

1. Bio-Molecule Detector

We've already seen how to build a bio-molecule detector where bio-molecules change the resistance between two electrodes. In this problem, we will explore a different scheme, where bio-molecules change the capacitance between two electrodes.

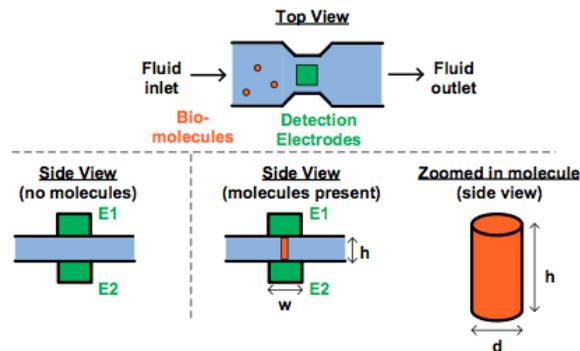
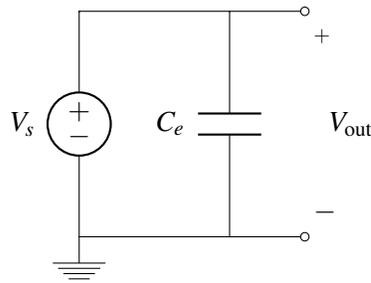


Figure 1: Bio-molecule detector.

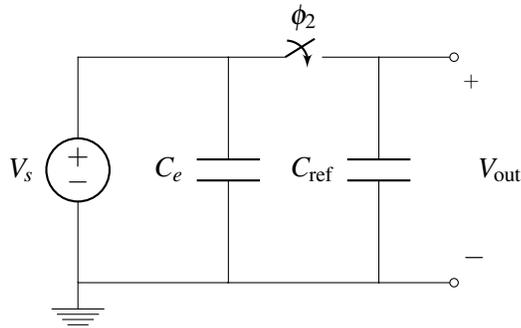
As shown in Figure 1, the detector works by flowing a liquid that may or may not contain the biomolecules through a region in the device that has electrodes on the top and bottom of the liquid channel. The electrodes (E1/E2 in Figure 1) are chemically “functionalized” (using e.g. some appropriately designed antibodies), so that if the specific bio-molecule of interest is present in the fluid sample, one or more of the molecules will get physically trapped between the two electrodes (bottom right of Figure 1). After all of the fluid has been cleared out of the device (i.e., so that if there are bio-molecules present, there is only air in between the two electrodes E1/E2), we can then figure out whether or not one or more bio-molecules were trapped by measuring the capacitance between the two electrodes.

- If no bio-molecules are present, what should the capacitance between E1/E2 be? Assume that the electrodes are $10\mu\text{m}$ on each side and that the electrodes are 100nm apart. Leave your answer in terms of ϵ_0 .
- As shown in Figure 1, if each bio-molecule is a cylinder with diameter $d = 2\mu\text{m}$, height $h = 100\text{nm}$, and permittivity $\epsilon_b = 4\epsilon_0$, what would the capacitance between E1 and E2 be if only a single bio-molecule is trapped? Note that you can assume that the trapped molecule is exactly vertically oriented when it is trapped – i.e., the top and bottom faces of the molecule are both aligned with surfaces of the electrodes.
- Using the same numbers for d , h , and ϵ_b as in part (b), as a function of the number of trapped bio-molecules $N_{\text{molecules}}$, what is the capacitance between E1 and E2? (Note that you can assume that $N_{\text{molecules}}$ is small enough that all of the molecules fit within the electrode area and that all of the molecules are still trapped in exactly vertical orientation.)
- The presence of bio-molecules changes the capacitance between the two electrodes. We would like to turn this change in capacitance into a change in voltage. Suppose that you design the circuit shown

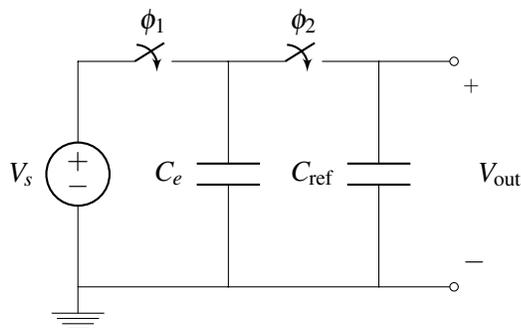
below to accomplish this, where C_e is the capacitance between the electrodes. If the value of C_e changes, does V_{out} change?



- (e) In order to measure the change in capacitance, we might try to use charge sharing with another capacitor whose capacitance we already know. Let's call this capacitor C_{ref} . Charge-sharing circuits generally have more than one phase. In the first phase, one set of switches might be closed, and in another phase, a different set of switches might be closed. Switches labeled ϕ_x are closed in phase x . We also need to know in what phase to measure the output voltage. Re-draw the circuit shown below in each phase. Does the voltage V_{out} depend on C_e ? Suppose that we measure V_{out} in phase 2.



- (f) Now we add another switch and come up with the circuit below. Re-draw the circuit in each phase. Assume that we measure V_{out} in phase 2 once again. Does V_{out} depend on the capacitance C_e ?



- (g) In the above circuit, how many times can you measure C_e ? Suppose that the circuit cycled through phase 1 and 2 again, could we measure C_e again? Modify the circuit, such that you can repeatedly use the circuit to measure C_e .
- (h) Finally, we have a circuit that can be used to measure C_e . Suppose that we now want to output 5 V when there are at least 5 biomolecules present. Otherwise, we want to output 0 V. Design a circuit that accomplishes this using a comparator, a voltage source, and your circuit from part (g).