1. Solving Systems of Equations

(a) Systems of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following system of equations, state whether or not a solution exists. If a solution exists, list all of them.

i. $\begin{cases} 49x+7y=49 \\ 42x+6y=42 \end{cases}$ ii. $\begin{cases} 5x+3y=-21 \\ 2x+y=-9 \end{cases}$ iii. $\begin{cases} 49x+7y=60 \\ 42x+6y=30 \end{cases}$

iv. $\begin{cases} 2x+2y+4z=-1 \\ y+z=-2 \\ x+2y+3z=2 \end{cases}$ v. $\begin{cases} 2x+2y+4z=6 \\ y+z=1 \\ x+2y+3z=4 \end{cases}$ vi. $\begin{cases} x+y+z=4 \\ 3z=6 \\ y+z=3 \end{cases}$

(b) Systems of equations can also be interpreted graphically. We will try to build a graphical intuition for the results you found in the previous part. Follow along as your TA walks through `dis1A.ipynb`.

2. Gaussian Elimination

Use Gaussian elimination to solve the following systems. Does a solution exist? Is it unique?

(a)

$$\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 4 & 2 \\ 1 & 2 & 8 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$