# EECS 16A Designing Information Devices and Systems I
# Spring 2018 Homework 10

## This homework is due April 9, 2018, at 23:59.
## Self-grades are due April 12, 2018, at 23:59.

**Submission Format**
Your homework submission should consist of **two** files.

- `hw10.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned)

Submit each file to its respective assignment on Gradescope.

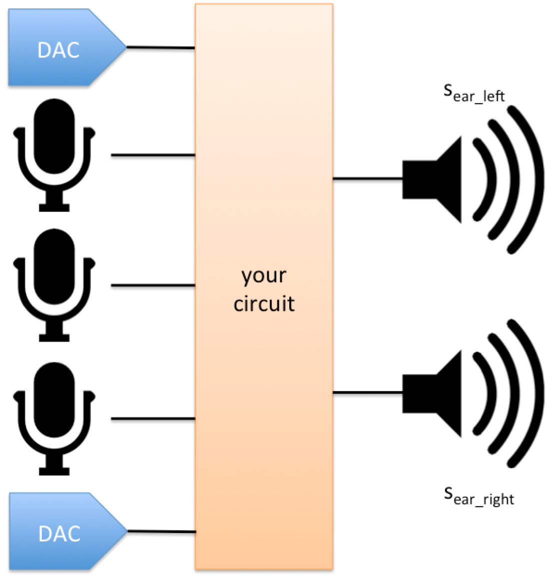
**1. Noise Cancelling Headphones Part 2**

Almost everyone has tried "noise cancelling" headphones at some point. The basic goal of noise cancelling headphones is for the user to hear only the desired audio signal and not any other sounds from external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources and then feeds a signal in to your speakers that cancels (subtracts out) that externally-generated sound.

**Solution:**

There are a lot of different solutions for this problem. This solution is aggressive and minimal, so be patient with your understanding. If your solution solves the same problem, you will receive credit.

(a) In discussion, we had just one speaker and one microphone, but almost all headphones today have two speakers (one for each ear). Adding an extra speaker that can be driven by a separate audio stream typically makes things sound more real to us. For similar reasons, having multiple microphones to pick up ambient sounds from multiple different locations can help us do a better job of cancellation, if we can use that information in the right way.




Let's now assume that our system has 3 microphones and 2 speakers, and that the source of our audio is stereo – i.e., we have two different audio streams $s_{\text{left}}$ and $s_{\text{right}}$ (produced by two different DACs) that represent the ideal sounds we would like the user to hear in their left and right ear. We have three microphone audio signals $s_{\text{mic1}}$, $s_{\text{mic2}}$, and $s_{\text{mic3}}$, and let's assume that without any active noise cancellation, some fraction of the signal picked up by each microphone would be heard by the user in each of their ears. For example, $a_{\text{1left}}$ would represent the fraction of the signal picked up by microphone 1 that will be heard in the user's left ear, $a_{\text{2right}}$ would represent the fraction of the signal picked up by microphone 2 that will be in the user's right ear, etc.

Let the vector $\vec{s}_{\text{noise}}$ represent the noise heard in each ear and $\vec{s}_{\text{mic}}$ represent the sound in each mic. Find a matrix $\mathbf{A}$ such that $\vec{s}_{\text{noise}} = \mathbf{A}\vec{s}_{\text{mic}}$.

**Solution:**

$$\begin{bmatrix} s_{\text{noise\_left}} \\ s_{\text{noise\_right}} \end{bmatrix} = \begin{bmatrix} a_{\text{1left}} & a_{\text{2left}} & a_{\text{3left}} \\ a_{\text{1right}} & a_{\text{2right}} & a_{\text{3right}} \end{bmatrix} \begin{bmatrix} s_{\text{mic1}} \\ s_{\text{mic2}} \\ s_{\text{mic3}} \end{bmatrix}$$

(b) Assume no noise canceling, find an equation for $\vec{s}_{\text{ear}}$, the sound heard in each ear in terms of the two audio streams and $\vec{s}_{\text{noise}}$.

**Solution:**

We can represent this as the matrix multiplication and addition below.

$$\begin{bmatrix} s_{\text{ear\_left}} \\ s_{\text{ear\_right}} \end{bmatrix} = \begin{bmatrix} a_{\text{1left}} & a_{\text{2left}} & a_{\text{3left}} \\ a_{\text{1right}} & a_{\text{2right}} & a_{\text{3right}} \end{bmatrix} \begin{bmatrix} s_{\text{mic1}} \\ s_{\text{mic2}} \\ s_{\text{mic3}} \end{bmatrix} + \begin{bmatrix} s_{\text{left}} \\ s_{\text{right}} \end{bmatrix}$$

(c) In order to cancel the noise, we want to create a signal that is the inverse of $\vec{s}_{\text{noise}}$. Let $\vec{s}_{\text{cancel}}$ be the vector representing the cancel signal in each headphone. Find a matrix $\mathbf{B}$ in terms of the matrix $\mathbf{A}$ such that $\vec{s}_{\text{cancel}} = \mathbf{B}\vec{s}_{\text{mic}}$.

**Solution:**

For the setup:

$$\begin{bmatrix} s_{\text{ear\_left}} \\ s_{\text{ear\_right}} \end{bmatrix} = \mathbf{A} \begin{bmatrix} s_{\text{mic1}} \\ s_{\text{mic2}} \\ s_{\text{mic3}} \end{bmatrix} + \mathbf{B} \begin{bmatrix} s_{\text{mic1}} \\ s_{\text{mic2}} \\ s_{\text{mic3}} \end{bmatrix} + \begin{bmatrix} s_{\text{left}} \\ s_{\text{right}} \end{bmatrix}$$

We want $\mathbf{B} = -\mathbf{A}$.

(d) Assume that the microphones can be modeled as voltage sources, whose value $v_{\text{micn}}$ is proportional to $s_{\text{micn}}$, design and sketch a circuit that would implement the cancellation matrix **B**. You should assume that this circuit has three voltage inputs $v_{\text{mic1}}$, $v_{\text{mic2}}$, and $v_{\text{mic3}}$ and two voltage outputs $v_{\text{cancel\_left}}$ and $v_{\text{cancel\_right}}$ (corresponding to the voltages that will be subtracted from the desired audio streams in order to cancel the externally-produced sounds). In order to simplify the problem, you can assume that all of the $v_{\text{mic}}$ voltages are already centered at 0 V (relative to the DAC ground). Furthermore, assume all entries of the **A** matrix are positive. You may use op-amps and resistors to implement your circuit. You do not have to pick specific resistor values, but write expressions for each resistor value.

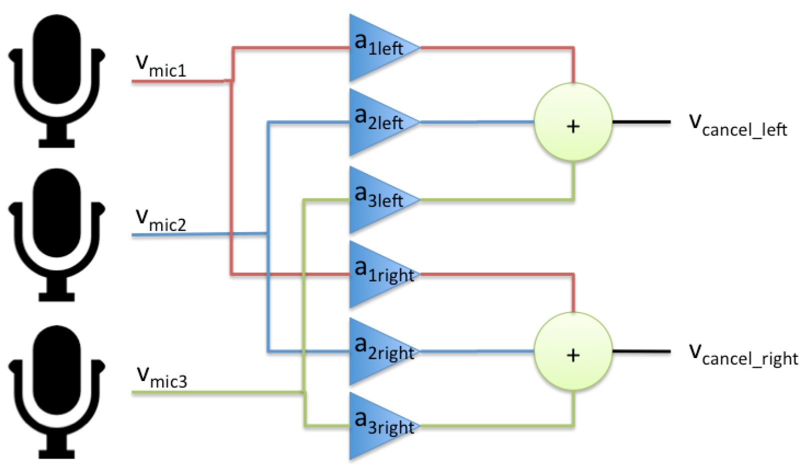
**Solution:**

Since we want to add $v_{\text{cancel\_left}}$ and $v_{\text{cancel\_right}}$ with the audio stream output to cancel noise, so we want these values to be

$$v_{\text{cancel\_left}} = -(a_{1\text{left}}\, v_{\text{mic1}} + a_{2\text{left}}\, v_{\text{mic2}} + a_{3\text{left}}\, v_{\text{mic3}})$$

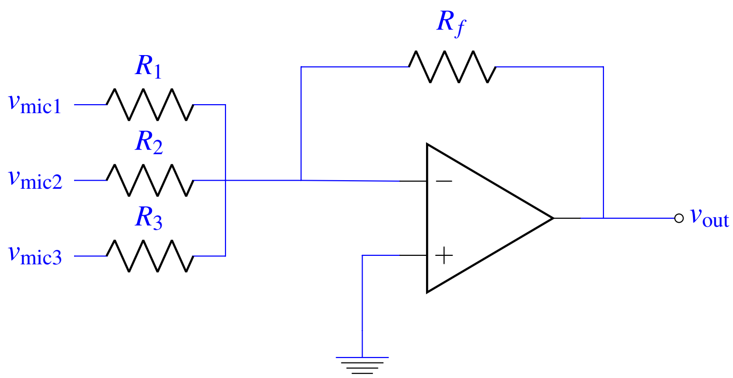
$$v_{\text{cancel\_right}} = -(a_{1\text{right}}\, v_{\text{mic1}} + a_{2\text{right}}\, v_{\text{mic2}} + a_{3\text{right}}\, v_{\text{mic3}})$$

Following the design process, we can draw a block diagram for the circuit.




