This homework is due March 12, 2018, at 23:59.
Self-grades are due March 15, 2018, at 23:59.

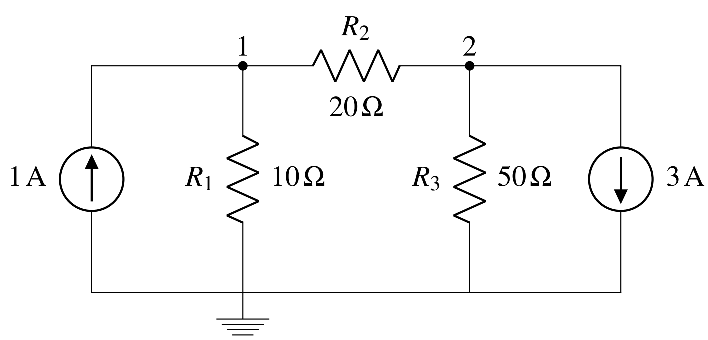
Submission Format
Your homework submission should consist of **one** file.

- `hw7.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. (PRACTICE) Circuit Analysis

Solve the circuit given below for currents and voltages.



Solution:

Method 1: Nodal Analysis

Applying KCL at Node 1, we get

$$\frac{V_1 - 0}{R_1} + \frac{V_1 - V_2}{R_2} - 1 = 0$$

$$\frac{V_1 - 0}{10} + \frac{V_1 - V_2}{20} - 1 = 0$$

which gives

$$2V_1 + V_1 - V_2 - 20 = 0$$

implying

$$3V_1 - V_2 = 20 \tag{1}$$

Applying KCL at Node 2, we get

$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - 0}{R_3} + 3 = 0$$

$$\frac{V_2 - V_1}{20} + \frac{V_2 - 0}{50} + 3 = 0$$

which gives

$$5V_2 - 5V_1 + 2V_2 + 300 = 0$$

implying

$$-5V_1 + 7V_2 = -300 \tag{2}$$

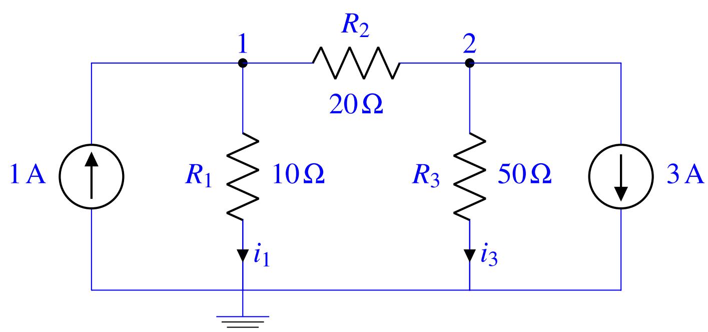
Writing equations 1 and 2 in matrix form, we get

$$\begin{bmatrix} 3 & -1 \\ -5 & 7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ -300 \end{bmatrix}$$

Solving the system of equations, we will get $V_1 = -10\,\text{V}$ and $V_2 = -50\,\text{V}$.

Method 2: Superposition

We define i_1 and i_3 as follows:



First, consider the effect of only the left 1 A current source on. Using current divider rule, we have

$$i_1 = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot 1\,\text{A}$$

$$i_1 = \frac{70}{10 + 70} \cdot 1\,\text{A} = 0.875\,\text{A}$$

and

$$i_3 = 1\,\text{A} - 0.875\,\text{A} = 0.125\,\text{A}$$

Therefore,

$$V_1^a = i_1 \cdot 10\,\Omega = 0.875\,\text{A} \cdot 10\,\Omega = 8.75\,\text{V}$$

and

$$V_2^a = i_3 \cdot 50\,\Omega = 0.125\,\text{A} \cdot 50\,\Omega = 6.25\,\text{V}$$

