EECS 16A Designing Information Devices and Systems I
Spring 2018 Homework 9

This homework is due April 2, 2018, at 23:59.
Self-grades are due April 5, 2018, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw9.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. Super-Capacitors

In order to enable small devices for the "Internet of Things" (IoT), many companies and researchers are currently exploring alternative means of storing and delivering electrical power to the electronics within these devices. One example of these are "super-capacitors" - the devices generally behave just like a "normal" capacitor but have been engineered to have extremely high values of capacitance relative to other devices that fit in to the same physical volume.

Your startup named **IoT4eva** is designing a new device that will revolutionize the process of making pizza, and you've been put in charge of selecting an energy source for it. You can't find a battery that quite suits your needs, so you decide to try out some super capacitors in various configurations. The super capacitors will be charged up to a certain voltage in the factory and will then act as the power supply (source of voltage) for the electronics in your device.

(a) Assuming that the electronics in your device can be modeled as drawing a constant current with a value of i_{load}, draw circuit models for your device using the following configurations of super-capacitors as the power supply for the electronics:

- Config 1: a single super-capacitor
- Config 2: two super-capacitors stacked in series
- Config 3: two super-capacitors connected in parallel

(b) If each super-capacitor is charged to an initial voltage v_{init} and has a capacitance of C_{sc}, for each of the three configurations above, write an expression for the voltage supplied to your electronics as a function of time after the device has been activated.

(c) Now let's assume that your electronics require some minimum voltage v_{min} in order to function properly. For each of the three super-capacitor configurations, write an expression you could use to calculate the lifetime of the device.

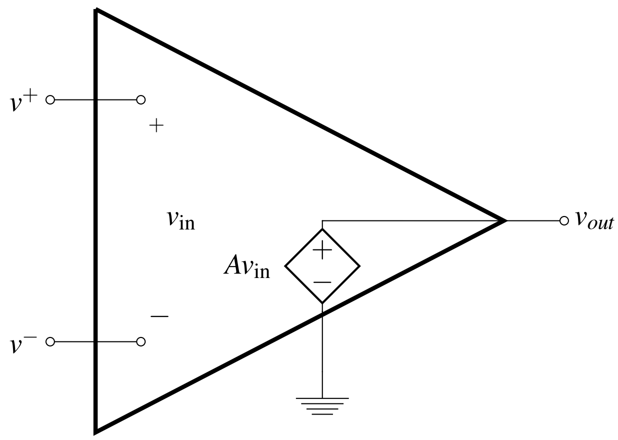
(d) Assuming that a single super-capacitor doesn't provide you sufficient lifetime and so you have to spend the extra money (and device volume) for another super-capacitor, which configuration would you pick and why would you pick one over the other?

- Config 2: two super-capacitors stacked in series

- Config 3: two super-capacitors connected in parallel

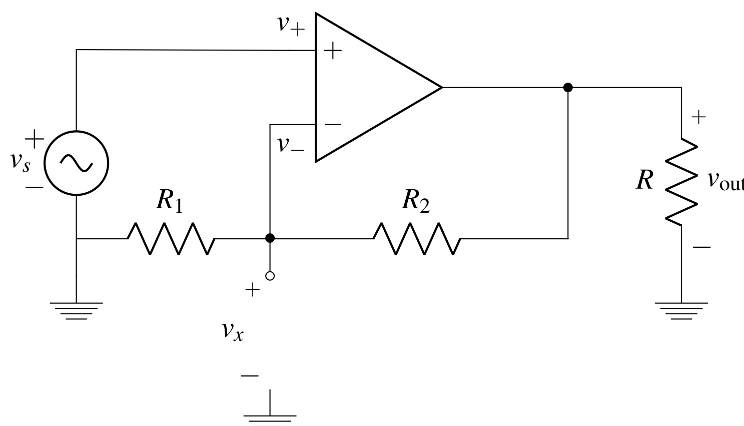
2. Op-Amp Golden Rules

In this question, we are going to show that the Golden Rules for op-amps hold by analyzing equivalent circuits and then taking the limit as the open-loop gain approaches infinity. Below is a picture of the equivalent model of an op-amp we are using for this question.