We can see here that the two channels are actually independent from each other. The only point where they meet is at the microphone voltage. Thus, we can start by designing one channel.

For each channel, we want to build a circuit that sums its inputs and negates them. We have many options to pick from; the easiest will be an inverting summer.




This topology gives us

$$V_{\text{out}} = -\left(\frac{R_f}{R_1}V_{\text{mic1}} + \frac{R_f}{R_2}V_{\text{mic2}} + \frac{R_f}{R_3}V_{\text{mic3}}\right)$$

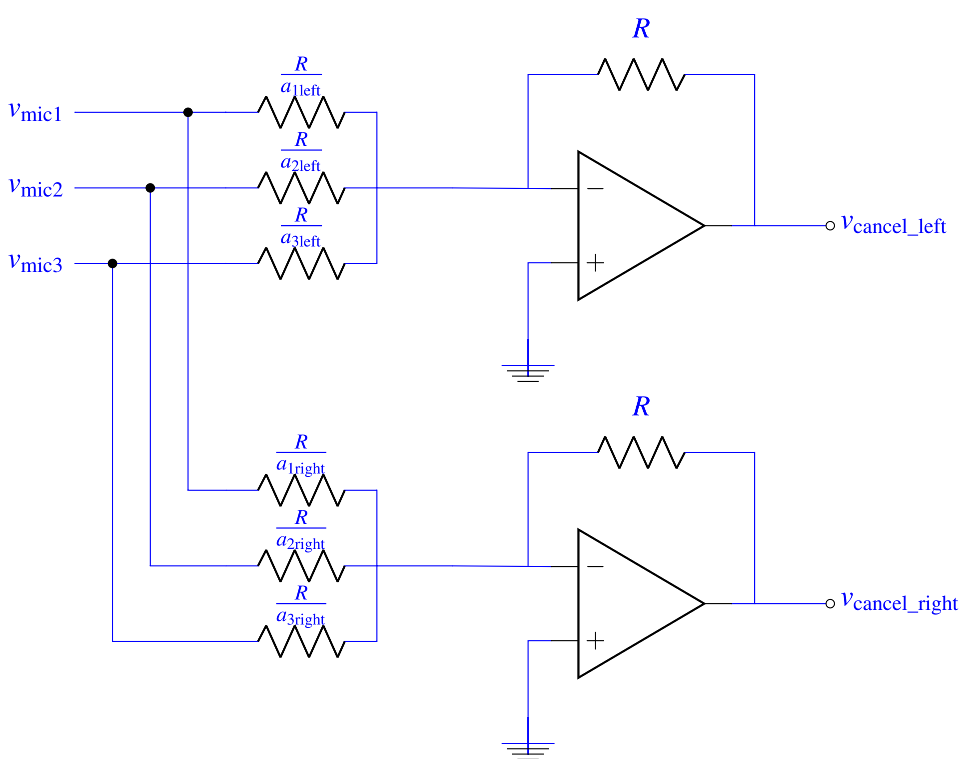
For the correct gains, we want

$$a_1 = \frac{R_f}{R_1}, a_2 = \frac{R_f}{R_2}, a_3 = \frac{R_f}{R_3}$$

We can pick $R_f$ arbitrarily. Then we set

$$R_1 = \frac{R_f}{a_1}, R_2 = \frac{R_f}{a_2}, R_3 = \frac{R_f}{a_3}$$

Now that we have all the building blocks we need, we can construct the two-channel noise cancelling circuit. $v_{\text{micn}}$ is connected to the output of the microphone buffers. We can choose an arbitrary value for $R$, for example, $1\,\text{k}\Omega$.

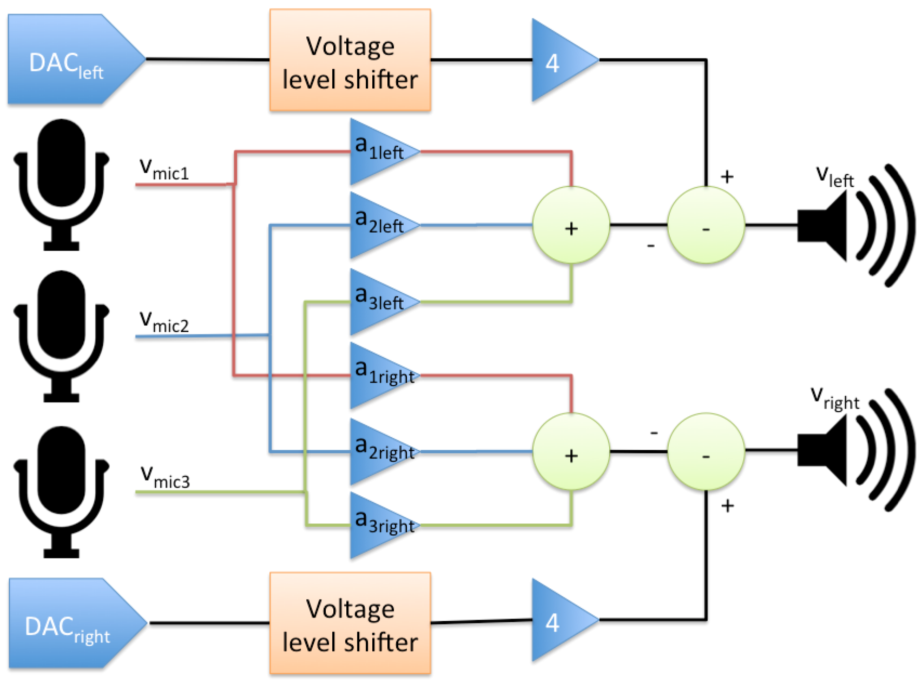


We just pick $R$ to be some value, we then calculate the appropriate resistances in the rest of the circuit, and we are good to go!

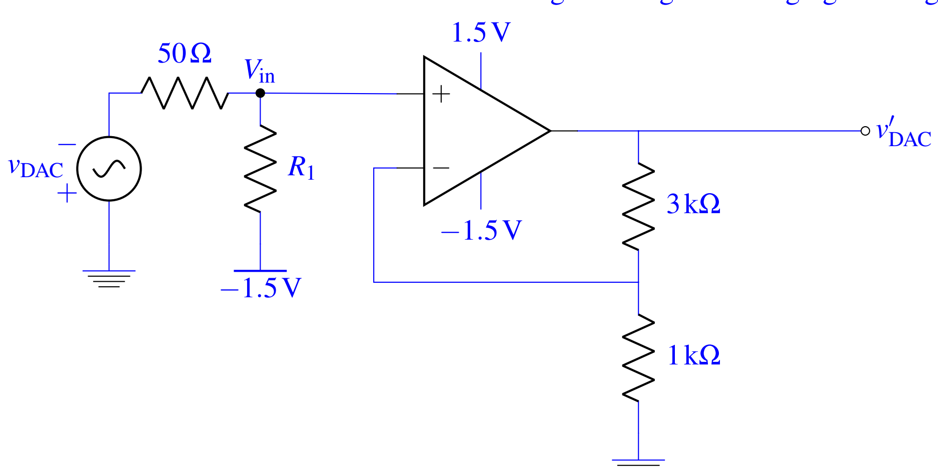
(e) **PRACTICE:** Building upon your solutions to all previous parts, and otherwise making the same assumptions about the relative voltage ranges of $v_{\text{mic1}}$, $v_{\text{mic2}}$, and $v_{\text{mic3}}$ and available supply voltages, sketch the complete circuit you would use to create the stereo audio on the two speakers while cancelling the noise picked up by the three microphones.

**Solution:**

We already have a circuit that does subtraction and a circuit that computes the noise cancelling signal. We just have to combine the two circuits such that it implements the matrix **B**.