Second, consider the effect of only the right 3 A current source on. Using current divider rule, we have

$$i_1 = -\frac{R_3}{R_3 + R_2 + R_1} \cdot 3\,\text{A}$$

$$i_1 = -\frac{50}{50 + 30} \cdot 3\,\text{A} = -1.875\,\text{A}$$

and

$$i_3 = -3\,\text{A} + 1.875\,\text{A} = -1.125\,\text{A}$$

Therefore,

$$V_1^b = i_1 \cdot 10\,\Omega = -1.875\,\text{A} \cdot 10\,\Omega = -18.75\,\text{V}$$

and

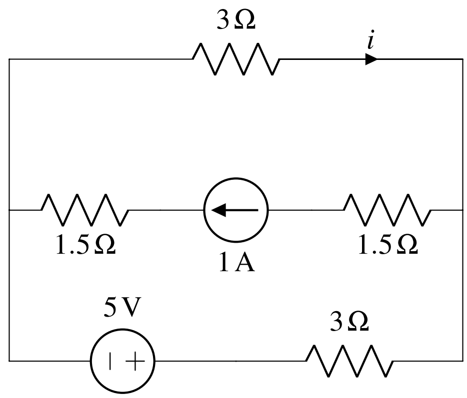
$$V_2^b = i_3 \cdot 50\,\Omega = -1.125\,\text{A} \cdot 50\,\Omega = -56.25\,\text{V}$$

Since the circuit is linear (i.e. we have linear elements and sources), we can use the *principle of superposition* to get $V_1 = V_1^a + V_1^b$ and $V_2 = V_2^a + V_2^b$. Therefore, we get $V_1 = 8.75\,\text{V} - 18.75\,\text{V}$ and $V_2 = 6.25\,\text{V} - 56.25\,\text{V}$. Finally, $V_1 = -10\,\text{V}$ and $V_2 = -50\,\text{V}$.

This solution agrees with the solution we obtained using nodal analysis.

2. (PRACTICE) Superposition

Solve the circuits shown below using superposition.

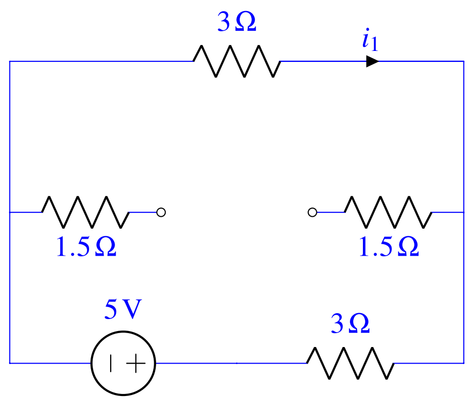


Solution:

$$i = -\frac{1}{3}\,\text{A}$$

Consider the circuits obtained by:

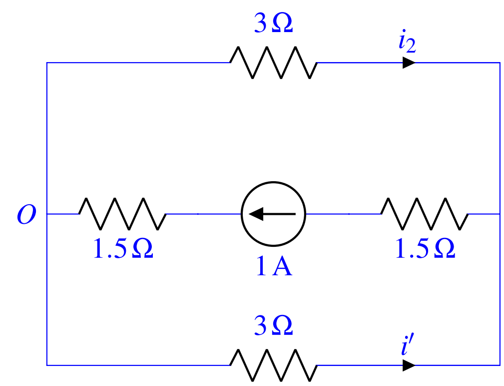
(a) Turning off the 1 A current source:



In the above circuit, no current is going to flow through the middle branch, as it is an open circuit. Thus this is just a 5 V voltage source connected to two 3 Ω resistors in series so

$$i_1 = -\frac{5}{6}\,\text{A}$$

(b) Turning off the 5 V voltage source:



In the above circuit, notice that the 3 Ω resistors are in parallel and therefore form a current divider. Since the values of the resistances are equal, the current flowing through them will also be equal, that is $i_2 = i'$. Applying KCL to node O, we get

