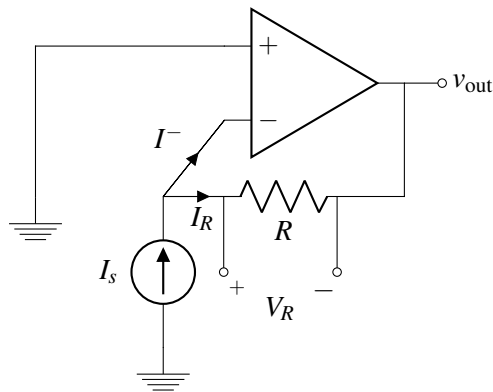


1. A Trans-Resistance Amplifier



- (a) Use the Golden Rules to calculate v_{out} as a function of I_s and R .

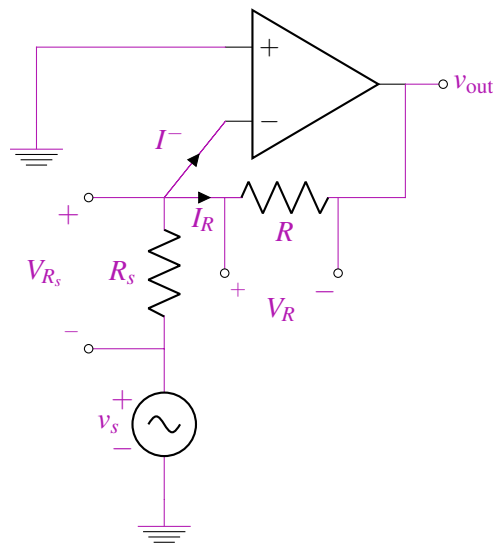
Answer:

$I^- = 0$, so $I_s = I_R$. Thus, $V_R = I_s R$.

Using Golden Rules, $V^- = 0V$, and thus $v_{out} = V^- - V_R = -I_s R$.

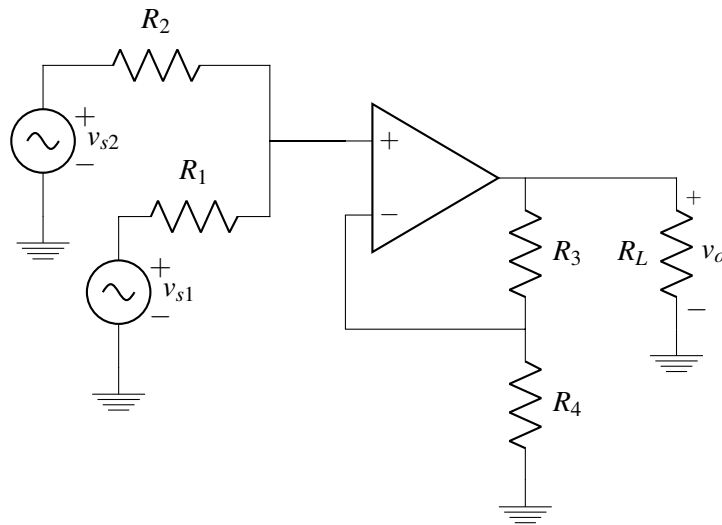
- (b) Use the Golden Rules to implement the same behavior as the above circuit (with a current source), but use a voltage source and a resistor instead.

Answer:



Remember, $V^- = 0V$ because of the Golden Rules. Thus, $V_{R_s} = v_s - 0V = v_s$, and so $I_R = \frac{v_s}{R_s}$. Hence, $v_{out} = -\frac{v_s}{R_s} R = -\frac{R}{R_s} v_s$. This is the **inverting amplifier**.

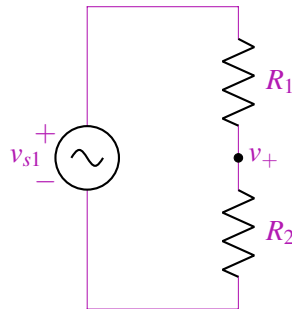
2. Multiple Inputs To One Op-Amp



(a) For the circuit above, find an expression for v_o . (Hint: Use superposition.)