Recall our circuit from last discussion for voltage shifting and changing the range of a DAC:

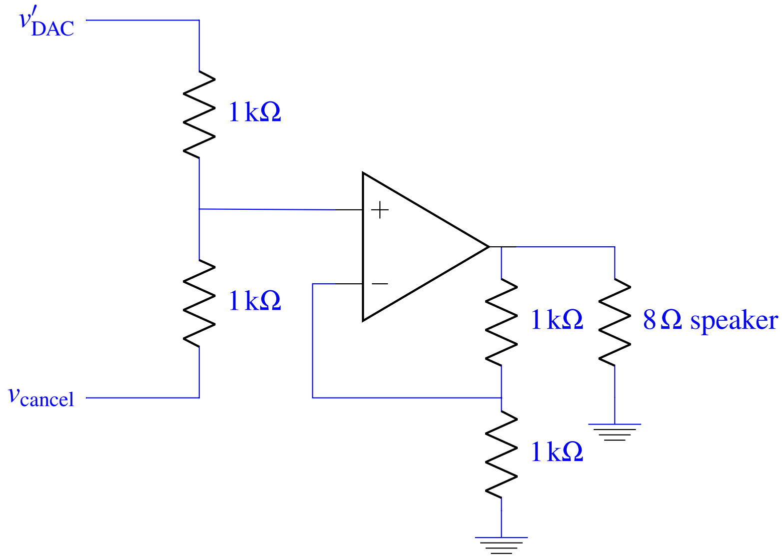



Now we want

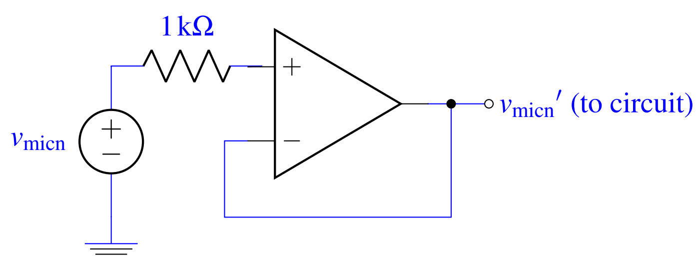
$$v_{\text{left}} = v'_{\text{DAC, left}} + v_{\text{cancel\_left}}$$

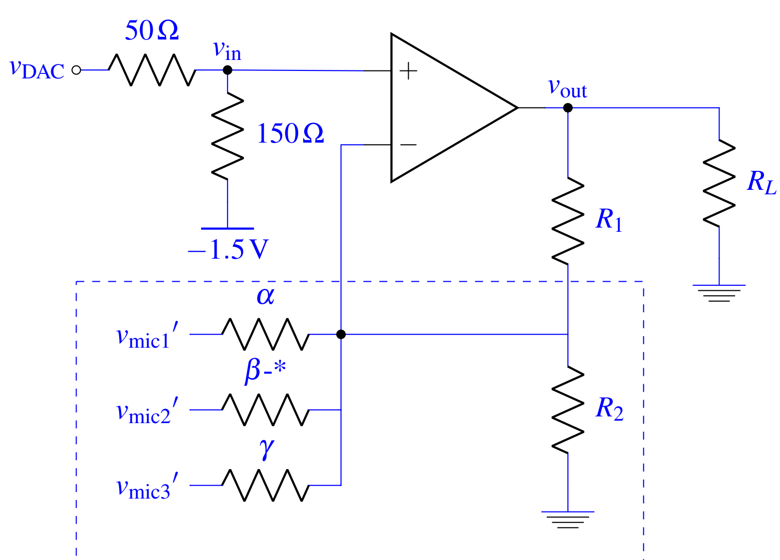
$$v_{\text{right}} = v'_{\text{DAC, right}} + v_{\text{cancel\_right}}$$

Note that $v_{\text{cancel}}$ already inverted the microphone signal so we are adding it to $v_{\text{DAC}}$. To sum, we can use a noninverting summer. Again, we consider the channels independently.




**Below is alternative circuit that does the same with one op-amp.** The approach here is to start with by attenuating the mic voltages and summing them with a voltage divider and solving for the correct resistances. Then we feed the the microphone voltage into the negative terminal and the DAC signal into the positive terminal of an op amp to subtract the signals.



Recall that the $v_{\text{in}}$ range is $-0.375\,\text{V}$ to $0.375\,\text{V}$, and it has to be amplified 4 times. We can write the KCL equation in the inverting input of the op-amp.

$$\frac{v_{\text{mic1}} - v_{\text{in}}}{\alpha} + \frac{v_{\text{mic2}} - v_{\text{in}}}{\beta} + \frac{v_{\text{mic3}} - v_{\text{in}}}{\gamma} + \frac{v_{\text{out}} - v_{\text{in}}}{R_1} + \frac{0 - v_{\text{in}}}{R_2} = 0$$

$$\frac{v_{\text{out}}}{R_1} = v_{\text{in}}\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{v_{\text{mic1}}}{\alpha} - \frac{v_{\text{mic2}}}{\beta} - \frac{v_{\text{mic3}}}{\gamma}$$

$$v_{\text{out}} = v_{\text{in}}\left(\frac{R_1}{\alpha} + \frac{R_1}{\beta} + \frac{R_1}{\gamma} + 1 + \frac{R_1}{R_2}\right) - \frac{R_1}{\alpha}v_{\text{mic1}} - \frac{R_1}{\beta}v_{\text{mic2}} - \frac{R_1}{\gamma}v_{\text{mic3}}$$

Just as before, we can compare this formula to the output we want. In this case, we want $v_{\text{out}} = 4v_{\text{in}} - a_1\,v_{\text{mic1}} - a_2\,v_{\text{mic2}} - a_3\,v_{\text{mic3}}$. Thus,

$$\frac{R_1}{\alpha} + \frac{R_1}{\beta} + \frac{R_1}{\gamma} + 1 + \frac{R_1}{R_2} = 4 \qquad \frac{R_1}{\alpha} = a_1 \qquad \frac{R_1}{\beta} = a_2 \qquad \frac{R_1}{\gamma} = a_3$$

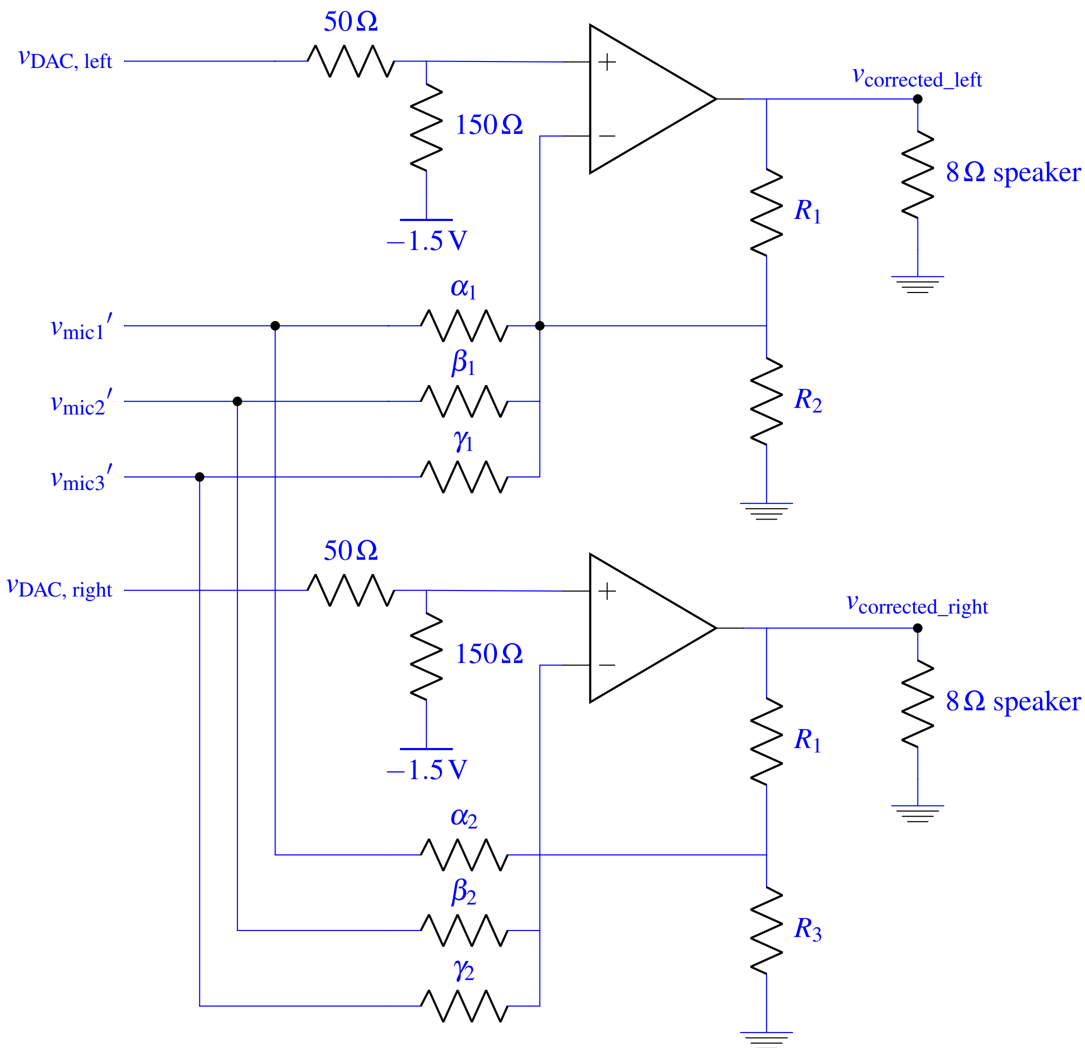
$$\alpha = \frac{R_1}{a_1} \qquad \beta = \frac{R_1}{a_2} \qquad \gamma = \frac{R_1}{a_3}$$

From the first equation,

$$a_1 + a_2 + a_3 + 1 + \frac{R_1}{R_2} = 4$$

$$R_2 = \frac{R_1}{3 - a_1 - a_2 - a_3}$$

Thus, if we pick a value for $R_1$, we can use the formulas above to calculate $\alpha$, $\beta$, $\gamma$ and $R_2$.
Now that we have a working circuit for one speaker, we can duplicate this circuit to have two speakers. Notice that in the circuit below we can use the same value for $R_1$ in the two channels, but we have to keep $R_2$ as a variable (hence it is replaced with $R_3$ in the right channel). This is because $R_1$ is a free variable. If we choose a value for $R_1$ arbitrarity, we can calculate what the other resistor values have to be with the equations we have derived.



