This homework is due March 12, 2018, at 23:59.
Self-grades are due March 15, 2018, at 23:59.

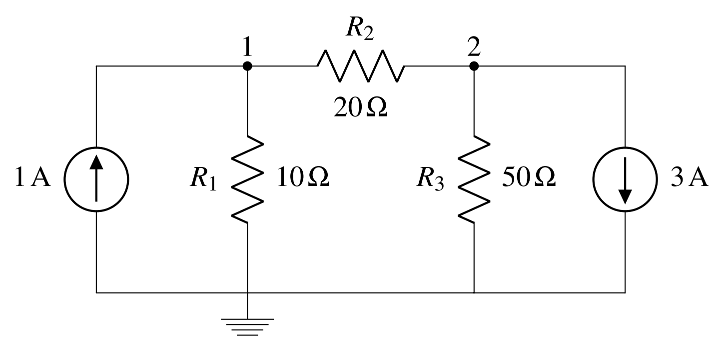
Submission Format
Your homework submission should consist of **one** file.

- `hw7.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

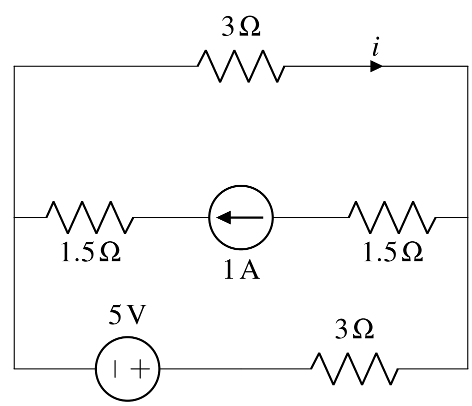
1. (PRACTICE) Circuit Analysis

Solve the circuit given below for currents and voltages.



2. (PRACTICE) Superposition

Solve the circuits shown below using superposition.

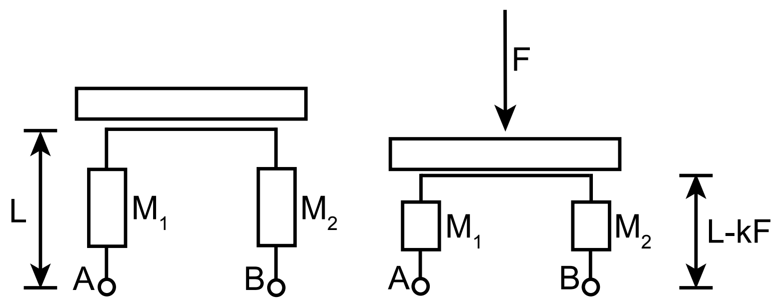


3. Fruity Fred

Fruity Fred just got back from Berkeley Bowl with a bunch of mangoes, pineapples, and coconuts. He wants to sort his mangoes in order of weight, so he decides to use his knowledge from EE16A to build a scale.

He finds two identical bars of material (M_1 and M_2) of length L (meters) and cross-sectional area A_c (meters^2), which are made of a material with resistivity ρ. He knows that the length of these bars decreases by k meters per Newton of force applied, while the cross-sectional area remains constant.

He builds his scale as shown below, where the top of the bars are connected with an ideal electrical wire. The left side of the diagram shows the scale at rest (with no object placed on it), and the right side shows it when the applied force is F (Newtons), causing the length to decrease by kF meters. Fred's mangoes are not very heavy, so $L \gg kF$.



(a) Let R_{AB} be the resistance between nodes A and B. Write an expression for R_{AB} as a function of A_c, L, ρ, F, and k.

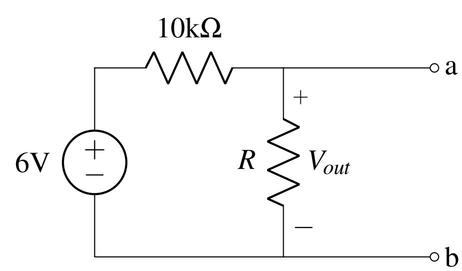
(b) Fred's scale design is such that the resistance R_{AB} changes depending on how much weight is placed on it. However, he really wants to measure a voltage rather than a resistance.