Answer:

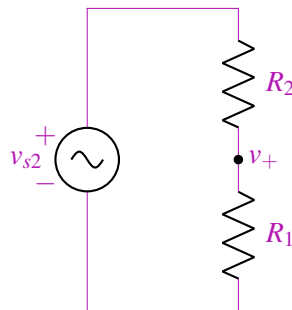
Let's call the voltage at the positive input of the op-amp v_+ . Using superposition, we first turn off v_{s2} and find v_+ . The circuit then looks like:



We recognize the above circuit as a voltage divider. Thus,

$$v_{+,v_{s1}} = \frac{R_2}{R_1 + R_2} v_{s1}$$

By symmetry, we expect v_{s2} to have a similar circuit and expression. The circuit for v_{s2} looks like:



The expression for v_+ with v_{s2} is then:

$$v_{+,v_{s2}} = \frac{R_1}{R_1 + R_2} v_{s2}$$

From superposition, we know the output must be the sum of these.

$$v_+ = \frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}$$

With v_+ determined, we can find the output voltage directly from the formula for a non-inverting amplifier. We can also derive it using the process below.

From the Golden Rules, $v_+ = v_-$. Using voltage dividers, we can express v_- in terms of v_o :

$$v_- = \frac{R_4}{R_3 + R_4}$$

$$v_o = \left(1 + \frac{R_3}{R_4}\right) v_- = \left(1 + \frac{R_3}{R_4}\right) v_+$$

Now, to find the final output, we can set v_+ to our earlier expression.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left(\frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}\right)$$

(b) How could you use this circuit to find the sum of different signals?

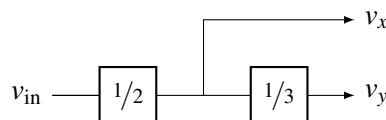
Answer:

The circuit already finds the weighted sum of two inputs. By setting $R_1 = R_2$ and $R_3 = R_4$, we can take the exact sum of two inputs.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left(\frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}\right) = (1 + 1) \left(\frac{1}{2} v_{s1} + \frac{1}{2} v_{s2}\right) = v_{s1} + v_{s2}$$

3. Modular Circuits

In this problem, we will explore the design of circuits that perform a set of (arbitrary) mathematical operations in order to elucidate some of the important properties and uses of op-amps in negative feedback. In the last discussion, we noticed that voltage dividers are not compose-able, so we will use op-amps instead. Again, recall that we want to implement the block diagram shown below:



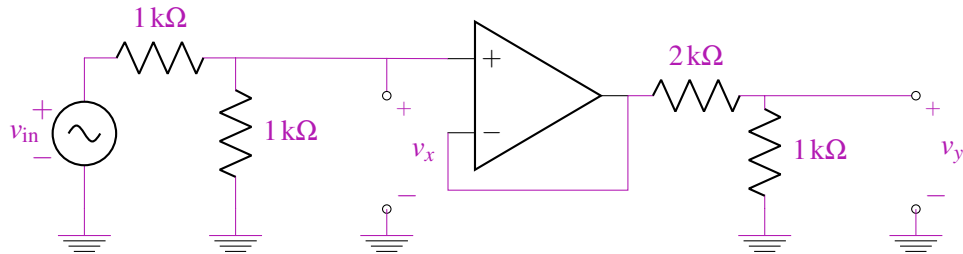
In other words, we want to implement a circuit with two outputs v_x and v_y , where $v_x = \frac{1}{2} v_{in}$ and $v_y = \frac{1}{3} v_x$.

(a) Using an ideal op-amp in negative feedback, modify the design of one of the two voltage divider circuits you built (i.e. the $1/2$ block or the $1/3$ block), so that the originally intended relationships between v_x and v_{in} as well as v_y and v_x are realized by the resulting overall circuit (where each block

is replaced by its individual implementation). Is this configuration enough by itself to attach loads at v_x and v_y ?