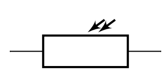

We have seen that if we choose values for $R_1$ and $R_3$ arbitrarily, we can find the other resistor values.

$$\alpha_1 = \frac{R_1}{a_{1\text{left}}} \qquad \beta_1 = \frac{R_1}{a_{2\text{left}}} \qquad \gamma_1 = \frac{R_1}{a_{3\text{left}}} \qquad R_2 = \frac{R_1}{3 - a_{1\text{left}} - a_{2\text{left}} - a_{3\text{left}}}$$

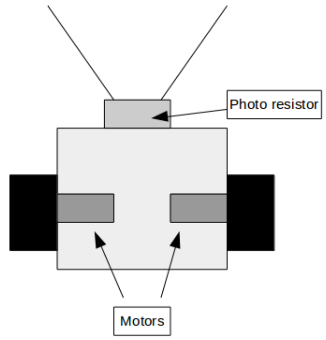
$$\alpha_2 = \frac{R_1}{a_{1\text{right}}} \qquad \beta_2 = \frac{R_1}{a_{2\text{right}}} \qquad \gamma_2 = \frac{R_1}{a_{3\text{right}}} \qquad R_3 = \frac{R_1}{3 - a_{1\text{right}} - a_{2\text{right}} - a_{3\text{right}}}$$

**2. PetBot Design**

In this problem, you will design circuits to control PetBot, a simple robot designed to follow light. PetBot measures light using photoresistors. A photoresistor is a light-sensitive resistor. As it is exposed to more light, its resistance decreases. Given below is the circuit symbol for a photoresistor.



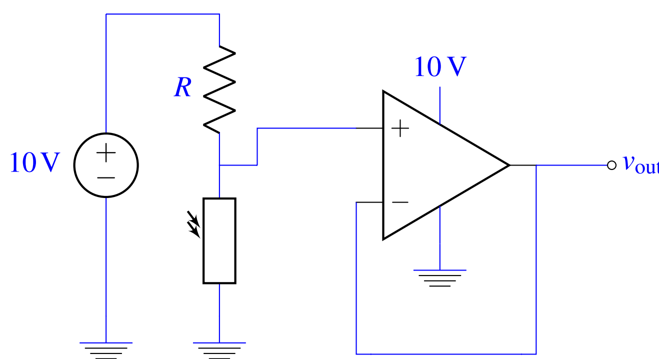
Below is the basic layout of the PetBot. It has one motor on each wheel. We will model each motor as a $1\,\Omega$ resistor. When motors have positive voltage across them, they drive forward; when they have negative voltage across them, they drive backward. At zero voltage across the motors, the PetBot stops. The speed of the motor is directly proportional to the magnitude of the motor voltage. The light sensor is mounted to the front of the robot.




(a) **Speed control** – Let us begin by first having PetBot decrease its speed as it drives toward the flashlight. Design a motor driver circuit that outputs a decreasing positive motor voltage as the PetBot drives toward the flashlight. The motor voltage should be at least 5 V far away from the flashlight. When far away from the flashlight, the photoresistor value will be 10 kΩ and dropping toward 100 Ω as it gets closer to the flashlight.

In your design, you may use any number of resistors and op-amps. You also have access to voltage sources of 10 V and −10 V. Based on your circuit, derive an expression for the motor voltage as a function of the circuit components that you used.

**Solution:**




The output of the above circuit is:

$$v_{\text{out}} = \frac{R_p}{R_p + R} \cdot 10\,\text{V}$$

$R_p$ represents the photoresistor, and $R \leq 10\,\text{k}\Omega$.

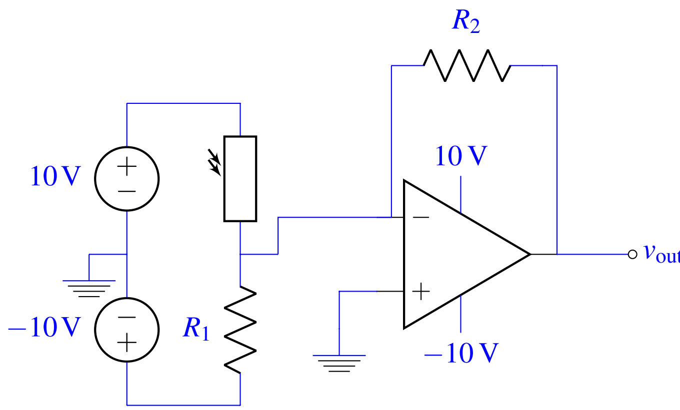
(b) **Distance control** – Let us now have PetBot drive up to a flashlight (or away from the flashlight) and stop at distance of 1 m away from the light. At the distance of 1 m from the flashlight, the photoresistor has a value 1 kΩ.

Design a circuit to output a motor voltage that is positive when the PetBot is at a distance greater than 1 m from the flashlight (making the PetBot move toward it), zero at 1 m from the flashlight (making the

PetBot stop), and negative at a distance of less than 1 m from the flashlight (making the PetBot back away from the flashlight.)

In your design, you may use any number of resisitors and op-amps. You also have access to voltage sources of 10 V and −10 V. Based on your circuit, derive an expression for the motor voltage as a function of the values of circuit components that you used.

**Solution:**




Choosing $R_1 = R = 1\,\text{k}\Omega$ we observe that the voltage $v_{out} = 0$ when Petbot is 1 m away as required. To find $v_{out}$ more generally, observe that $V_- = V_+ = 0$, so we need to find the current $i$ going through $R_2$.

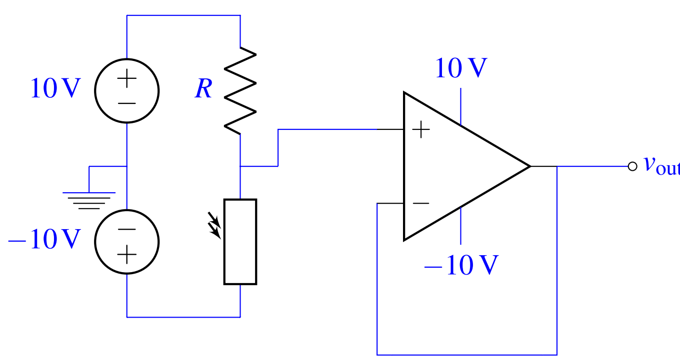
$$v_{out} = -iR_2 = -(\frac{10}{R_p} + \frac{-10}{R_1})R_2$$

From this we see that when PetBot is greater than 1 m away we have

$$R_p > R_1 = 1\,\text{k}\Omega$$
$$\frac{1}{R_p} - \frac{1}{R_1} < 0$$
$$\Rightarrow -(\frac{1}{R_p} - \frac{1}{R_1}) \cdot 10R_2 > 0$$

Similarly when the Petbot is less than 1 m away, we have that the motor voltage will be negative.

Alternatively:




Here observe that $v_{out} = v_+$. Using superposition we find that

$$v_{out} = v_+ = 10\frac{R_p}{R+R_p} - 10\frac{R}{R+R_p} = 10\frac{R_p - R}{R+R_p}$$

To satisfy the condition that $v_{out} = 0$ when Petbot is 1 m away, we have that $R = 1\,\text{k}\Omega$. Similar to the previous design, we can do the analysis for when the Petbot is far away and close by. We will show how to do it when the Petbot is close by here.

$$R_p < R = 1\,\text{k}\Omega$$
$$\Rightarrow R_p - R < 0$$
$$\Rightarrow v_{out} = 10\frac{R - R_p}{R + R_p} < 0$$

**3. Island Karaoke Machine**

After a plane crash, you're stuck on a desert island and everyone is bored out of their minds. Fortunately, you have your EE16A lab kit with op-amps, wires, resistors, and your handy breadboard. You decide to build a karaoke machine. You recover one speaker from the crash remains and use your iPhone as your source. You know that many songs put instruments on either the "left" or the "right" channel, but the vocals are usually present on both channels with equal strength.

