EECS 16A Designing Information Devices and Systems I
Spring 2018 Homework 6

This homework is due March 5th, 2018, at 23:59.
Self-grades are due March 8th, 2018, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

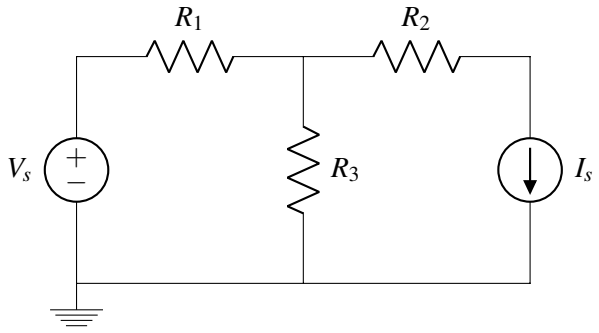
- `hw6.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

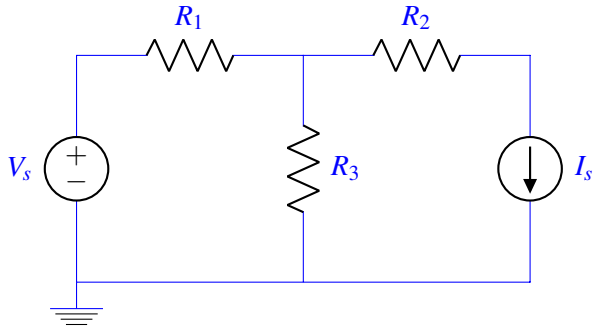
1. Circuit Analysis

Using the steps outlined in lecture, solve the following circuits for the currents through each branch and the voltages at each node. Use the ground node labelled for you. You may use a numerical tool, such as IPython.

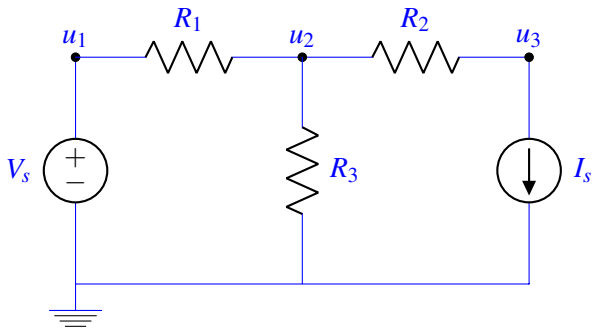
(a) $V_s = 5\,\text{V}, I_s = 2\,\text{A}, R_1 = R_2 = 2\,\Omega, R_3 = 4\,\Omega$



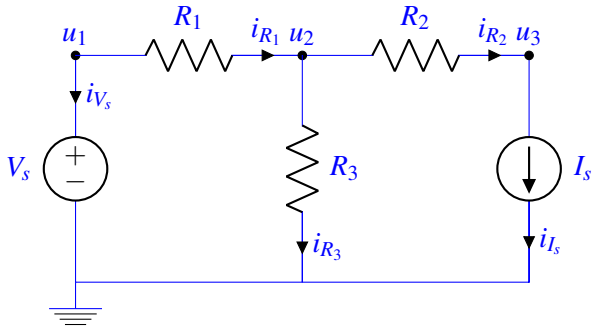
Solution:
First, select a ground node.



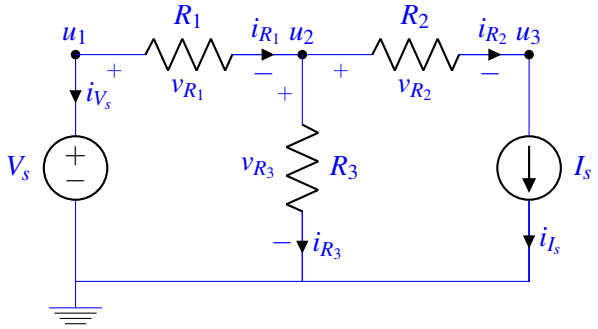
Then label all of the node potentials.



Now we label all of the branch currents.



Now, we label all of the voltages across the elements according to passive sign convention.