Answer:

Use a voltage buffer. Note that this configuration's outputs would change with the addition of a load. As a follow-up, think about ways to make the outputs agnostic to the loads attached. If we used the latter half of the circuit as a fractional divider block, we would need to buffer the output.



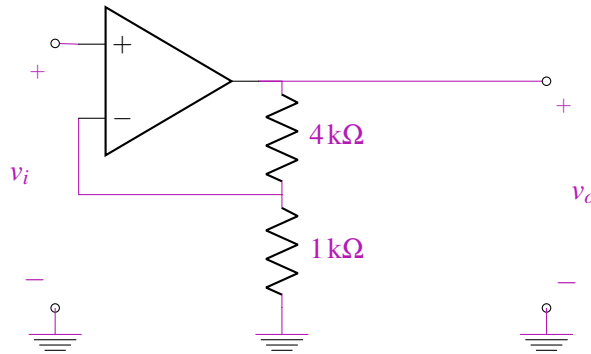
(b) Now let's assume that we want to expand our toolbox of circuits that implement mathematical operations. In particular, design blocks that implement:

- i. $v_o = 5 v_i$
- ii. $v_o = -2 v_i$
- iii. $v_o = v_{i_1} + v_{i_2}$

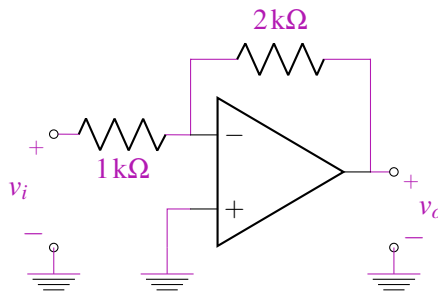
Pay careful attention to the way you design these blocks, so that connecting any one block to any other block does not modify the intended functionality of any of the blocks.

Answer:

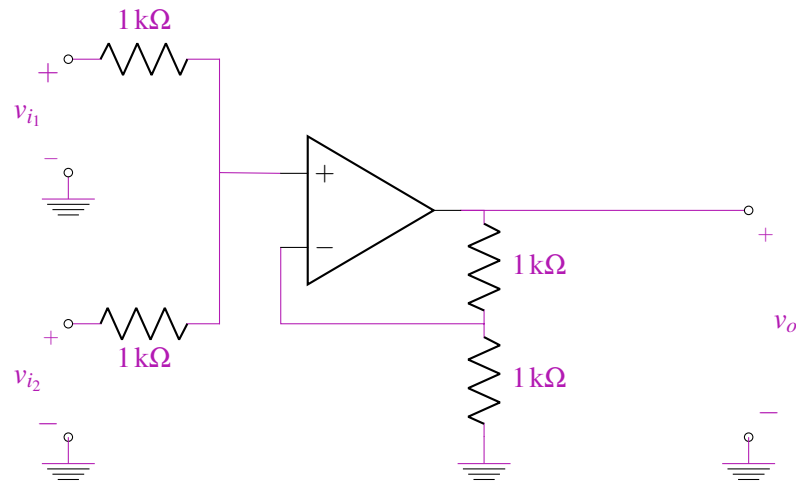
i.



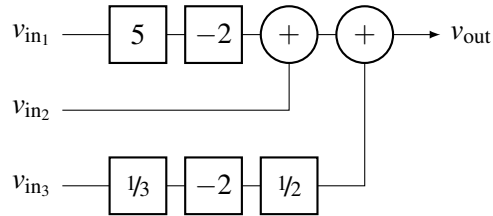
ii.



iii.



(c) Check that your designs from part (b) indeed enable a library of compose-able elements (i.e., that you can connect any block to any other block without having the intended functionality be modified) by implementing the block diagram shown below.



Answer:

If op-amps are used properly to drive the output of each stage, there will be no load on any stage due to the previous stage. This will give us the behavior we want. We get:

$$v_{out} = -10v_{in1} + v_{in2} - \frac{1}{3}v_{in3}$$