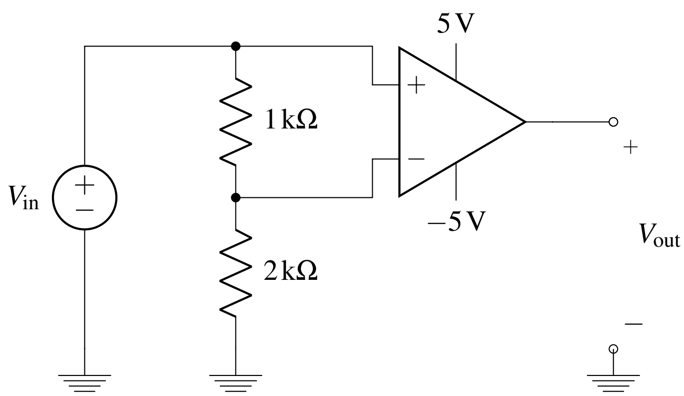
1. Op-Amps As Comparators

For each of the circuits shown below, plot V_{out} for V_{in} ranging from $-10\,\text{V}$ to $10\,\text{V}$ for part (a) and from $0\,\text{V}$ to $10\,\text{V}$ for part (b). Let $A = 100$ for your plots.

(a)



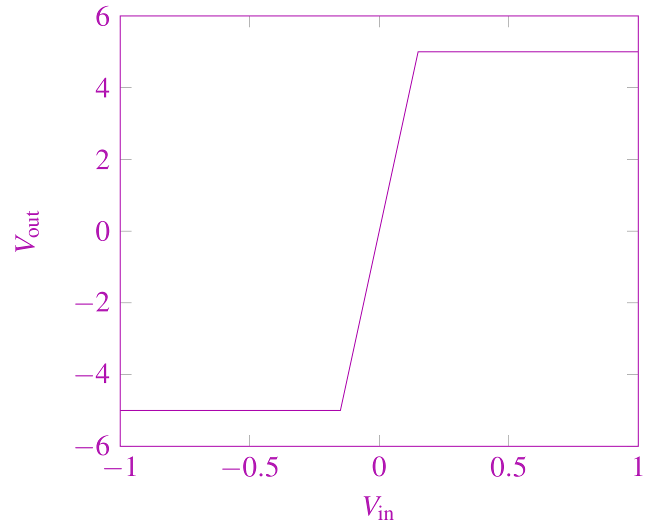
Answer:

$$V_+ = V_{\text{in}}$$
$$V_- = \frac{2\,\text{k}\Omega}{1\,\text{k}\Omega + 2\,\text{k}\Omega} V_{\text{in}} = \frac{2}{3} V_{\text{in}}$$
$$V_{\text{out}} = A(V_+ - V_-) + \frac{V_S^+ + V_S^-}{2}$$
$$= AV_{\text{in}}\left(1 - \frac{2}{3}\right) = \frac{1}{3} AV_{\text{in}}$$

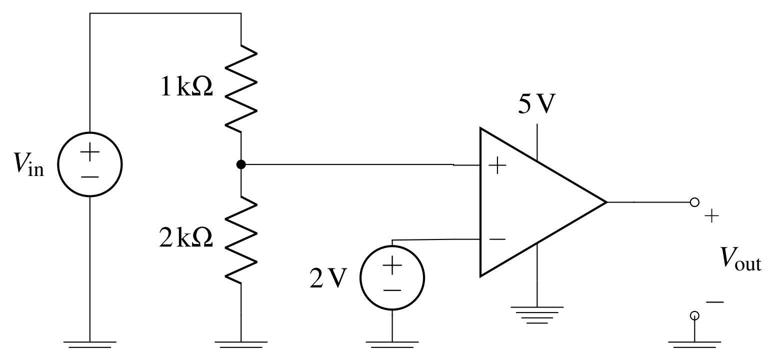
The op-amp satisfies the linear relation above for $V_S^- \leq V_{out} \leq V_S^+$.

$$V_S^- \leq V_{\text{out}} \leq V_S^+$$
$$V_S^- \leq \frac{1}{3} AV_{\text{in}} \leq V_S^+$$
$$3\frac{V_S^-}{A} \leq V_{\text{in}} \leq 3\frac{V_S^+}{A}$$
$$3\frac{-5\,\text{V}}{100} \leq V_{\text{in}} \leq 3\frac{5\,\text{V}}{100}$$
$$-0.15\,\text{V} \leq V_{\text{in}} \leq 0.15\,\text{V}$$

The op-amp saturates outside of this range.



