EECS 16A Designing Information Devices and Systems I
Spring 2018 Homework 8

This homework is due March 19, 2018, at 23:59.
Self-grades are due March 22, 2018, at 23:59.

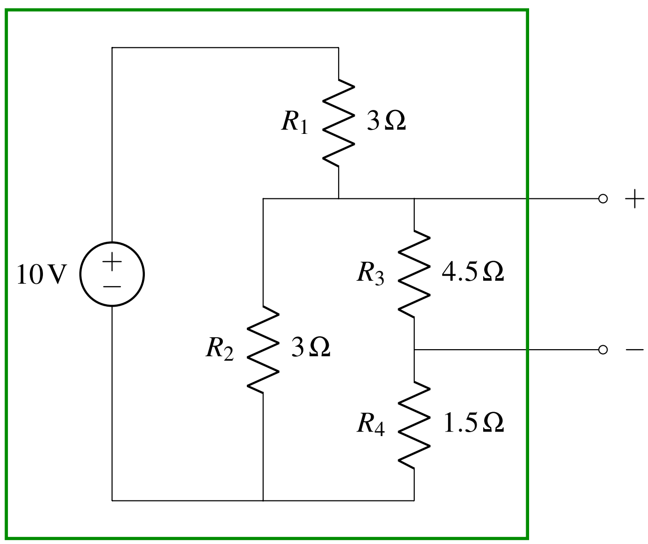
Submission Format
Your homework submission should consist of **one** file.

- `hw8.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

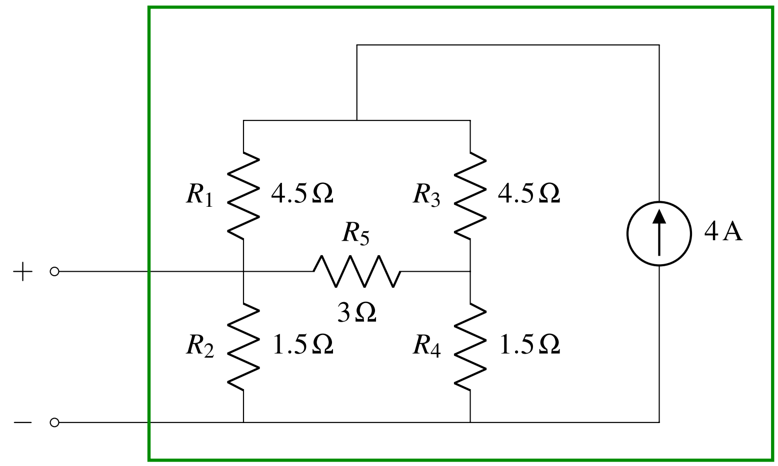
Submit the file to the appropriate assignment on Gradescope.

1. (PRACTICE) Thévenin and Norton Equivalent Circuits

(a) Find the Thévenin and Norton equivalent circuits seen from outside of the box.

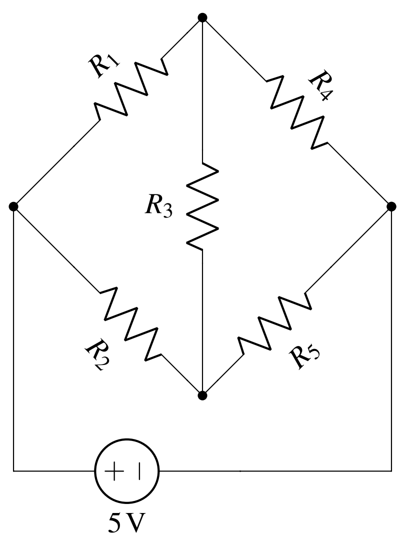


(b) Find the Thévenin and Norton equivalent circuits seen from outside of the box.

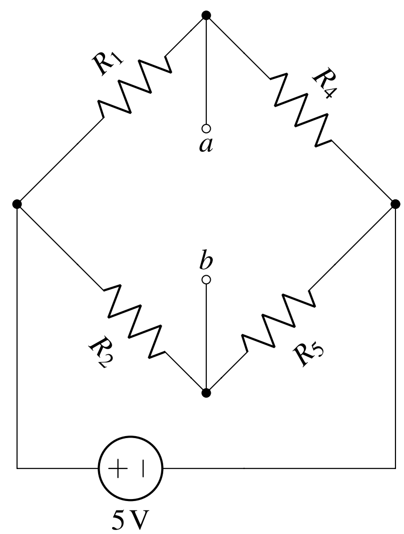


2. Wheatstone Bridge

Thévenin equivalence is a powerful technique we can use to solve the Wheatstone bridge circuit shown below. This circuit is used in many sensor application where a sensing element is the "bridge" resistor, R_3. It is often useful to find the current through the bridge resistor or the voltage across the bridge resistor. Intuitively, knowing I_{R_3} or V_{R_3} allows us to solve the rest of the circuit. In this problem, we want to find the current I_{R_3} flowing through the bridge resistor R_3.