$$1 - i_2 - i' = 0$$

which gives us

$$i_2 = \frac{1}{2}\,\text{A}$$

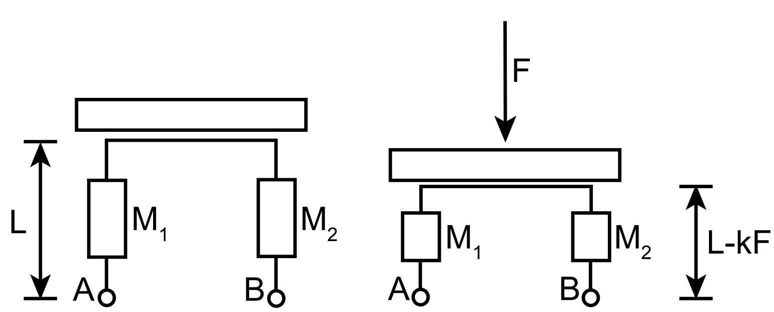
Now, applying the principle of superposition, we have $i = i_1 + i_2 = -\frac{5}{6}\,\text{A} + \frac{1}{2}\,\text{A} = -\frac{1}{3}\,\text{A}$.

3. Fruity Fred

Fruity Fred just got back from Berkeley Bowl with a bunch of mangoes, pineapples, and coconuts. He wants to sort his mangoes in order of weight, so he decides to use his knowledge from EE16A to build a scale.

He finds two identical bars of material (M_1 and M_2) of length L (meters) and cross-sectional area A_c (meters^2), which are made of a material with resistivity ρ. He knows that the length of these bars decreases by k meters per Newton of force applied, while the cross-sectional area remains constant.

He builds his scale as shown below, where the top of the bars are connected with an ideal electrical wire. The left side of the diagram shows the scale at rest (with no object placed on it), and the right side shows it when the applied force is F (Newtons), causing the length to decrease by kF meters. Fred's mangoes are not very heavy, so $L \gg kF$.



(a) Let R_{AB} be the resistance between nodes A and B. Write an expression for R_{AB} as a function of A_c, L, ρ, F, and k.

Solution:

The length of each spring as a function of F is $L - kF$.

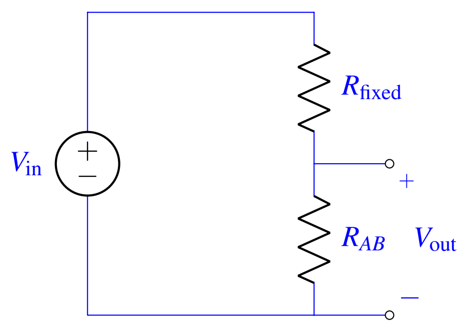
The combination of R_1 and R_2 has a resistance $R_{AB} = R_1 + R_2 = \frac{2\rho(L-kF)}{A_c}$.

(b) Fred's scale design is such that the resistance R_{AB} changes depending on how much weight is placed on it. However, he really wants to measure a voltage rather than a resistance.

Design a circuit for Fred that outputs a voltage that is some function of the weight. Your circuit should include R_{AB}, and you may use any number of voltage sources and resistors in your design. Be sure to label where the voltage should be measured in your circuit. Also provide an expression relating the output voltage of your circuit to the force applied on the scale.

Solution:

One possible solution: use a voltage divider.



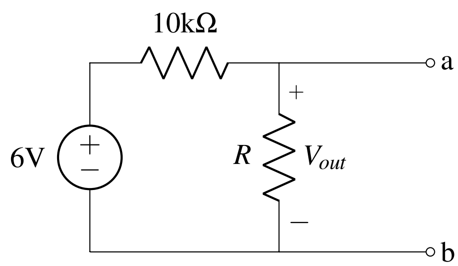
$$V_{\text{out}} = \frac{R_{AB}}{R_{\text{fixed}} + R_{AB}} V_{\text{in}}$$

$$V_{\text{out}} = \frac{\frac{2\rho(L-kF)}{A_c}}{R_{\text{fixed}} + \frac{2\rho(L-kF)}{A_c}} V_{\text{in}} = \frac{2\rho(L-kF)}{R_{\text{fixed}} A_c + 2\rho(L-kF)} V_{\text{in}}$$

4. Resistive Voltage "Regulator"

In this problem, we will design a circuit that provides an approximately constant voltage divider across a range of loads. We will use a resistive voltage divider circuit. The goal is to design a circuit that, from a source voltage of 6V, would yield an output voltage within 5% of 4V for loads in the range of 1kΩ to 100kΩ.