(a) Now consider the circuit below.

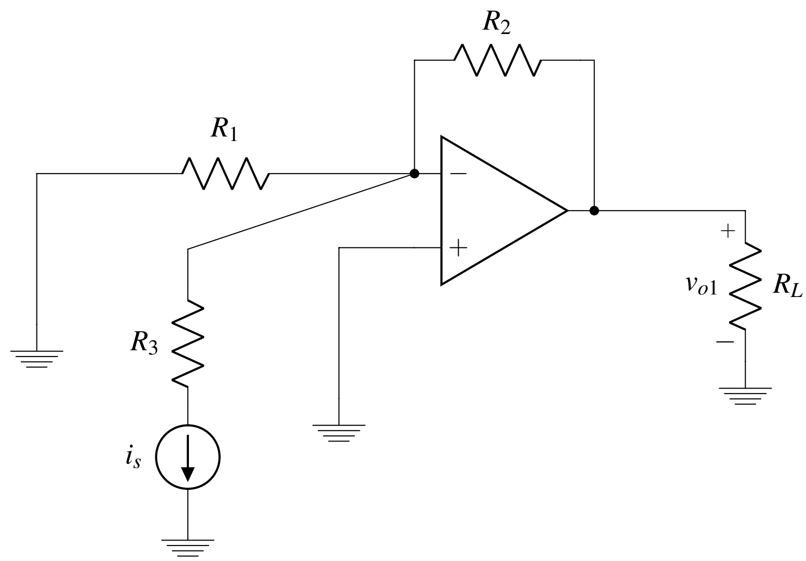
Draw an equivalent circuit by replacing the op-amp with the op-amp model shown above and calculate v_{out} and v_x in terms of A, v_s, R_1, R_2 and R. Is the magnitude of v_x larger or smaller than the magnitude of v_s? Do these values depend on R?



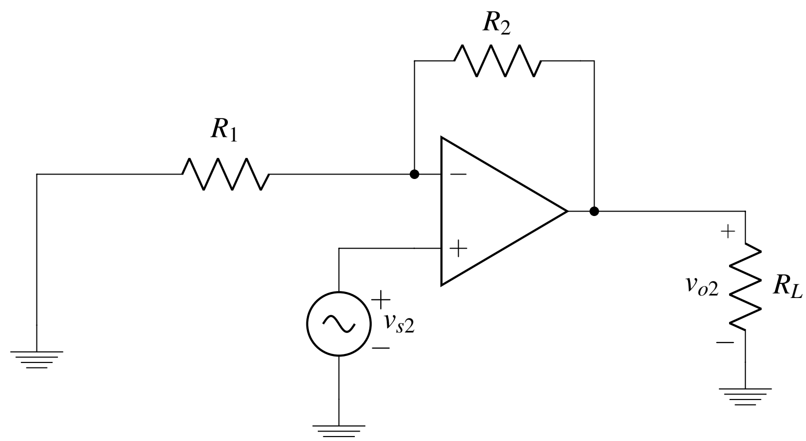
(b) Using your solution to part (a), calculate the limits of v_{out} and v_x as $A \to \infty$. Do you get the same answers if you apply the Golden Rules ($v_+ = v_-$ when there is negative feedback)?