(a) First, let's remove the bridge resistor R_3. Calculate the Thévenin equivalent voltage V_{th} between the two terminals a and b, for the circuit shown below, where the bridge resistor has been removed.



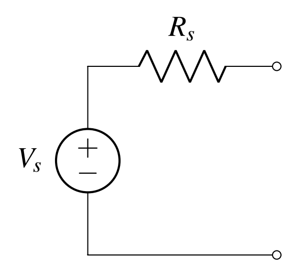
(b) Is the Thévenin voltage V_{th} you found in part (a) equal to the actual voltage V_{R_3} across the bridge resistor? Why or why not?

(c) Find the Thévenin resistance R_{th} between the two terminals a and b for the above circuit. Draw the Thévenin equivalent between the terminals a and b for the circuit above.

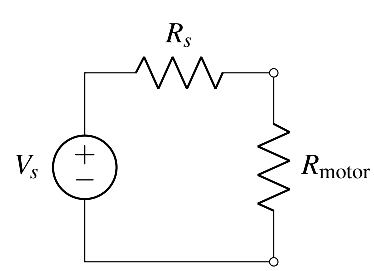
(d) With this equivalent circuit, calculate the current I_{R_3} through the bridge resistor and the voltage V_{R_3} across the bridge resistor.

3. Maximum Horsepower

You are an engineer working on an electric car. Your job is to design a motor to be used the car. Specifically you are designing the resistance of this motor. The battery used by this car has some series resistance, and it is modeled by the circuit shown below.



You attach your motor to the battery as shown below.



(a) Calculate the power P_s delivered by the voltage source in terms of V_s, R_s, and R_{motor}.

(b) Now calculate the power P_{motor} dissipated by the load resistor in terms of V_s, R_s, and R_{motor}.

(c) Suppose we wanted to maximize the power dissipated across the load. Find the optimal value for R_{motor} in terms of R_s.

Hint: Use calculus.

(d) Now you've swiched teams to designing the battery. Your job is now to pick the optimal R_s for maximizing the power delivered to the motor. What value of R_s should you pick?

Hint: Don't use calculus.

4. Digital to Analog Converter (DAC)

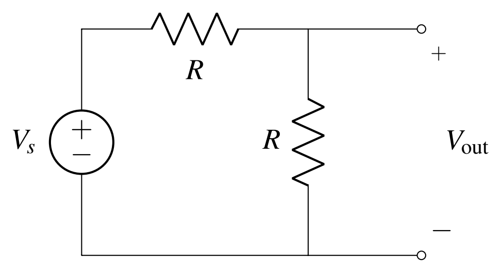
For some outputs, such as audio applications, we need to produce an analog output, or a continuous voltage from 0 to V_s. These analog voltages must be produced from digital voltages, that is sources, that can only be V_s or 0. A circuit that does this is known as a Digital to Analog Converter. It takes a binary representation of a number and turns it into an analog volage.

The output of a DAC can be represented with the equation shown below:

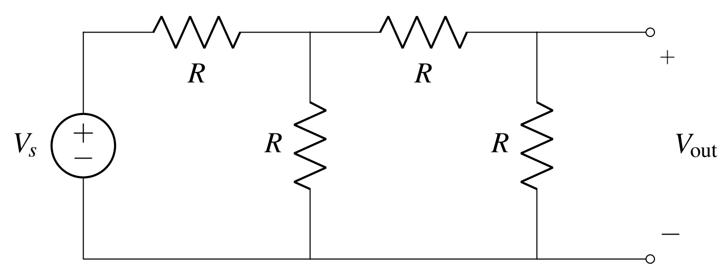
$$V_{\text{out}} = V_s \sum_{n=0}^{N} \frac{1}{2^n} \cdot b_n$$

where each binary digit b_n is multiplied by $\frac{1}{2^n}$.