(b)



Answer:

$$V_+ = \frac{2\,\mathrm{k\Omega}}{1\,\mathrm{k\Omega} + 2\,\mathrm{k\Omega}} V_{\mathrm{in}} = \frac{2}{3} V_{\mathrm{in}}$$

$$V_- = 2\,\mathrm{V}$$

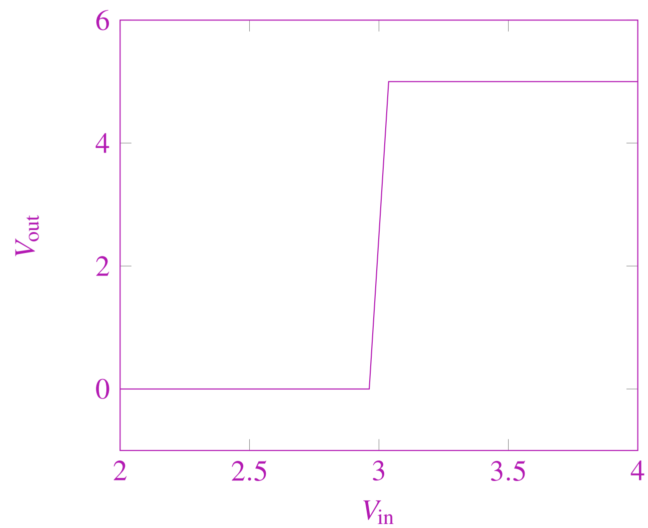
$$V_{\mathrm{out}} = A(V_+ - V_-) + \frac{V_S^+ + V_S^-}{2}$$

$$= A\left(\frac{2}{3}V_{\mathrm{in}} - 2\right) + 2.5$$

The op-amp satisfies the linear relation above for $V_S^- \leq V_{out} \leq V_S^+$.

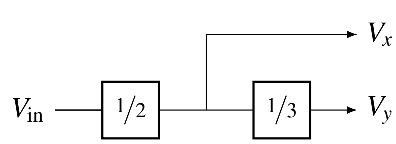
$$V_S^- \leq V_{\text{out}} \leq V_S^+$$
$$V_S^- \leq A\left(\frac{2}{3}V_{\text{in}} - 2\right) + 2.5 \leq V_S^+$$
$$\frac{3}{2}\left(\frac{V_S^- - 2.5}{A} + 2\right) \leq V_{\text{in}} \leq \frac{3}{2}\left(\frac{V_S^+ - 2.5}{A} + 2\right)$$
$$\frac{3}{2}\left(\frac{-2.5\,\text{V}}{100} + 2\right) \leq V_{\text{in}} \leq \frac{3}{2}\left(\frac{5\,\text{V} - 2.5\,\text{V}}{100} + 2\right)$$
$$2.9625\,\text{V} \leq V_{\text{in}} \leq 3.0375\,\text{V}$$

The op-amp saturates outside of this range.