3. Amplifier with Multiple Inputs

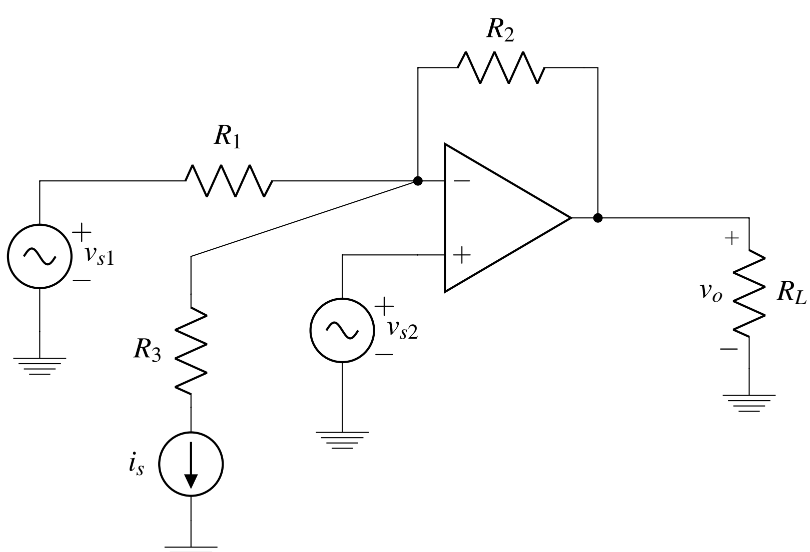
(a) Use the Golden Rules to find v_{o1} for the circuit below.



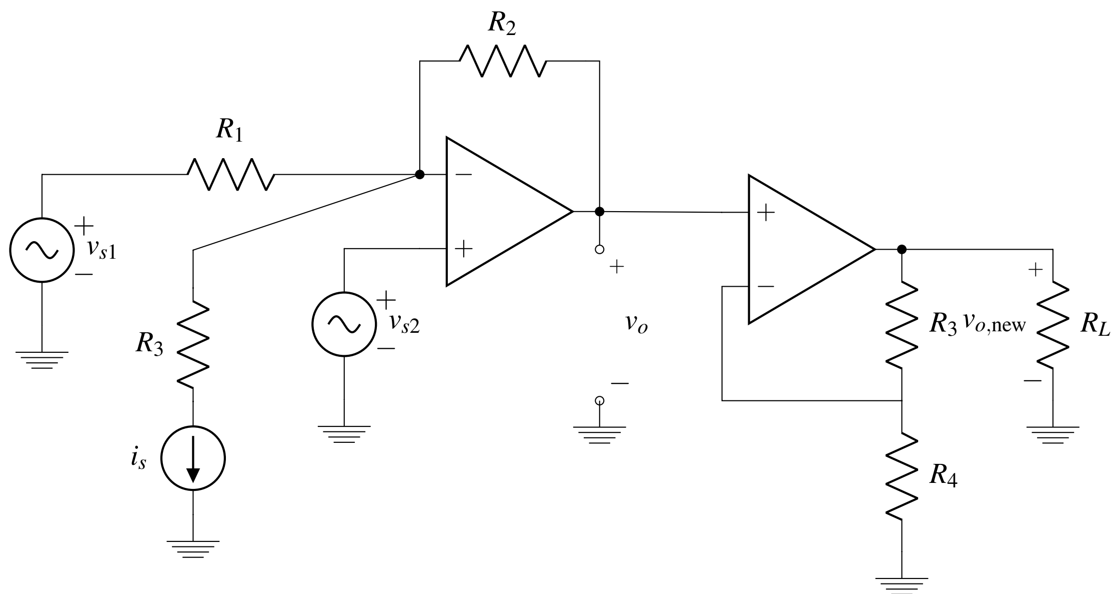
(b) Use the Golden Rules to find v_{o2} for the circuit below.



(c) Use the Golden Rules to find the output voltage v_o for the circuit shown below.

