

### 1. Mechanical Inverses

In each part, determine whether the inverse of  $\mathbf{A}$  exists. If it exists, find it.

(a)  $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

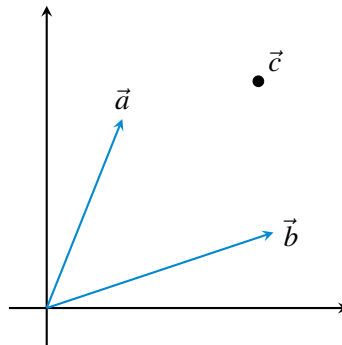
(b)  $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$

(c)  $\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 0 & 4 \end{bmatrix}$

(d)  $\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$

### 2. Visualizing Span

We are given a point  $\vec{c}$  that we want to get to, but we can only move in two directions:  $\vec{a}$  and  $\vec{b}$ . We know that to get to  $\vec{c}$ , we can travel along  $\vec{a}$  for some amount  $\alpha$ , then change direction, and travel along  $\vec{b}$  for some amount  $\beta$ . We want to find these two scalars  $\alpha$  and  $\beta$ , such that we reach point  $\vec{c}$ . That is,  $\alpha\vec{a} + \beta\vec{b} = \vec{c}$ .



- (a) First, consider the case where  $\vec{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ , and  $\vec{c} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ . Find the two scalars  $\alpha$  and  $\beta$ , such that we reach point  $\vec{c}$ . What if  $\vec{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $\vec{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ ?
- (b) Now formulate the general problem as a system of linear equations and write it in matrix form.

### 3. Span Proofs

Given some set of vectors  $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ , show the following:

(a)

$$\text{span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\} = \text{span}\{\alpha\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}, \text{ where } \alpha \text{ is a non-zero scalar}$$

In other words, we can scale our spanning vectors and not change their span.

(b)

$$\text{span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\} = \text{span}\{\vec{v}_2, \vec{v}_1, \dots, \vec{v}_n\}$$

In other words, we can swap the order of our spanning vectors and not change their span.

(c)

$$\text{span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\} = \text{span}\{\vec{v}_1 + \vec{v}_2, \vec{v}_2, \dots, \vec{v}_n\}$$

In other words, we can replace one vector with the sum of itself and another vector and not change the span.