1. Op-Amps As Comparators

For each of the circuits shown below, plot $V_{\text{out}}$ for $V_{\text{in}}$ ranging from $-10 \text{ V}$ to $10 \text{ V}$ for part (a) and from $0 \text{ V}$ to $10 \text{ V}$ for part (b). Let $A = 100$ for your plots.

(a)

![Circuit Diagram](image)

Answer:

$$V_+ = V_{\text{in}}$$

$$V_- = \frac{2k\Omega}{1k\Omega + 2k\Omega}V_{\text{in}} = \frac{2}{3}V_{\text{in}}$$

$$V_{\text{out}} = A(V_+ - V_-) + \frac{V^+_{S} + V^-_{S}}{2}$$

$$= AV_{\text{in}} \left( 1 - \frac{2}{3} \right) = \frac{1}{3}AV_{\text{in}}$$

The op-amp satisfies the linear relation above for $V^-_{S} \leq V_{\text{out}} \leq V^+_{S}$.

$$V^-_{S} \leq V_{\text{out}} \leq V^+_{S}$$

$$V^-_{S} \leq \frac{1}{3}AV_{\text{in}} \leq V^+_{S}$$

$$\frac{3V^-_{S}}{A} \leq V_{\text{in}} \leq 3\frac{V^+_{S}}{A}$$

$$\frac{3\cdot -5 \text{ V}}{100} \leq V_{\text{in}} \leq \frac{3 \cdot 5 \text{ V}}{100}$$

$$-0.15 \text{ V} \leq V_{\text{in}} \leq 0.15 \text{ V}$$

The op-amp saturates outside of this range.
Answer:

\[
V_+ = \frac{2 \Omega}{1 \Omega + 2 \Omega} V_{in} = \frac{2}{3} V_{in}
\]

\[
V_- = 2 \text{ V}
\]

\[
V_{out} = A(V_+ - V_-) + \frac{V_S^+ + V_S^-}{2}
\]

\[
= A \left( \frac{2}{3} V_{in} - 2 \right) + 2.5
\]

The op-amp satisfies the linear relation above for \(V_S^- \leq V_{out} \leq V_S^+\).
\[ V_S^- \leq V_{out} \leq V_S^+ \]
\[ V_S^- \leq A \left( \frac{2}{3} V_{in} - 2 \right) + 2.5 \leq V_S^+ \]
\[ \frac{3}{2} \left( \frac{V_S^- - 2.5}{A} + 2 \right) \leq V_{in} \leq \frac{3}{2} \left( \frac{V_S^+ - 2.5}{A} + 2 \right) \]
\[ \frac{3}{2} \left( \frac{-2.5V}{100} + 2 \right) \leq V_{in} \leq \frac{3}{2} \left( \frac{5V - 2.5V}{100} + 2 \right) \]
\[ 2.9625 \text{ V} \leq V_{in} \leq 3.0375 \text{ V} \]

The op-amp saturates outside of this range.

2. Modular Circuits

In this problem, we will explore the design of circuits that perform a set of (arbitrary) mathematical operations. (Note that the so-called analog signal processing – where these kinds of mathematical operations are performed on continuously-valued voltages by analog circuits – is extremely common in real-world applications; without this capability, essentially none of our radios or sensors would actually work.) Specifically, let’s assume that we want to implement the block diagram shown below:

![Block Diagram](image)

In other words, we want to implement a circuit with two outputs \( V_x \) and \( V_y \), where \( V_x = \frac{1}{2} V_{in} \) and \( V_y = \frac{1}{3} V_x \).

(a) Design two voltage divider circuits that each independently would implement the two multiplications shown in the block diagram above (i.e., multiply by \( \frac{1}{2} \) and multiply by \( \frac{1}{3} \)). Note that you do not need to include the input voltage sources in your design – you can simply define the input to each block as being at the appropriate potential (e.g., \( V_{in} \) or \( V_x \)).
(b) Assuming that \( V_{\text{in}} \) is created by an ideal voltage source, implement the original block diagram as a circuit by directly replacing each block with the designs you came up with in part (a).