The Thevenin equivalent model of the iPhone audio jack and speakers is shown below. We assume that the audio signals $v_{left}$ and $v_{right}$ have equivalent source resistance of the left/right audio channels of $R_{left} = R_{right} = 3\,\Omega$. The speaker has an equivalent resistance of $4\,\Omega$.

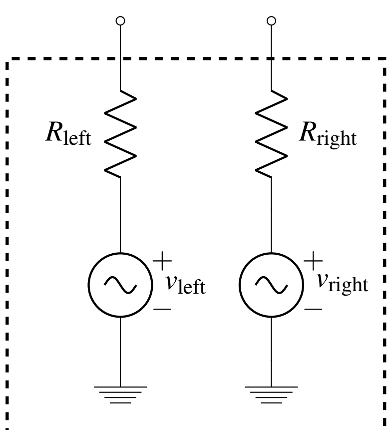
For this problem, we'll assume that

$$v_{left} = v_{vocals}$$
$$v_{right} = v_{vocals} + v_{instrument},$$

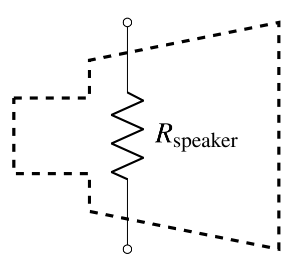
where the voltage source $v_{vocals}$ can have values anywhere in the range of $\pm 120\,\text{mV}$ and $v_{instrument}$ can have values anywhere in the range of $\pm 50\,\text{mV}$.

That is, the vocals are present on the left and right channel, but the instrument is present only on the right channel.

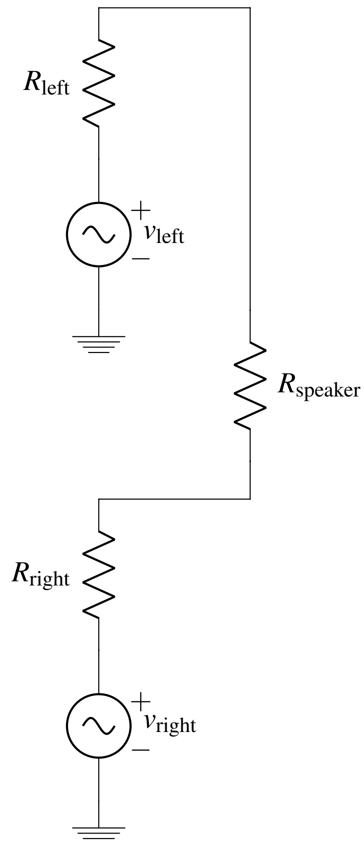
What is the goal of a karaoke machine? The ultimate goal is to *remove* the vocals from the audio output. We're going to do this by first building a circuit that takes the left and right outputs of the smartphone audio output and then takes its difference. Let's see what happens.




iPhone

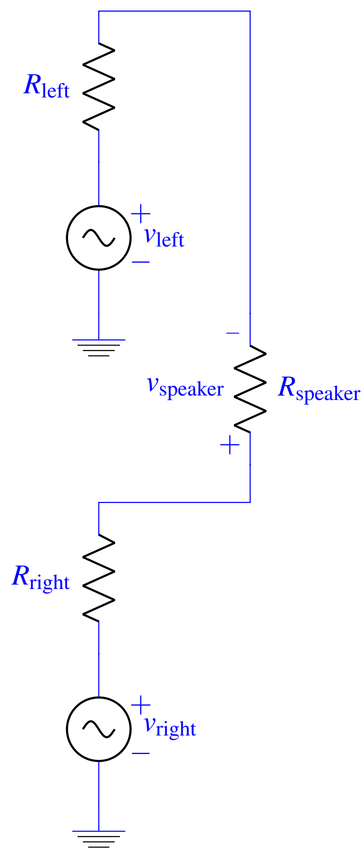


(a) One of your island survivors suggests the following circuit to do this. Calculate the voltage across the speaker as a function of $v_{\text{vocals}}$ and $v_{\text{instruments}}$. What do you notice? Does the voltage across the speaker depend on $v_{\text{vocals}}$? What do you think the islanders will hear – vocals, instruments, or both?




**Solution:**

Let's mark the voltage across the speaker, $v_{\text{speaker}}$, from bottom to top as in the figure:




We can apply the principle of superposition to solve for $v_{\text{speaker}}$. First, we solve for the voltage across the speaker when only $v_{\text{left}}$ is on. Let's call this $v_{\text{speaker,left}}$. Notice that the circuit becomes a voltage divider. Therefore, we get

$$-v_{\text{speaker,left}} = \frac{v_{\text{left}} R_{\text{speaker}}}{R_{\text{speaker}} + R_{\text{left}} + R_{\text{right}}} = \frac{4 v_{\text{vocals}}}{10} = 0.4 v_{\text{vocals}},$$

giving

$$v_{\text{speaker,left}} = -0.4 v_{\text{vocals}}.$$

Similarly, we solve for the voltage across the speaker when only $v_{\text{right}}$ is on. Let's call this $v_{\text{speaker,right}}$. Again, notice that the circuit becomes a voltage divider. Therefore, we get

$$v_{\text{speaker,right}} = \frac{v_{\text{right}} R_{\text{speaker}}}{R_{\text{speaker}} + R_{\text{left}} + R_{\text{right}}} = \frac{4(v_{\text{vocals}} + v_{\text{instrument}})}{10} = 0.4(v_{\text{vocals}} + v_{\text{instrument}}).$$

Superposition tells us that $v_{\text{speaker}} = v_{\text{speaker,left}} + v_{\text{speaker,right}} = 0.4 v_{\text{instrument}} = 0.4 \cdot 50\,\text{mV} = 20\,\text{mV}$. What did you notice? The vocals got canceled out! The islanders will only hear the instruments, just as they wanted.

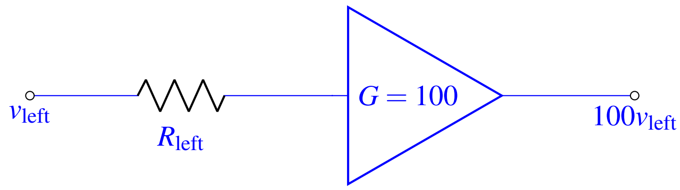
(b) Clearly, we need to boost the sound level to get the party going. We can do this by *amplifying* both $v_{\text{left}}$ and $v_{\text{right}}$. Keep in mind that we could use inverting or non-inverting amplifiers.

Let's assume, just for this part, that we have already implemented circuits that amplify $v_{\text{left}}$ and $v_{\text{right}}$ by some factor $G$. We now have two voltage sources, $v_{\text{Gl}}$ and $v_{\text{Gr}}$ that are $v_{\text{left}}$ multiplied by $G$ and $v_{\text{right}}$ multiplied by $G$. Use these voltage sources to get $G \times v_{\text{instruments}}$ across $R_{\text{speaker}}$.

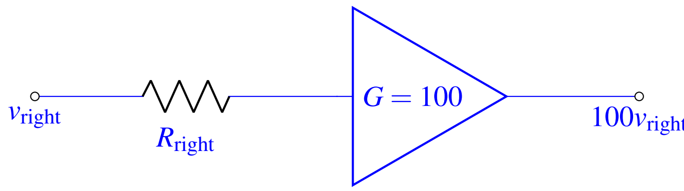
**Solution:**

Basically, we have three components of the circuit we want to build that we already know:

- The part of the circuit that amplifies $v_{\text{left}}$ to $100v_{\text{left}}$, which we can draw as below:

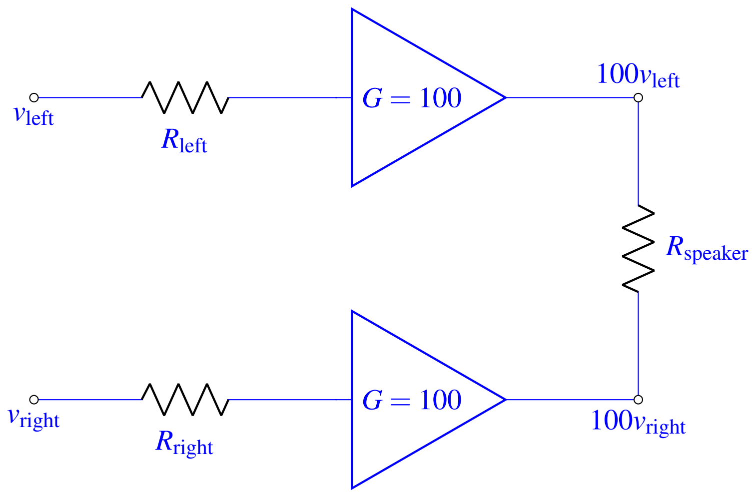



- The part of the circuit that amplifies $v_{\text{right}}$ to $100v_{\text{right}}$, which we can draw as below:




- The speaker

If we want to take the difference of the two amplified outputs across the speaker, all we need to do is connect the output terminals of the first two components to the terminals of the speaker as shown below:




You can see this solution taking inspiration from part (a). Why do we get exactly $100(v_{\text{right}} - v_{\text{left}})$ across the speaker? Why does the voltage not divide as before?

