EECS 16A Designing Information Devices and Systems I
Spring 2018 Discussion 5B

1. Diagonalization

One of the most powerful ways to think about matrices is to think of them in diagonal form [1].

(a) Consider a matrix $\mathbf{A}$, a matrix $\mathbf{V}$ whose columns are the eigenvectors of $\mathbf{A}$, and a diagonal matrix $\mathbf{\Lambda}$ with the eigenvalues of $\mathbf{A}$ on the diagonal (in the same order as the eigenvectors (or columns) of $\mathbf{V}$). From these definitions, show that

$$\mathbf{AV} = \mathbf{V\Lambda}$$

(b) We now multiply both sides on the right by $\mathbf{V}^{-1}$ and get $\mathbf{A} = \mathbf{V\Lambda V}^{-1}$, the diagonal form of $\mathbf{A}$. Consider the action of $\mathbf{A}$ on a coordinate vector $\vec{x}_u$ in the standard basis. Interpret each step of the following calculation in terms of coordinate transformations and scaling by eigenvalues.

$$\mathbf{A}\vec{x}_u = \mathbf{V\Lambda V}^{-1}\vec{x}_u$$

2. Matrix Powers

One of the most powerful things about matrix diagonalization is that it gives us some insight into polynomial functions of matrices.

(a) Write $\mathbf{A}^N$ using the diagonalization of $\mathbf{A}$ and simplify your result as much as possible. What do you get?

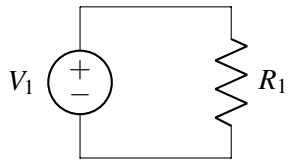
(b) How could you find $\mathbf{A}$ raised to any power while only doing three matrix multiplications?

(c) Can you suggest an easy way to compute any polynomial function of $\mathbf{A}$?

[1] Not all matrices can be put in this form but most can. The ones that can't be diagonalized can be put in a similar form called the Jordan form.

3. A Simple Circuit

For the circuit shown below, find the voltages across all the elements and the currents through all the elements.



(a) In the above circuit, pick a ground node. Does your choice of ground matter?

(b) With your choice of ground, label the node potentials for every node in the circuit.

(c) Label all of the branch currents. Does the direction you pick matter?

(d) Draw the $+/-$ labels on every element. What convention must you follow?

(e) Set up a matrix equation in the form $\mathbf{A}\vec{x} = \vec{b}$ to solve for the unknown node potentials and currents. What are the dimensions of the matrix $\mathbf{A}$?

(f) Use KCL to find as many equations as you can for the matrix.

(g) Use *IV* relations to find the remaining the equations for the matrix.

(h) Solve the system of equations if $V_1 = 5\,\text{V}$ and $R_1 = 5\,\Omega$.