2. Modular Circuits

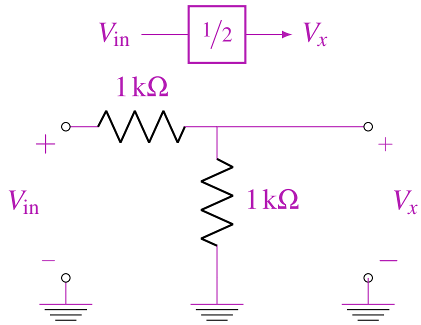
In this problem, we will explore the design of circuits that perform a set of (arbitrary) mathematical operations. (Note that the so-called analog signal processing – where these kinds of mathematical operations are performed on continuously-valued voltages by analog circuits – is extremely common in real-world applications; without this capability, essentially none of our radios or sensors would actually work.) Specifically, let's assume that we want to implement the block diagram shown below:

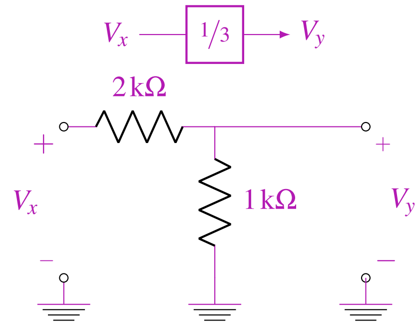


In other words, we want to implement a circuit with two outputs V_x and V_y, where $V_x = \frac{1}{2}V_{\text{in}}$ and $V_y = \frac{1}{3}V_x$.

(a) Design two voltage divider circuits that each independently would implement the two multiplications shown in the block diagram above (i.e., multiply by 1/2 and multiply by 1/3). Note that you do not need to include the input voltage sources in your design – you can simply define the input to each block as being at the appropriate potential (e.g., V_{in} or V_x).

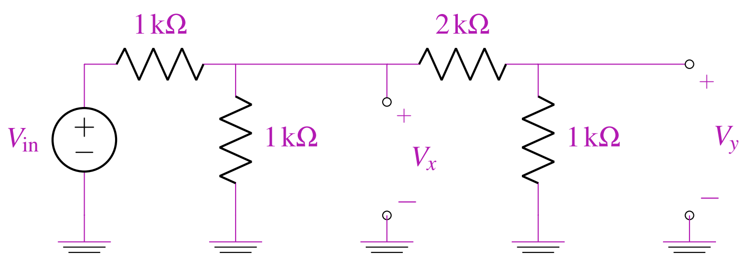
Answer:





(b) Assuming that V_{in} is created by an ideal voltage source, implement the original block diagram as a circuit by directly replacing each block with the designs you came up with in part (a).

Answer:



(c) For the circuit from part (b), do you get the desired relationship between V_y and V_x? How about between V_x and V_{in}? Be sure to explain why or why not each block retains its desired functionality.

Answer:

The relationship between V_y and V_x will be correct. The relationship between V_x and V_{in} will not be correct. The 1/3 block will be an additional load on the 1/2 block. Both blocks have non-zero Thevenin resistance, so the current/voltage at the load can change. Applying parallel equivalence, we get $V_x = \frac{750\,\Omega}{750\,\Omega + 1\,\text{k}\Omega} \cdot V_{in} = \frac{3}{7} V_{in}$.

(d) Now let's assume that we have discovered compose-able circuits that implement mathematical operations. In particular, we have these blocks that implement:

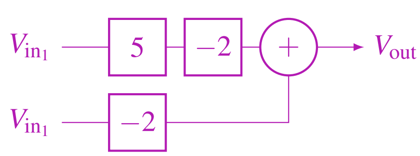
i. $V_o = 5V_i$
ii. $V_o = -2V_i$
iii. $V_o = V_{i_1} + V_{i_2}$

Using just these blocks, draw the block diagram that implements:

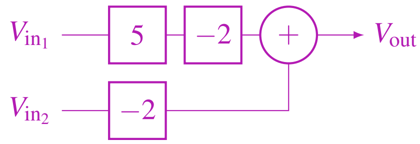
i. $V_o = -12V_{in_1}$
ii. $V_o = -10V_{in_1} - 2V_{in_2}$
iii. $V_o = -V_{in_1} + V_{in_2}$

Answer:

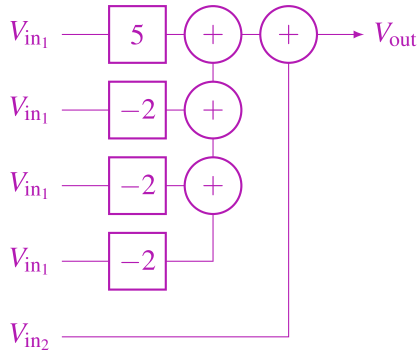
i.



ii.