To answer this, look back at the circuit for the non-inverting amplifier. If we solve for the Thevenin output resistance of this circuit, we will find that it is zero. Furthermore, the Thevenin voltage will be $100v_{\text{left}}$ (or $100v_{\text{right}}$). This implies that, no matter what $R_{\text{left}}$ or $R_{\text{right}}$ is, we are going to only see $100v_{\text{left}}$ or $100v_{\text{right}}$ at the output.

(c) Now, design a circuit that takes in $v_{\text{left}}$ and $v_{\text{right}}$ and outputs an amplified version of $v_{\text{instrument}}$ across the speaker load. You want $\pm 2\,\text{V}$ across the speaker to get the party going. You can use up to three op-amps, and each of them can be inverting or non-inverting.

**Solution:**

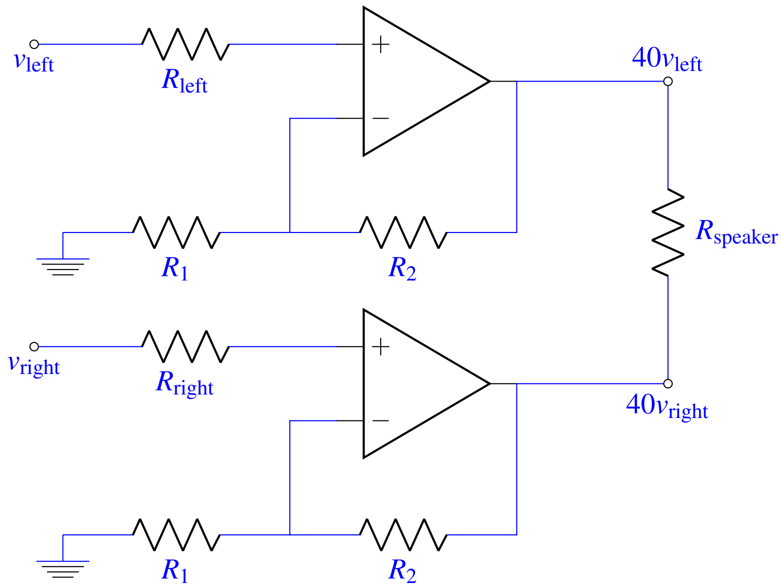
Solution using only two op-amps:

Simply feed the non-ideal voltage source $\{v_{\text{left}}, R_{\text{left}}\}$ into a non-inverting amplifier with gain $G$ and the non-ideal voltage $\{v_{\text{right}}, R_{\text{right}}\}$ into another non inverting amplifier with gain $G$. (We have a different gain from the previous part, which we need to determine.) Then connect the two outputs across $R_{\text{speaker}}$ as shown in the previous part.

In this circuit, we will get $v_{\text{speaker}} = G \cdot 50\,\text{mV} = 0.05GV$, which will give us $P_{\text{speaker}} = \frac{0.0025G^2}{4}\,\text{W}$. We want $P_{\text{speaker}} = 1\,\text{W}$, implying that we need $\frac{0.0025G^2}{4}\,\text{W} = 1\,\text{W}$. This gives us $G = 40$.

Therefore, we want to design for a non-inverting opamp with voltage gain of 40.

We can use the circuit schematic from part (c), but now, we just need to design the non-inverting amplifier to have gain 40. We get the equivalent circuit below:

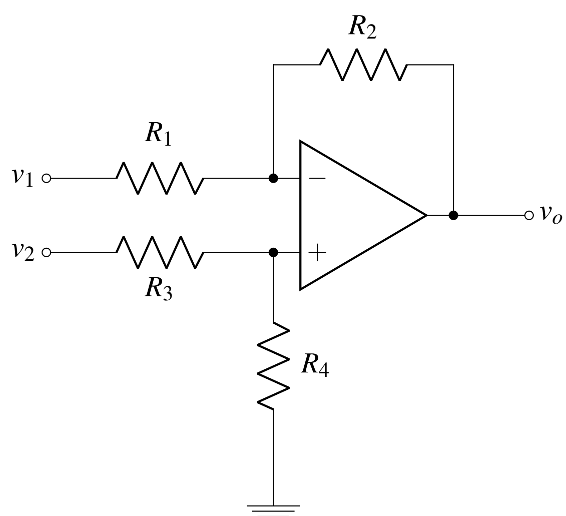



Now, we need to find $R_1$ and $R_2$.

$$G = 1 + \frac{R_2}{R_1}$$

Therefore, we can then choose any $R_1$ and $R_2$ such that $\frac{R_2}{R_1} = 39$. Note that there are multiple ways of choosing them. One such choice is $R_1 = 1\,\text{k}\Omega$ and $R_2 = 39\,\text{k}\Omega$, for instance.

(d) The trouble with the previous part is the number of op-amps required. Let's say you only have one op-amp with you. What would you do? One night in your dreams, you have an inspiration. Why not combine the inverting and non-inverting amplifier into one, as shown below!



If we set $v_2 = 0\,\text{V}$, what is the output $v_o$ in terms of $v_1$? (This is the inverting path.)

**Solution:**

If we set $v_2 = 0\,\text{V}$, we would get $v_+ = 0\,\text{V}$. Applying the Golden Rules, we will get $v_- = v_+ = 0\,\text{V}$. Writing the node equation at the $-$ terminal of the op-amp, we get

$$\frac{v_1 - 0}{R_1} = \frac{0 - v_{o,1}}{R_2},$$

which gives

$$v_{o,1} = \frac{-v_1 R_2}{R_1}.$$

(e) If we set $v_1 = 0\,\text{V}$, what is the output $v_o$ in terms of $v_2$? (This is the non-inverting path.)

**Solution:**

If we set $v_1 = 0\,\text{V}$, we would get $v_+ = \frac{v_2 R_4}{R_3 + R_4} = v_-$. Writing the node equation at the $-$ terminal gives

$$\frac{0 - v_-}{R_1} = \frac{v_- - v_{o,2}}{R_2},$$

which gives

$$v_{o,2} = v_- \left(1 + \frac{R_2}{R_1}\right) = v_2 \left(\frac{R_4}{R_3 + R_4}\right)\left(1 + \frac{R_2}{R_1}\right).$$

(f) Now, determine $v_o$ in terms of $v_1$ and $v_2$. (*Hint:* Use superposition.) Choose values for $R_1$, $R_2$, $R_3$ and $R_4$, such that the speaker has $\pm 2\,\text{V}$ across it.

**Solution:**

By the principle of superposition,

$$v_o = v_{o,1} + v_{o,2}.$$

If we set $v_1 = v_{\text{left}}$ and $v_2 = v_{\text{right}}$, we'd ideally want $v_o = -40v_1 + 40v_2$. We can choose $R_1$, $R_2$, $R_3$ and $R_4$, so that this happens.

How do we do this? Let's do this in steps. First, note that, looking for the expression for $v_{o,1}$, we'll want $\frac{R_2}{R_1} = 40$. Therefore, we can choose any values of $R_2$ and $R_1$, such that this happens. One such choice is $R_1 = 1\,\text{k}\Omega$ and $R_2 = 40\,\text{k}\Omega$. Then, plug that into the expression of $v_{o,2}$, and the condition we now want is

$$\left(\frac{R_4}{R_3 + R_4}\right)\left(1 + \frac{R_2}{R_1}\right) = 40,$$

which gives us

$$\frac{R_4}{R_3 + R_4} = \frac{40}{41}.$$

Thus, we need to choose $R_3$ and $R_4$. As before, we can choose these values in many ways. One such choice is $R_4 = 40\,\text{k}\Omega$ and $R_3 = 1\,\text{k}\Omega$.

**Note:** Keep in mind that, for this problem, we actually assumed that $v_1 = v_{\text{left}}$ and $v_2 = v_{\text{right}}$, which would mean that we are ideally connecting $v_{\text{left}}$ and $v_{\text{right}}$ as inputs. However, in reality, we're actually connecting the outputs from the iPhone as inputs. This means that $R_{\text{left}}$ and $R_{\text{right}}$ will also actually affect the output.