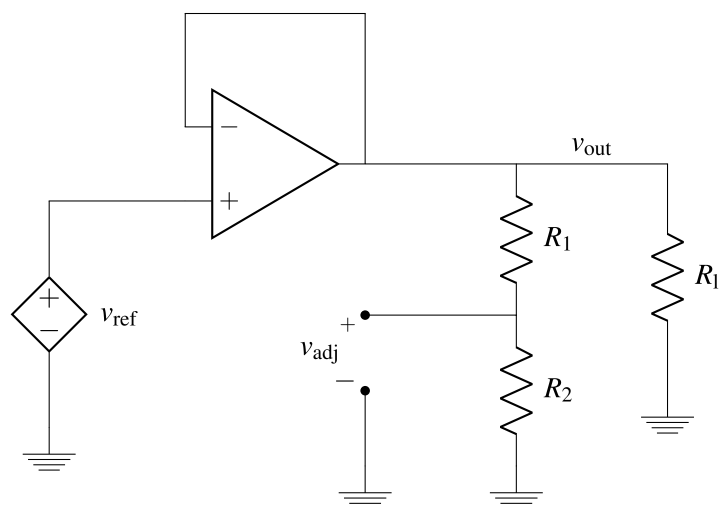


(d) Now add a second stage as shown below. What is $v_{o,\text{new}}$? Does v_o change between part (c) and this part? Does the voltage $v_{o,\text{new}}$ depend on R_L?



4. Op-Amps and State Transition Matrices

Consider the following circuit that where we are given that $v_{\text{ref}} = v_{\text{adj}} + 1.25$.



(a) Express v_{out} in terms of the other voltages and resistor values. Then express v_{adj} in terms of v_{out}.

(b) Let us model the nodal voltages as updating once every dt units of time to see how the long term steady state of the system behaves in this circuit. Use the state vector given below and construct a state transition matrix for the circuit. More precisely find the matrix $\vec{s}(t+dt) = A\vec{s}(t)$.

$$\vec{s} = \begin{bmatrix} v_{\text{out}} \\ v_{\text{adj}} \\ v_{\text{ref}} \\ 1.25 \end{bmatrix}$$

(c) Now find the eigenvalues of the matrix. You may use an online tool like Wolfram Alpha for this part. What do these eigenvalues say about the existence steady state of the system? If there exists a steady state, write it down.

5. Cool For The Summer

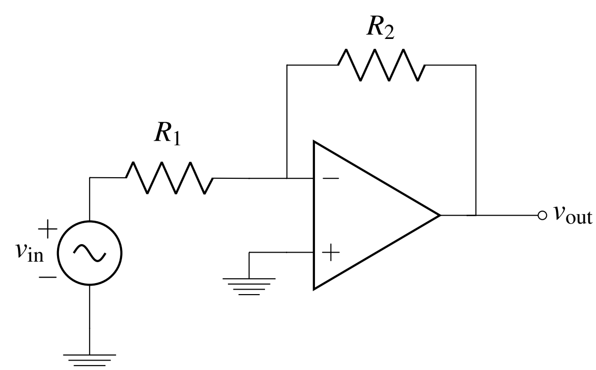
You and a friend want to make a box that helps control an air conditioning unit. You both have dials that display a voltage: 0 means that you want to leave the temperature as it is. Negative voltages mean that you want to reduce the temperature. (It's hot, so we will assume that you never want to increase the temperature – so, we're not talking about a Berkeley summer...)

Your air conditioning unit, however, responds to positive voltages. The higher the magnitude of the voltage, the stronger it runs. At zero, it is off.

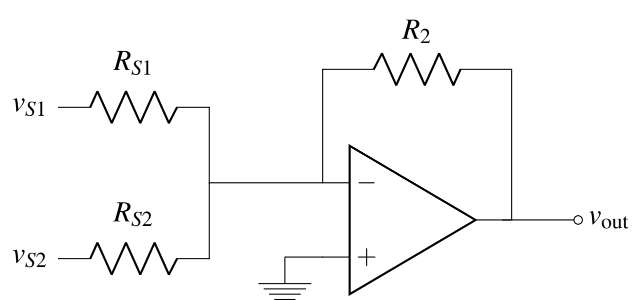
Therefore, you need a box that is an inverting summer – it outputs a weighted sum of two voltages where the weights are both negative. The sum is weighted because each of you has your own subjective sense of how much to turn the dial down, so you need to compensate for this.