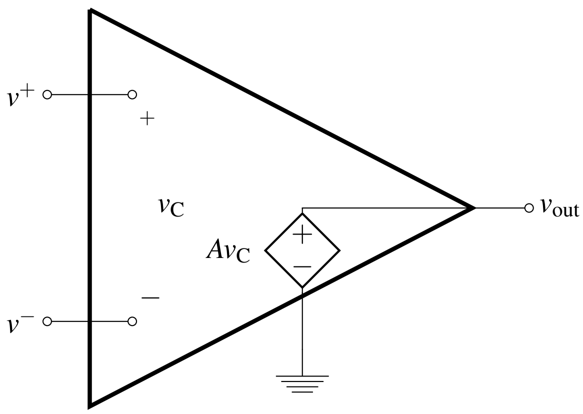


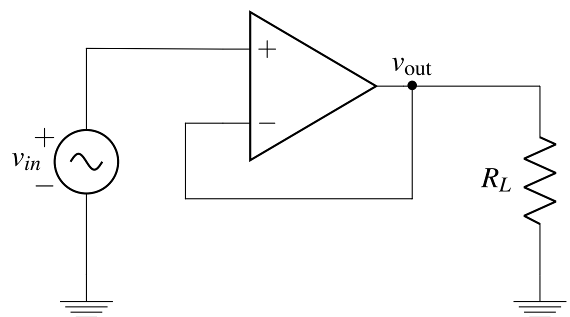
iii.



3. Op-Amp Golden Rules

On the left is the equivalent circuit of an op-amp for reference.





(a) What are the currents flowing into the positive and negative terminals of the op-amp (i.e., what are I^+ and I^-)? What are some of the advantages of your answer with respect to using an op-amp in your circuit designs?

Answer:

The v^+ and v^- terminals have no closed circuit connection between them, and therefore no current can flow into or out of them. This is very good because we can connect an op-amp to any other circuit, and the op-amp will not disturb that circuit in any way because it does not load the circuit (it is an open circuit).

(b) Suppose we add a resistor of value R_L between v_{out} and ground. What is the value of v_{out}? Does your answer depend on R_L? In other words, how does R_L affect Av_C? What are the implications of this with respect to using op-amps in circuit design?

Answer:

Notice that v_{out} is connected directly to a controlled/dependent voltage source, and therefore v_{out} (relative to ground) will always have to be equal to Av_C regardless of what R_L is connected to the op-amp. This is very advantageous because it means that the output of the op-amp can be connected to any other circuit (except a voltage source), and we will always get the desired/expected voltage out of the op-amp.

(c) Now consider the circuit on the right. Assuming that this is an ideal op-amp, what is v_{out}?

Answer:

Recall for an ideal op-amp in negative feedback, we know from the Golden Rules that $v^+ = v^-$. In this case, $v^- = v_{\text{out}} = v^+$.

(d) Draw the equivalent circuit for this op-amp and calculate v_{out} in terms of A, v_{in}, and R_L. Does v_{out} depend on R_L? What is v_{out} in the limit as $A \to \infty$?

Answer:

Notice that the op-amp can be modeled as a voltage-controlled voltage source. Thus, we have the following equation:

$$\begin{aligned}
v_{\text{out}} &= A(v_{in} - v_{\text{out}}) \\
v_{\text{out}} + Av_{\text{out}} &= Av_{in} \\
v_{\text{out}} &= v_{in}\frac{A}{1+A}
\end{aligned}$$

Thus, as $A \to \infty$, $v_{\text{out}} \to V$. This is the same as what we get after applying the op-amp Golden Rules.

Notice that output voltage does not depend on R. Thus, this circuit acts like a voltage source that provides the same voltage read at v^+ without drawing any current from the terminal at v^+. This is why the circuit is often referred to as a "unity gain buffer," "voltage follower," or just "buffer."