Now, we have 8 unknowns in this system: $u_1, u_2, u_3, i_{V_s}, i_{I_s}, i_{R_1}, i_{R_2}, i_{R_3}$.
Therefore, our vector of unknowns is of size 8, and the matrix will be 8×8.
Using KCL, we get the following equations:

$$\begin{aligned} i_{V_s} + i_{R_1} &= 0 \\ -i_{R_1} + i_{R_2} + i_{R_3} &= 0 \\ -i_{R_2} + i_{I_s} &= 0 \end{aligned}$$

From the *IV* relations for all of the elements, we find the following equations:

$$\begin{aligned} u_1 - 0 &= V_s \\ i_{I_s} &= I_s \\ u_1 - u_2 - R_1 i_{R_1} &= 0 \\ u_2 - u_3 - R_2 i_{R_2} &= 0 \\ u_2 - 0 - R_3 i_{R_3} &= 0 \end{aligned}$$

Note that we now have 8 equation for 8 unknowns. Thus, we set up the following matrix relation:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -R_1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -R_2 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -R_3 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{V_s} \\ i_{R_1} \\ i_{R_2} \\ i_{R_3} \\ i_{I_s} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_s \\ I_s \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

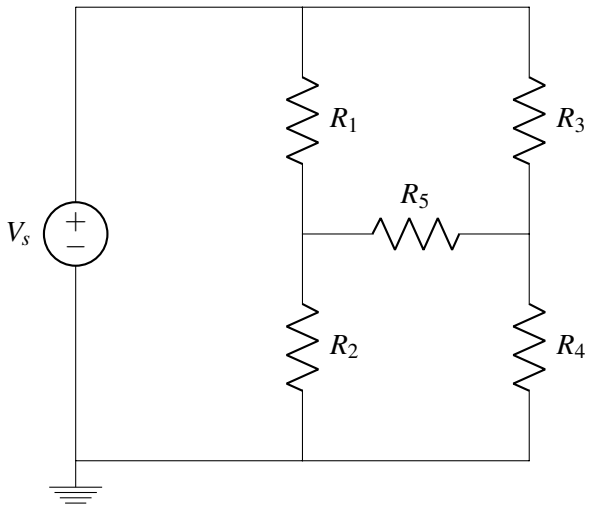
Finally, we plug in the values we were given into the matrix above and use Gaussian elimination to find the vector of unknowns.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -4 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{V_s} \\ i_{R_1} \\ i_{R_2} \\ i_{R_3} \\ i_{I_s} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We find that:

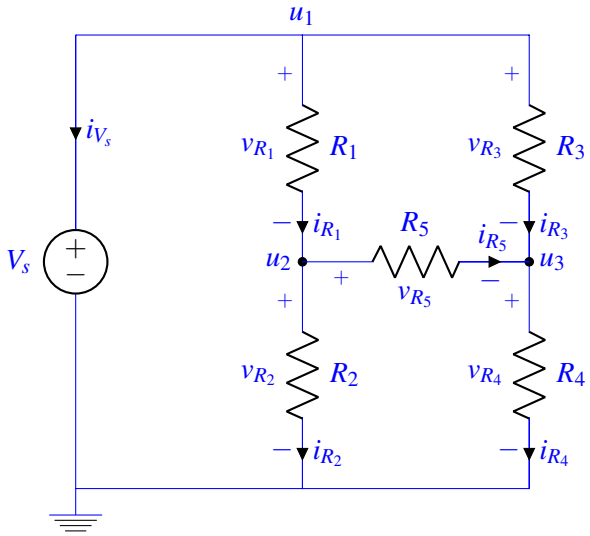
$$\begin{bmatrix} i_{V_s} \\ i_{R_1} \\ i_{R_2} \\ i_{R_3} \\ i_{I_s} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -2.167 \\ 2.167 \\ 2 \\ 0.167 \\ 2 \\ 5 \\ 0.667 \\ -3.33 \end{bmatrix}$$

(b) $V_s = 5\,\text{V}, R_1 = 1\,\Omega, R_2 = 2\,\Omega, R_3 = 3\,\Omega, R_4 = 4\,\Omega, R_5 = 5\,\Omega$



Solution:

Here, we will skip showing all of the individual steps. Below is the circuit with our choice of ground and current directions.