Design a circuit for Fred that outputs a voltage that is some function of the weight. Your circuit should include R_{AB}, and you may use any number of voltage sources and resistors in your design. Be sure to label where the voltage should be measured in your circuit. Also provide an expression relating the output voltage of your circuit to the force applied on the scale.

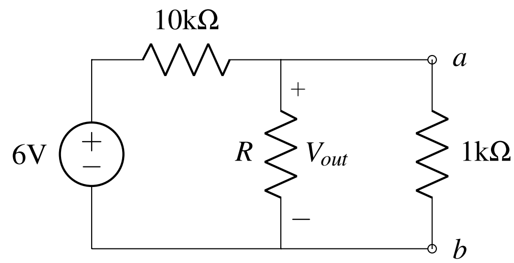
4. Resistive Voltage "Regulator"

In this problem, we will design a circuit that provides an approximately constant voltage divider across a range of loads. We will use a resistive voltage divider circuit. The goal is to design a circuit that, from a source voltage of 6V, would yield an output voltage within 5% of 4V for loads in the range of 1kΩ to 100kΩ.

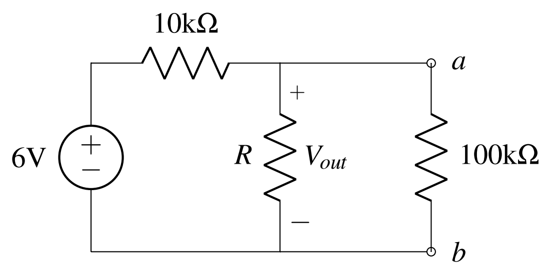
(a) First, consider the resistive voltage divider in the following circuit. What resistance R would achieve a voltage V_{out} of 4V?



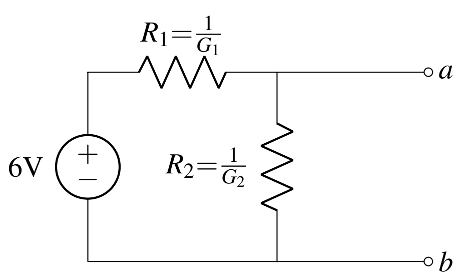
(b) Now using the same resistor R as calculated in part (a), consider loading the circuit with a resistor of $1\text{k}\Omega$ as depicted in the following circuit. What is the voltage V_{out} now?

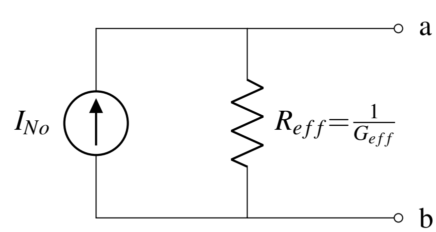


(c) Now using the same resistor R as calculated in part (a), consider loading the circuit with a resistor of $100\text{k}\Omega$, instead, as depicted in the following circuit. What is the voltage V_{out} now?

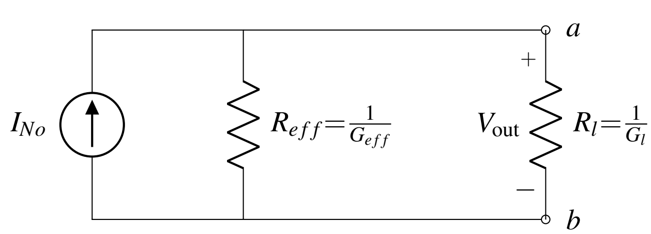


(d) Now we would like to design a divider that would keep the voltage V_{out} regulated for loads for a range of loads R_l. By that, we would like the voltage to remain within a 5% window of 4V. That is, we would like to design the following circuit such that $3.80\text{V} \leq V_{\text{out}} \leq 4.20\text{V}$ for a range of loads R_l. As a first step, what is the Norton equivalent of the circuit on the left? Write I_{No} and G_{eff} in terms of conductance values $G_1 = \frac{1}{R_1}$ and $G_2 = \frac{1}{R_2}$.