(a) We know how to take an input voltage and divide it by 2:



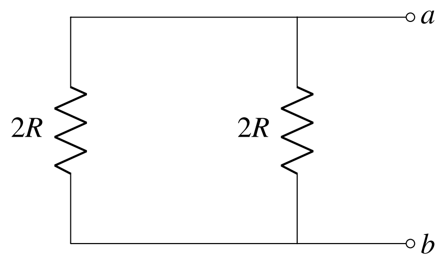
To divide by larger powers of two, we might hope to just "cascade" the above voltage divider. For example, consider:



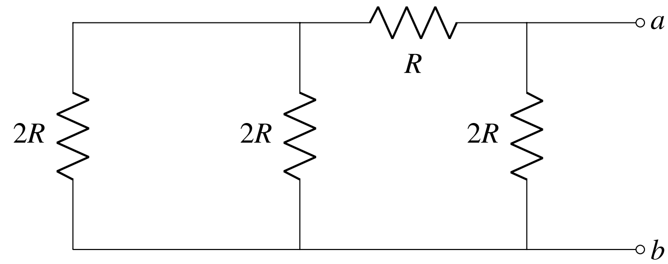
Calculate V_{out} in the above circut. Is $V_{\text{out}} = \frac{1}{4}V_s$?

(b) The R-$2R$ ladder, shown below, has a very nice property. For each of the circuits shown below, find the equivalent resistance looking in from points a and b. Do you see a pattern?

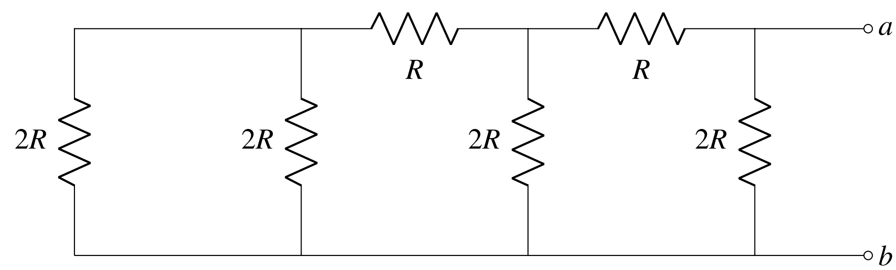
i.



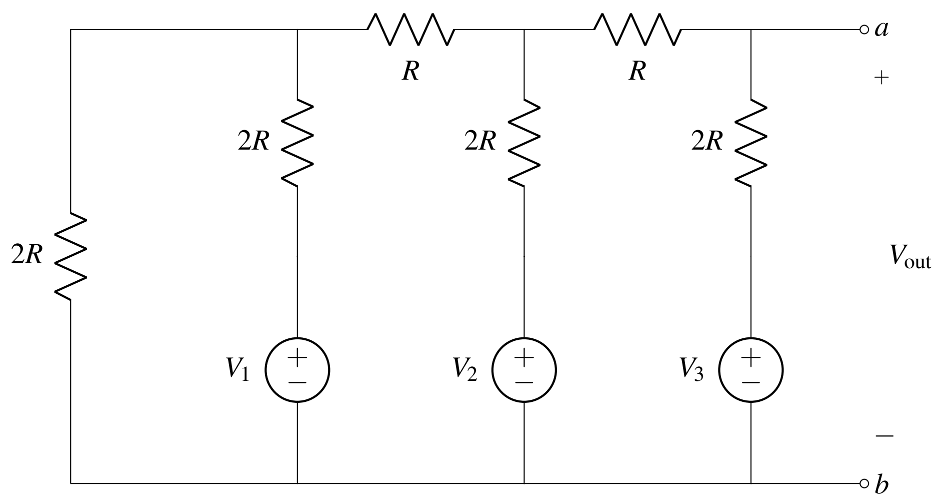
ii.



iii.



(c) The following circuit is an R-$2R$ DAC. To understand its functionality, use superposition to find V_{out} in terms of each V_k in the circuit.



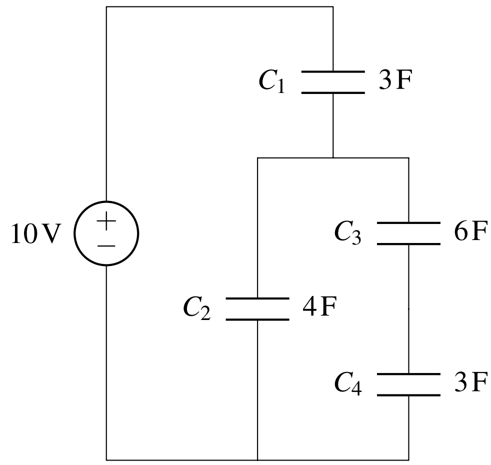
(d) We've now designed a 3-bit R-$2R$ DAC. What is the output voltage V_{out} if $V_2 = 1\,\text{V}$ and $V_1 = V_3 = 0\,\text{V}$?