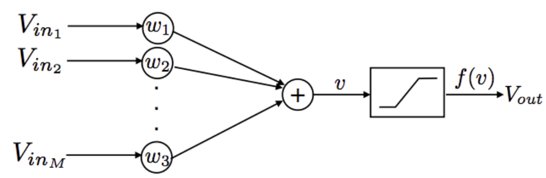
With this effect, we will actually get

$$v_o = -\frac{v_1 R_2}{R_{1,eq}} + v_2 \left(\frac{R_4}{R_{3,eq} + R_4}\right)\left(1 + \frac{R_2}{R_{1,eq}}\right),$$

where $R_{1,eq} = R_1 + R_{\text{left}}$ and $R_{3,eq} = R_3 + R_{\text{right}}$.
Therefore, we can just *fold in* the effect of $R_{\text{left}}$ and $R_{\text{right}}$ into these. For instance, we want to set $R_{3,eq} = 1\,\text{k}\Omega$. Now, we can actually make $R_3 = R_{3,eq} - 3\,\Omega = 997\,\Omega$ and $R_1 = R_{1,eq} - 3\,\Omega = 997\,\Omega$.
Give yourself full credit even if you didn't notice this, but keep this in mind!
*Bonus:* Can you now see why we wanted to keep $R_1$ and $R_3$ in the order of k$\Omega$ or larger?

**4. Brain-on-a-Chip with 16A Neurons**

Neurelic Inc, is a hot new startup building chips that emulate some of the brain functions (for example associative memory). As an intern, fresh out of 16A you get to implement the neural network circuits on this chip. The neural network consists of neurons that consist of the following blocks shown on the figure below.

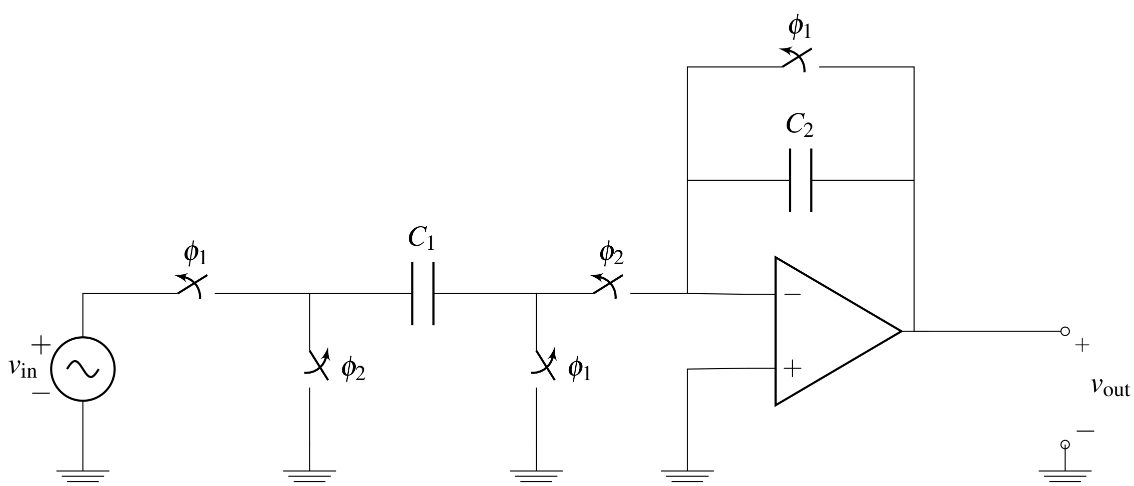



Input signals $v_{in_i}$ are voltages from other neurons, which are multiplied by a constant weight $w_i$ in each synapse and summed in the neuron. Each neuron also contains a nonlinear function (called a sigmoid) which is defined as

$$f(v) = \begin{cases} -1, & v \leq -1 \\ v, & -1 < v < 1 \\ +1, & v \geq +1 \end{cases}$$

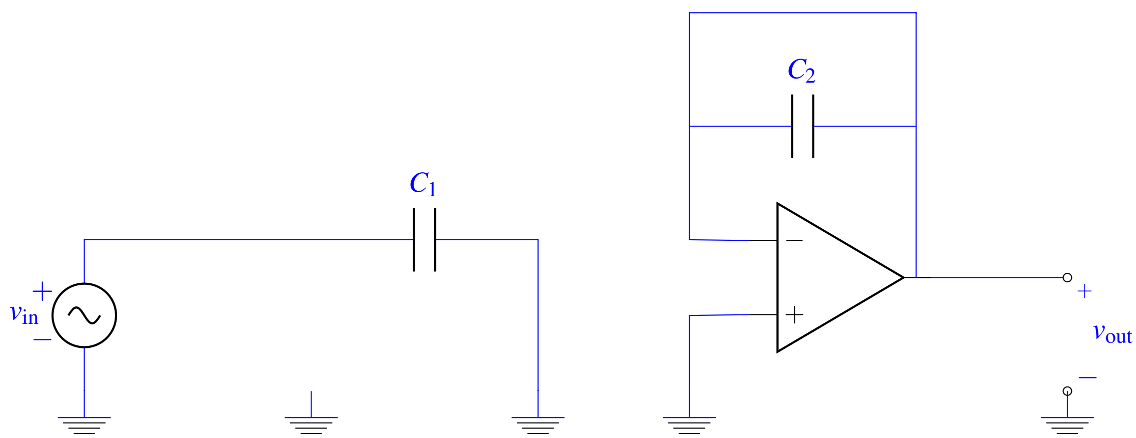
where $v$ is the internal neuron voltage after the synapse summer and $f(v)$ is the neuron voltage output.

(a) Your mentor suggests that you warm-up first by analyzing the circuit below to use as neuron with a single synapse. $\phi_1$ and $\phi_2$ are non-overlapping clock phases that control the circuit switches.



i. Draw an equivalent circuit during $\phi_1$ and write an expression for $v_{out}$ as a function of $v_{in}$, $C_1$ and $C_2$.

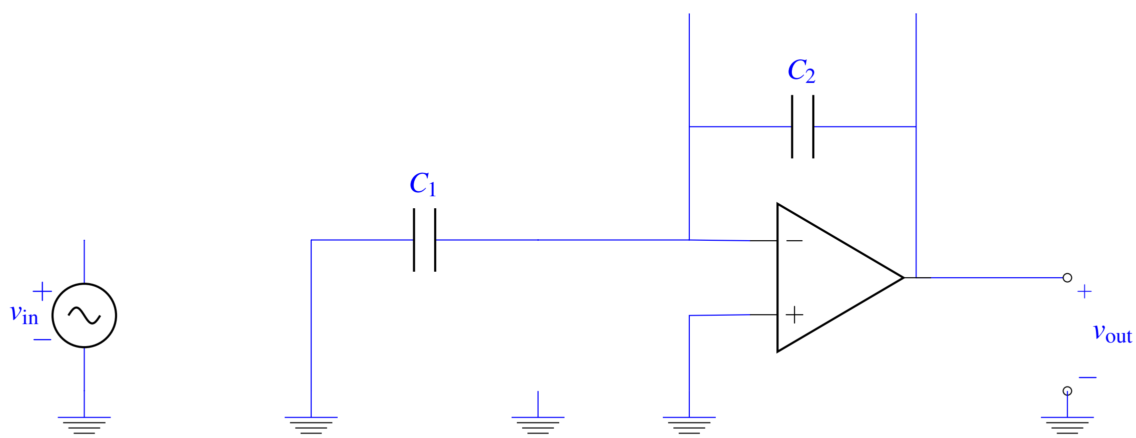
**Solution:** $v_{out} = 0$




ii. Draw an equivalent circuit during $\phi_2$ and write an expression for $v_{out}$ as a function of $v_{in}$, $C_1$ and $C_2$.

**Solution:**

$$v_{in}C_1 = v_{out}C_2$$
$$v_{out} = \frac{C_1}{C_2}v_{in}$$

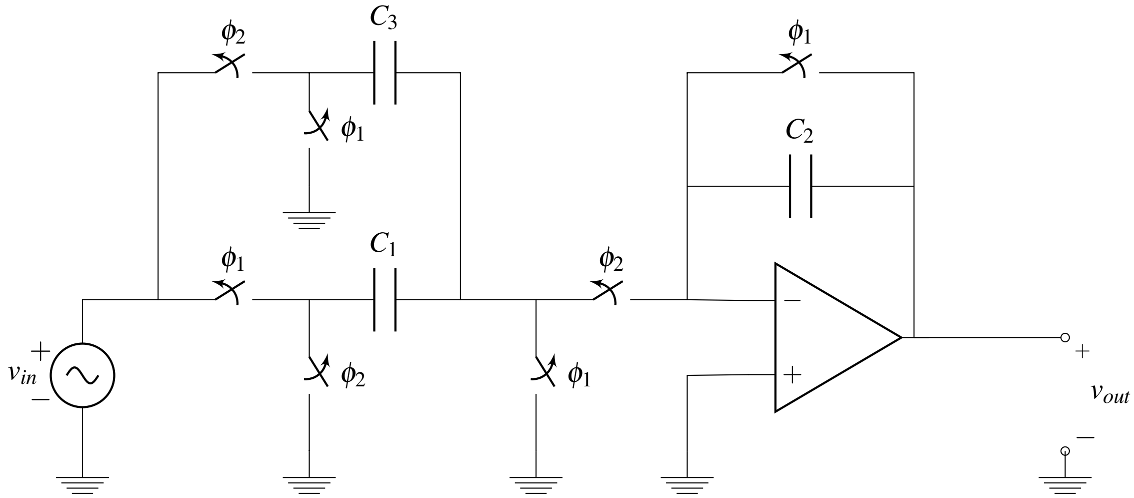



(b) Write an equation for $v_{out}$ during $\phi_2$ as a function of $v_{in}$ for $C_1 = C_2$ and op-amp supply voltages of $\pm 1$ V. Briefly explain how this circuit implements the sigmoid function.