(e) For the second step, using the Norton equivalent circuit you found in part (d), what is the range of G_{eff} that achieves $3.80\text{V} \leq V_{\text{out}} \leq 4.20\text{V}$ in terms of I_{No} and G_l?



(f) Translate the range of G_{eff} in terms of I_{No} and G_l (that you found in part (e)) into a range on G_2 in terms of G_1 and G_l.

(g) Say we want to support loads in the range $1\text{k}\Omega \leq R_l \leq 100\text{k}\Omega$ with approximately constant voltage as described above (that is, $3.80\text{V} \leq V_{\text{out}} \leq 4.20\text{V}$). What is the range of G_2 in terms of G_1 now? Translate the range of G_2 in terms of G_1 into a range of R_2 in terms of R_1.

(h) Note that conductance is always non-negative. From the bounds on G_2 you found in the previous part, derive a bound on G_1 that ensures that G_2 is always non-negative and non-empty (that is, the whole range of possible G_2 values is non-negative and is not empty). Translate this range into a range of possible R_1 values.

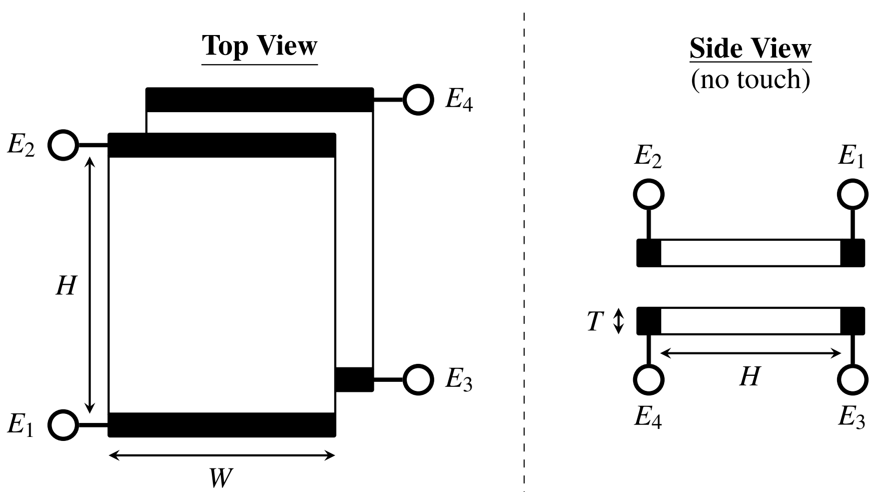
Hint: In addition to the conductance being non-negative, also make sure that the range for G_2 is non-empty.

(i) Pick the values of R_1 and R_2 that achieve $3.80\text{V} \leq V_{\text{out}} \leq 4.20\text{V}$ for $1\text{k}\Omega \leq R_l \leq 100\text{k}\Omega$ while minimizing the power consumed by the voltage divider circuit in open circuit (when there is no load attached to the output). What are these values R_1 and R_2? How much power is consumed in this case? Calculate and report this power consumption using both the original circuit and the Norton equivalent circuit. Are the power you calculated using the original circuit and the power you calculated using the Norton equivalent circuit equal?

(j) Now using the same values R_1 and R_2 from the previous part, load the circuit with a load of $51\text{k}\Omega$. How much power is consumed by each of the three resistors, R_1, R_2 and R_l (use the original circuit to compute the power)?