(e) Draw the Thévenin equivalent of the above circuit, looking in from the terminals a and b with $V_2 = 1\,\text{V}$ and $V_1 = V_3 = 0\,\text{V}$.

(f) Suppose that we now attach a speaker to the DAC with a resistance of $\frac{R}{3}$. Why is the voltage across the speaker lower than what we computed in part (d)? What is the actual output voltage?

5. Mechanical Circuits with Capacitors and Resistors

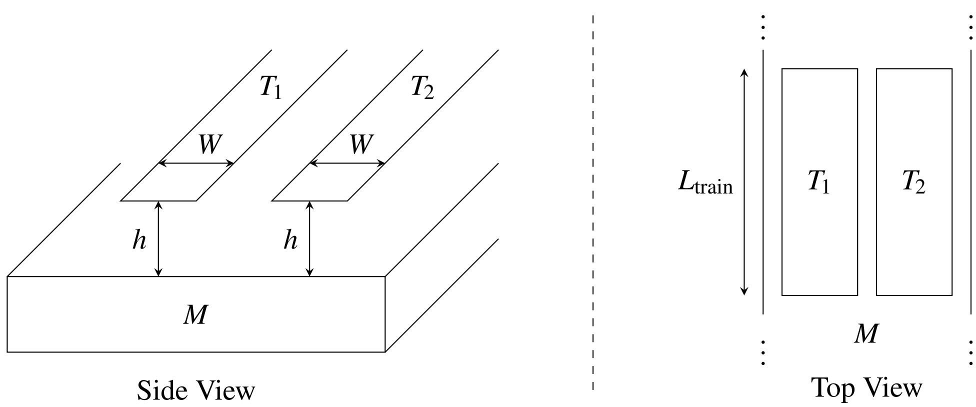
Find the voltages across and currents flowing through all of the capacitors at steady state.



6. Maglev Train Height Control System

One of the fastest forms of land transportation are trains that actually travel slightly elevated from ground using magnetic levitation (or "maglev" for short). Ensuring that the train stays at a relatively constant height above its "tracks" (the tracks in this case are what provide the force to levitate the train and propel it forward) is critical to both the safety and fuel efficiency of the train. In this problem, we'll explore how the maglev trains use capacitors to keep them elevated. (Note that real maglev trains may use completely different and much more sophisticated techniques to perform this function, so if you e.g. get a contract to build such a train, you'll probably want to do more research on the subject.)

(a) As shown below, let's imagine that all along the bottom of the train, we put two parallel strips of metal (T_1, T_2), and that on the ground below the train (perhaps as part of the track), we have one solid piece of metal (M).



Assuming that the entire train is at a uniform height above the track and ignoring any fringing fields (i.e., we can use the simple equations developed in lecture to model the capacitance), as a function of L_{train} (the length of the train), W (the width of T_1/T_2), and h (the height of the train off of the track), what is the capacitance between T_1 and M? How about the capacitance between T_2 and M?

(b) Any circuit on the train can only make direct contact at T_1 and T_2. Thus, to detect the height of the train, it would only be able to measure the equivalent capacitance between T_1 and T_2. Draw a circuit model showing how the capacitors between T_1 and M and between T_2 and M are connected to each other.

(c) Using the same parameters as in part (a), provide an expression for the equivalent capacitance between T_1 and T_2.

(d) Let's assume that instead of just detecting the height (by measuring the equivalent capacitance between T_1 and T_2), we also want to control it. Let's assume that the device we use to control the height takes in only one of only two commands: increase the height, or decrease the height. In particular, this device is controlled by an input voltage. If that voltage is greater than 2.5 V, it will push the train higher above the track, and if it is less than 2.5 V, it will let the train move down closer to the track.

Assuming that the train is 100 m long ($L_{\text{train}} = 100\,\text{m}$) and that the T_1/T_2 metals are each 1 cm wide ($W = 1\,\text{cm}$), design a circuit that will feed a voltage into the control device to make the train levitate 1 cm above the track. Be sure to show how your circuit is connected to T_1 and T_2, and be as specific as possible in terms of the component values you would use. You can use any combination of switches, voltage sources, current sources, resistors, and capacitors that you would like to implement this circuit.

(e) So far we've assumed that the height of the train off of the track is uniform along its entire length, but in practice, this may not be the case. Suggest and sketch a modification to the basic sensor design (i.e., the two strips of metal T_1/T_2 along the entire bottom of the train) that would allow you to measure the height at the train at 4 different locations.

7. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?