C_2}$$

(d)

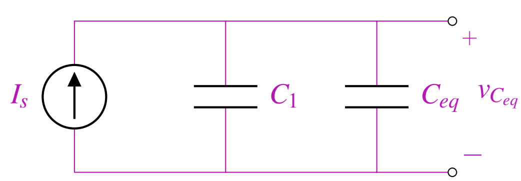



**Answer:**

Instead of finding $v_{\text{out}}$, let's first find the voltage $v_{C_{eq}}$ across $C_2$ and $C_3$.




To do this, we replace $C_2$ and $C_3$ with their equivalent capacitance $C_{eq} = C_2 \parallel C_3 = \frac{C_2 C_3}{C_2 + C_3}$.




From part (b), we know that to solve for $v_{C_{eq}}$, we can find the equivalent capacitance of $C_1$ and $C_{eq}$ first, which is $C_1 + C_{eq}$. Since the capacitors are initially uncharged, $V_0 = 0$.

$$v_{C_{eq}}(t) = \int \frac{I_s}{C_1 + C_{eq}} dt = \frac{I_s t}{C_1 + C_{eq}} + V_0 = \frac{I_s t}{C_1 + C_{eq}}$$

Now that we know that voltage across the equivalent capacitor $C_{eq}$, we can find the current flowing through the equivalent capacitor $C_{eq}$.

$$i_{C_{eq}}(t) = C_{eq}\frac{dv_{C_{eq}}(t)}{dt} = \frac{C_{eq}I_s}{C_1 + C_{eq}}$$

Note that the current $i_{C_{eq}}$ is equal to the current flowing through $C_3$ since $C_2$ and $C_3$ were originally connected in series.

$$i_{C_3}(t) = i_{C_{eq}}(t) = \frac{C_{eq}I_s}{C_1 + C_{eq}}$$

Since $v_{\text{out}}$ is the voltage across the capacitor $C_3$, we integrate to find $v_{\text{out}}$. Again, since all capacitors are initially uncharged, $V_0 = 0$.

$$i_{C_3}(t) = C_3\frac{dv_{\text{out}}(t)}{dt}$$

$$v_{\text{out}}(t) = \int \frac{C_{eq}I_s}{C_3(C_1 + C_{eq})}dt = \frac{C_{eq}I_st}{C_3(C_1 + C_{eq})} + V_0 = \frac{\frac{C_2C_3}{C_2+C_3}I_st}{C_3\left(C_1 + \frac{C_2C_3}{C_2+C_3}\right)} = \frac{C_2I_st}{C_1C_2 + C_1C_3 + C_2C_3}$$