This problem walks you through this using an op-amp.

(a) As a first step, find v_{out} in terms of R_2, R_1, v_{in}.



(b) Now we will add a second input to this circuit as shown below. Find v_{out} in terms of v_{S1}, v_{S2}, R_{S1}, R_{S2} and R_2.



(c) Let's suppose that you want $v_{out} = -\left(\frac{1}{4}v_{S1} + 2v_{S2}\right)$ where v_{S1} and v_{S2} represent the input voltages from you and your friend. Select resistor values such that the circuit implements this desired relationship.

(d) Now suppose that you have another AC unit that you want to add to the same room. This unit however, functions opposite to the already existing unit; it responds to negative voltages. You want to run both units at the same time. Add another op-amp based circuit to your existing circuit to create an output for the second AC unit.

6. IoT4eva Revisited

After guiding them to make an intelligent selection for their super-capacitors, IoT4eva was so happy with your performance that you got a promotion! The good news is that you're getting paid more, but the "bad" news is that you have more responsibilities too. In particular, you are now responsible not only for selecting the super-capacitors used to power the device, but also for building the rest of the circuitry associated with the power supply.

In practice, many real circuits (especially sensors that are trying to detect very small signals) don't like to operate with supply voltages that vary substantially over time. Remembering that the voltage on our super capacitors drops linearly as we pull current out of them. This means that if we want to use these super capacitors for our device, we need to build another circuit. This circuit is powered by the super-capacitor and produces a constant voltage at its output, where this voltage will then be used to supply power to rest of the device. These circuits are often referred to as "voltage regulators," and in this problem we'll explore how to build the simplest form of such a voltage regulator.

(a) The first problem we have to solve to realize such a voltage regulator is to figure out how to build a reference that would allow us to set the voltage at the output of our regulator to a known absolute value. Fortunately, someone else in the company has already built one of those and made it available to you – the Thévenin equivalent of this circuit is a voltage source whose value is $0.8\,\text{V}$ and a resistance of $1\,\text{k}\Omega$. (The internals of this voltage reference circuit aren't important for this problem, but as you should see shortly, this circuit by itself is not appropriate for supplying power to the rest of the device.)

Now that we have a reference, we can focus on the core of the voltage regulator itself. Using this reference circuit, an op-amp, and resistors, design a circuit that is powered by the super-capacitor voltage V_{sc} (which for now you can assume is always high enough for the circuit to work) and that would produce a constant $1.2\,\text{V}$ supply voltage for the rest of the device. Note that you can model the load from the rest of the device as a $10\,\text{mA}$ current source; please be sure to choose specific values for any resistors you use in your circuit as well.

Hint: Remember that the op-amp itself needs to be supplied with power, and the only source of power we have available is the super-capacitor.

(b) Now that we've built the voltage regulator and we know that we want its output voltage to stay fixed at $1.2\,\text{V}$, what is the minimum voltage we need on our super capacitors $V_{sc,min}$ to ensure that the regulator can indeed produce a fixed $1.2\,\text{V}$ output?

(c) One of the most important things to evaluate about a voltage regulator is its efficiency – i.e., the power dissipated by the load circuits (in this case, the rest of the IoT4eva device) divided by the total amount of power delivered by the power supply. Continuing to model the rest of the IoT4eva device as a $10\,\text{mA}$ current source, how much power is dissipated by the $10\,\text{mA}$ current source? Assuming that all of the IoT4eva's $10\,\text{mA}$ current flows through the super-capacitor, and that no other current is added by the op-amp itself, how much power is delivered by the super-capacitor? In this case, what is the overal efficiency of our design?

7. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?