**Solution:** From part (a)(ii) we know $v_{out} = \frac{C_1}{C_2}v_{in}$.Setting $C_1 = C_2$, we find $v_{out} = v_{in}$. Because of the rails of the op amp, once the $v_{in}$ exceeds 1 V, the output will be 1 V. Similarly when $v_{in}$ is less than $-1$ V, the output will be $-1$ V. At intermediate values we have $v_{out} = v_{in}$ from the analysis above. From this, we see the circuit implements the sigmoid function:

$$v_{out} = \begin{cases} -1, & v_{in} \leq -1 \\ v_{in}, & -1 < v_{in} < 1 \\ +1, & v_{in} \geq +1 \end{cases}$$

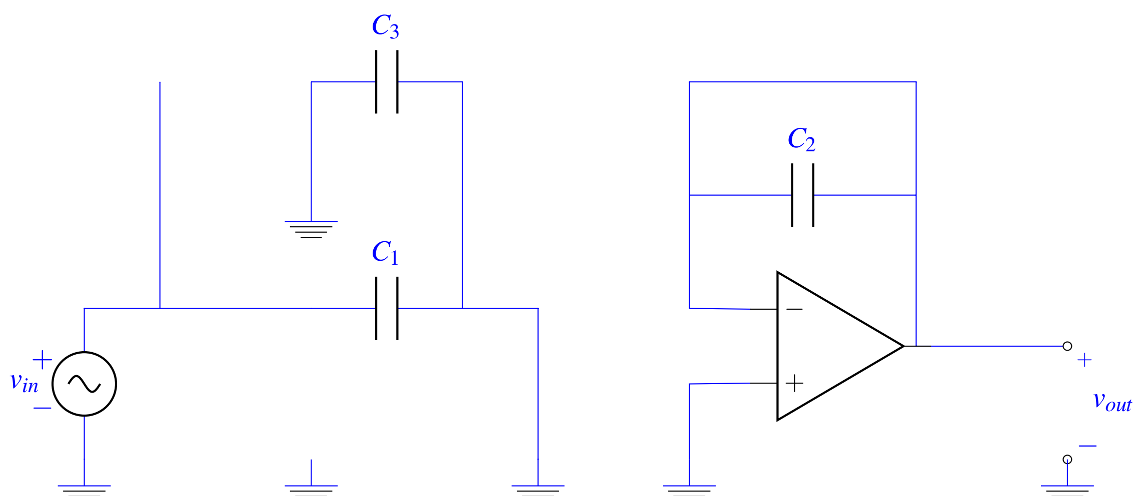
(c) Then, your mentor shows you the following neuron circuit, which can realize both positive and negative synapse weight and create $v_{out} = w_1 v_{in}$ in $\phi_2$.




i. Draw an equivalent circuit during $\phi_1$ and write an expression for $v_{\text{out}}$ as a function of $v_{\text{in}}$, $C_1$, $C_2$, and $C_3$.

**Solution:**

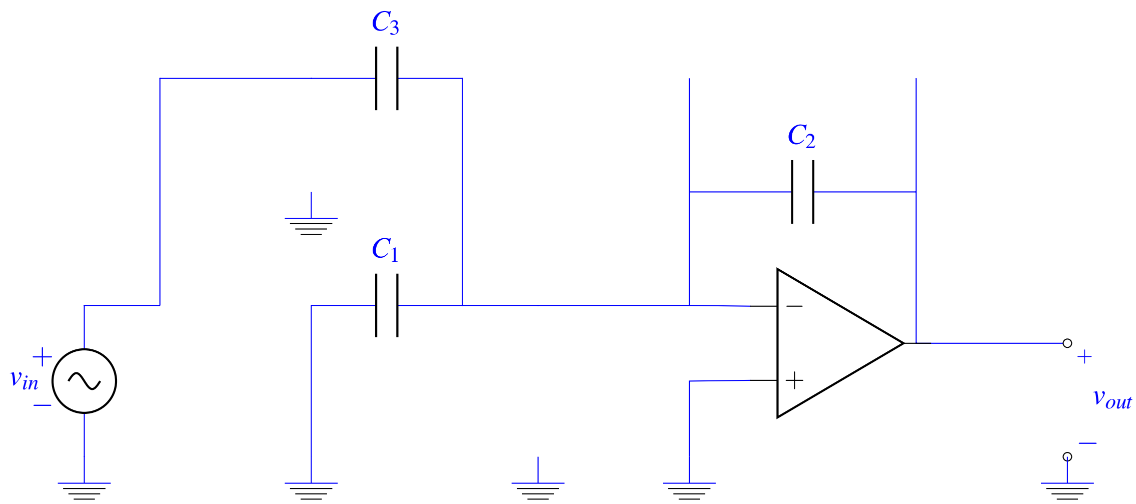
In phase 1, $v_{\text{out}} = 0$.




ii. (5 points) Draw an equivalent circuit during $\phi_2$ and write an expression for $v_{out}$ as a function of $v_{in}$, $C_1$, $C_2$, and $C_3$.

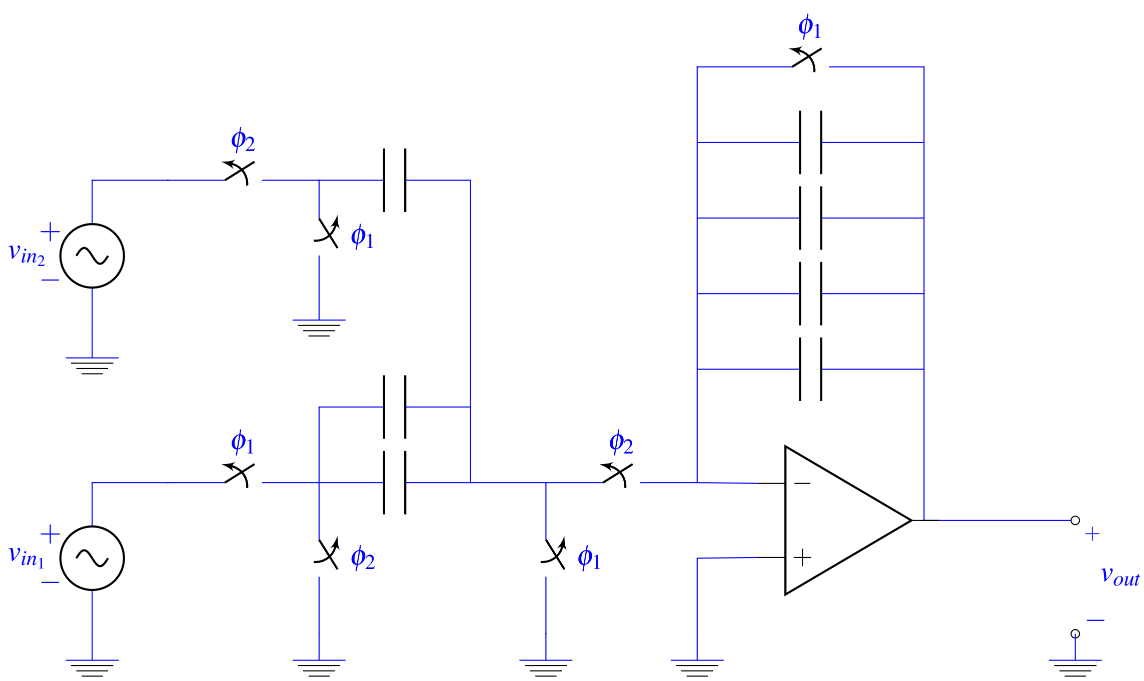
**Solution:**

In phase 2, $v_{\text{out}} = \frac{C_1 - C_3}{C_2} v_{\text{in}}$.




(d) Now it is your turn to implement a neuron that realizes the following function $v_{out} = w_1 v_{in_1} + w_2 v_{in_2}$. Draw the circuit, such that $w_1 = 1/2$ and $w_2 = -1/4$. Label all circuit elements appropriately. You should use a single op-amp and as many capacitors and switches as you need. All capacitors must be of size $C_{unit}$. Assume that the op-amp power supplies are $\pm 1$V (no need to draw them in the circuit). The circuit should operate in 2 phases, with $v_{out} = w_1 v_{in_1} + w_2 v_{in_2}$ in the second phase ($\phi_2$), and reset in $\phi_1$.

**Solution:**




## 5. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

**Solution:**

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.