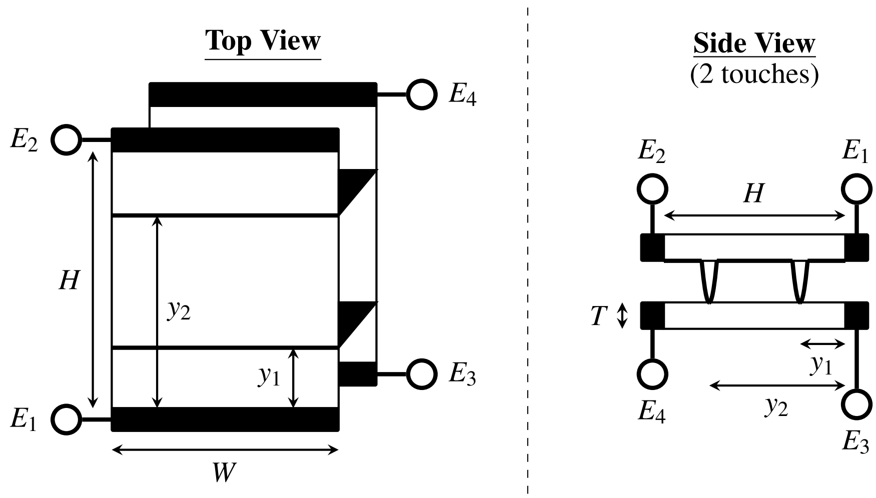
5. Multitouch Resistive Touchscreen

In this problem, we will look at a simplified version of the multitouch resistive touchscreen. In particular, rather than measuring the position of two potential touch points in both dimensions (i.e. a pair of coordinates (x_1, y_1) and (x_2, y_2) corresponding to two touch positions), let's think about a version where we are interested in measuring only the vertical position of the two touch points (i.e. y_1 and y_2). Therefore, unlike the touchscreens we looked at in class, both of the resistive plates (i.e. both the top and the bottom plate) would have conductive strips placed along their top and bottom edges, as shown below.

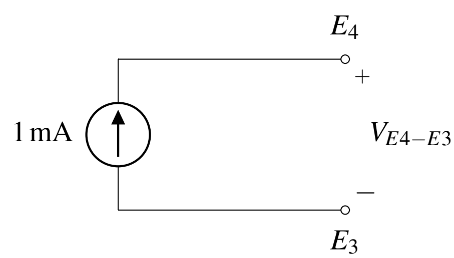


(a) Assuming that both of the plates are made out of a material with $\rho = 1\,\Omega\text{m}$ and that the dimensions of the plates are $W = 3\,\text{cm}$, $H = 12\,\text{cm}$, and $T = 1\,\text{mm}$, with no touches at all, what is the resistance between terminals E_1 and E_2 (which would be the same as the resistance between terminals E_3 and E_4)?

(b) Now let's look at what happens when we have two touch points. Let's assume that at wherever height the touch occurs, a perfect contact is made between the top plate and the bottom plate along the entire width of the plates (i.e. you don't have to worry about any lateral resistors), but that otherwise none of the electrical characteristics of the plates change. Defining the bottom of the plate as being $y = 0\,\text{cm}$ (i.e. a touch at E_1 would be at $y = 0\,\text{cm}$), let's assume that the two touches happen at $y_1 = 3\,\text{cm}$ and $y_2 = 7\,\text{cm}$ and that your answer to part (a) was $8\,\text{k}\Omega$ (which may or may not be the right answer). Draw a model with 6 resistors that captures the electrical connections between E_1, E_2, E_3, and E_4 and calculate their resistances. Note that for clarity, the system has been redrawn below to depict this scenario.



(c) Using the same assumptions as part (b), if you drove terminals E_3 and E_4 with a 1 mA current source (as shown below) but left terminals E_1 and E_2 open-circuited, what is the voltage you would measure across $E_4 - E_3$ (i.e. V_{E4-E3})?



(d) Now let's try to generalize the situation by assuming that the two touches can happen at any two arbitrary points y_1 and y_2, but with y_1 defined to always be less than y_2 (i.e. y_1 is always the bottom touch point). Leaving the setup the same as in part (c) except for the arbitrary y_1 and y_2, by measuring only the voltage between E_4 and E_3, what information can you extract about the two touch positions? Please be sure to provide an equation relating V_{E4-E3} to y_1 and y_2 as a part of your answer, and note that you may want to redraw the model from part (b) to help you with this.

(e) One of your colleagues claims that by measuring the appropriate voltages, not only can they extract what both y_1 and y_2 are in this system, but they can even do so by formulating a system of three independent voltage equations related to y_1 and y_2. As we will see later, this will allow us to gain some robustness to noise in the voltage measurements.

In order to facilitate this, write equations relating V_{E4-E2} and V_{E1-E3} to y_1 and y_2. (The third voltage we'll use is V_{E4-E3}, which you should have already derived an equation for in the previous part of the problem.)

6. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?