**Answer:**

\[
\begin{align*}
V_{\text{in}} &\rightarrow \frac{1}{2} \rightarrow V_x \\
\frac{1}{2} \rightarrow& \quad \frac{1}{2} \rightarrow V_x \\
&\quad \frac{1}{2} \rightarrow V_x
\end{align*}
\]

\[
\begin{align*}
V_{\text{in}} &\rightarrow \frac{1}{3} \rightarrow V_y \\
\frac{1}{3} \rightarrow& \quad \frac{1}{3} \rightarrow V_y \\
&\quad \frac{1}{3} \rightarrow V_y
\end{align*}
\]

(c) For the circuit from part (b), do you get the desired relationship between \( V_y \) and \( V_x \)? How about between \( V_x \) and \( V_{\text{in}} \)? Be sure to explain why or why not each block retains its desired functionality.

**Answer:**

The relationship between \( V_y \) and \( V_x \) will be correct. The relationship between \( V_x \) and \( V_{\text{in}} \) will not be correct. The \( \frac{1}{3} \) block will be an additional load on the \( \frac{1}{2} \) block. Both blocks have non-zero Thevenin resistance, so the current/voltage at the load can change. Applying parallel equivalence, we get \( V_x = \frac{750\Omega}{750\Omega + 1\Omega} \cdot V_{\text{in}} = \frac{7}{8} V_{\text{in}} \).

(d) Now let’s assume that we have discovered compose-able circuits that implement mathematical operations. In particular, we have these blocks that implement:

i. \( V_o = 5V_i \)
ii. \( V_o = -2V_i \)
iii. \( V_o = V_{i_1} + V_{i_2} \)

Using just these blocks, draw the block diagram that implements:

i. \( V_o = -12V_{\text{in}_1} \)
ii. \( V_o = -10V_{\text{in}_1} - 2V_{\text{in}_2} \)
iii. \( V_o = -V_{\text{in}_1} + V_{\text{in}_2} \)

**Answer:**

i. 

\[
\begin{align*}
V_{\text{in}_1} &\rightarrow 5 \rightarrow -2 \rightarrow V_{\text{out}} \\
V_{\text{in}_1} &\rightarrow -2 \rightarrow V_{\text{out}}
\end{align*}
\]
iii.

3. Op-Amp Golden Rules

On the left is the equivalent circuit of an op-amp for reference.

(a) What are the currents flowing into the positive and negative terminals of the op-amp (i.e., what are $I^+$ and $I^-$)? What are some of the advantages of your answer with respect to using an op-amp in your circuit designs?

Answer:
The $v^+$ and $v^-$ terminals have no closed circuit connection between them, and therefore no current can flow into or out of them. This is very good because we can connect an op-amp to any other circuit, and the op-amp will not disturb that circuit in any way because it does not load the circuit (it is an open circuit).

(b) Suppose we add a resistor of value $R_L$ between $v_{out}$ and ground. What is the value of $v_{out}$? Does your answer depend on $R_L$? In other words, how does $R_L$ affect $AV_c$? What are the implications of this with respect to using op-amps in circuit design?

Answer:
Notice that $v_{out}$ is connected directly to a controlled/dependent voltage source, and therefore $v_{out}$ (relative to ground) will always have to be equal to $AV_c$ regardless of what $R_L$ is connected to the op-amp. This is very advantageous because it means that the output of the op-amp can be connected to any other circuit (except a voltage source), and we will always get the desired/expected voltage out of the op-amp.
(c) Now consider the circuit on the right. Assuming that this is an ideal op-amp, what is $v_{\text{out}}$?

**Answer:**
Recall for an ideal op-amp in negative feedback, we know from the Golden Rules that $v^+ = v^-$. In this case, $v^- = v_{\text{out}} = v^+$. 

(d) Draw the equivalent circuit for this op-amp and calculate $v_{\text{out}}$ in terms of $A$, $v_{\text{in}}$, and $R_L$. Does $v_{\text{out}}$ depend on $R_L$? What is $v_{\text{out}}$ in the limit as $A \to \infty$?

**Answer:**
Notice that the op-amp can be modeled as a voltage-controlled voltage source. Thus, we have the following equation:

\[
v_{\text{out}} = A(v_{\text{in}} - v_{\text{out}})
\]

\[
v_{\text{out}} + Av_{\text{out}} = Av_{\text{in}}
\]

\[
v_{\text{out}} = v_{\text{in}} \frac{A}{1 + A}
\]

Thus, as $A \to \infty$, $v_{\text{out}} \to V$. This is the same as what we get after applying the op-amp Golden Rules. Notice that output voltage does not depend on $R$. Thus, this circuit acts like a voltage source that provides the same voltage read at $v^+$ without drawing any current from the terminal at $v^+$. This is why the circuit is often referred to as a “unity gain buffer,” “voltage follower,” or just “buffer.”