From the above circuit, we get the following KCL equations:

$$\begin{aligned} i_{V_s} + i_{R_1} + i_{R_3} &= 0 \\ -i_{R_1} + i_{R_2} + i_{R_5} &= 0 \\ -i_{R_3} + i_{R_4} - i_{R_5} &= 0 \end{aligned}$$

Using the *IV* relations for each element, we find 6 more equations:

$$\begin{aligned} u_1 - 0 &= V_s \\ u_1 - u_2 - i_{R_1} R_1 &= 0 \\ u_2 - 0 - i_{R_2} R_2 &= 0 \\ u_1 - u_3 - i_{R_3} R_3 &= 0 \\ u_3 - 0 - i_{R_4} R_4 &= 0 \\ u_2 - u_3 - i_{R_5} R_5 &= 0 \end{aligned}$$

Note that we now have 9 equation for 9 unknowns. Thus, we set up the following matrix relation:

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -R_1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -R_2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -R_3 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -R_4 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -R_5 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_{V_s} \\ i_{R_1} \\ i_{R_2} \\ i_{R_3} \\ i_{R_4} \\ i_{R_5} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

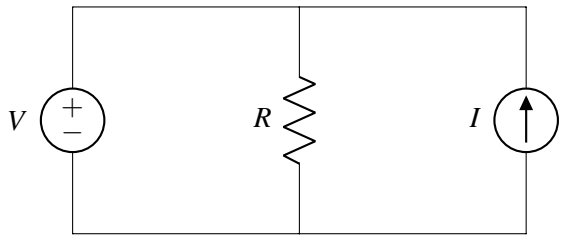
Finally, we plug in the values we were given into the matrix above and use Gaussian elimination to

find the vector of unknowns.

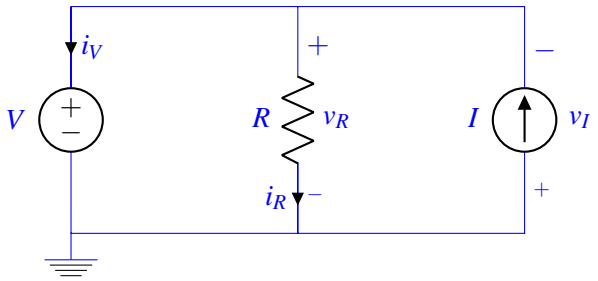
$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -5 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_{V_s} \\ i_{R_1} \\ i_{R_2} \\ i_{R_3} \\ i_{R_4} \\ i_{R_5} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i_{V_s} \\ i_{R_1} \\ i_{R_2} \\ i_{R_3} \\ i_{R_4} \\ i_{R_5} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -2.38 \\ 1.709 \\ 1.645 \\ 0.677 \\ 0.741 \\ 0.0645 \\ 5 \\ 3.29 \\ 2.968 \end{bmatrix}$$

2. **Power Analysis** Find the power dissipated by each element here. Remember to label voltages using passive sign convention.



Solution: We label a ground node, and then solve for the currents i_1, i_2 and the voltages V_1, V_2.



Solving the above circuit using nodal analysis, we get

$$i_R = \frac{V}{R}$$

$$i_V = I - \frac{V}{R}$$
$$v_I = -V$$
$$v_R = V$$

Using this we can calculate

$$P_V = V i_V = IV - \frac{V^2}{R}$$
$$P_I = I v_I = -IV$$
$$P_R = i_R v_R = \frac{V^2}{R}$$

Note that $P_V + P_I + P_R = 0$, i.e. energy provided is energy dissipated, which verifies our intuition about conservation of energy.

3. **Midterm Problem 3**
   Redo Midterm Problem 3.

   Solution: See midterm solutions.

4. **Midterm Problem 4**
   Redo Midterm Problem 4.

   Solution: See midterm solutions.

5. **Midterm Problem 5**
   Redo Midterm Problem 5.

   Solution: See midterm solutions.

6. **Midterm Problem 6**
   Redo Midterm Problem 6.

   Solution: See midterm solutions.

7. **Midterm Problem 7**
   Redo Midterm Problem 7.

   Solution: See midterm solutions.

8. **Midterm Problem 8**
   Redo Midterm Problem 8.

   Solution: See midterm solutions.

9. **Midterm Problem 9**
   Redo Midterm Problem 9.

   Solution: See midterm solutions.

10. **Midterm Problem 10**
    Redo Midterm Problem 10.

    Solution: See midterm solutions.

11. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.