(a) First, consider the resistive voltage divider in the following circuit. What resistance R would achieve a voltage V_{out} of 4V?

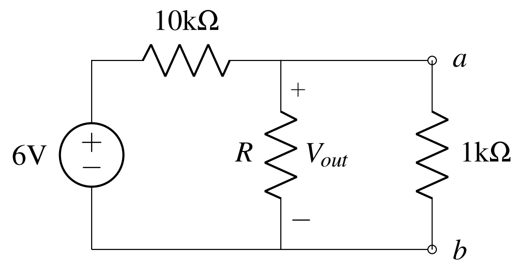


Solution:

The current in the circuit is $I = \frac{6\text{V}}{R+10\text{k}\Omega}$ which means that the output voltage can be calculated as $V_{\text{out}} = I \cdot R = 6\text{V} \cdot \frac{R}{R+10\text{k}\Omega}$. By constraining $V_{\text{out}} = 4\text{V}$, we get

$$\begin{aligned}
\frac{4\text{V}}{6\text{V}} &= \frac{R}{R+10\text{k}\Omega} \\
\frac{4\text{V}}{6\text{V}}(R+10\text{k}\Omega) &= R \\
\frac{4\text{V}}{6\text{V}} \cdot 10\text{k}\Omega &= R\left(1-\frac{4\text{V}}{6\text{V}}\right) \\
\frac{4\text{V}}{6\text{V}\left(1-\frac{4\text{V}}{6\text{V}}\right)} \cdot 10\text{k}\Omega &= R \\
\frac{4\text{V}}{6\text{V}-4\text{V}} \cdot 10\text{k}\Omega &= R \\
20.00\text{k}\Omega &= R
\end{aligned}$$

(b) Now using the same resistor R as calculated in part (a), consider loading the circuit with a resistor of 1kΩ as depicted in the following circuit. What is the voltage V_{out} now?

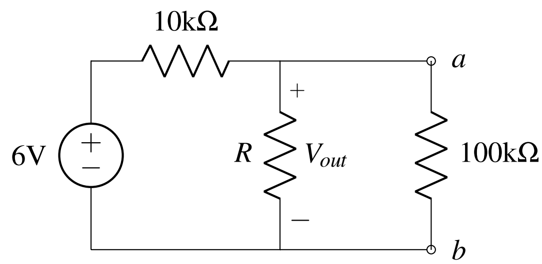


Solution:

The value from part (a) is $R = 20.00\text{k}\Omega$. This means that the effective resistance is $(20.00\text{k}\Omega \parallel 1\text{k}\Omega) = \frac{20.00 \cdot 1}{20.00+1} = 0.95\text{k}\Omega$. The current is therefore $I = \frac{6\text{V}}{10\text{k}\Omega+0.95\text{k}\Omega}$, which yields:

$$\begin{aligned}
V_{\text{out}} &= \frac{6\text{V} \cdot 0.95\text{k}\Omega}{10\text{k}\Omega + 0.95\text{k}\Omega} \\
V_{\text{out}} &= 0.52\text{V}
\end{aligned}$$

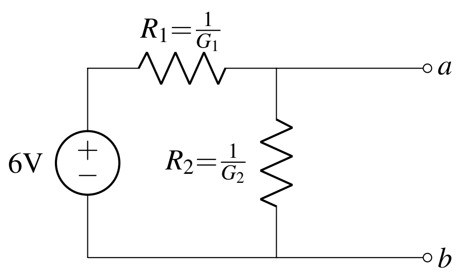
(c) Now using the same resistor R as calculated in part (a), consider loading the circuit with a resistor of 100kΩ, instead, as depicted in the following circuit. What is the voltage V_{out} now?

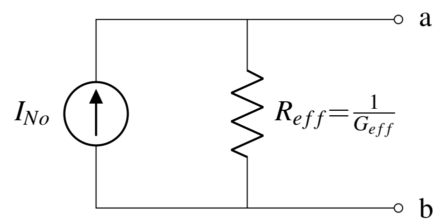


Solution:
The value from part (a) is $R = 20.00\text{k}\Omega$. This means that the effective resistance is $(20.00\text{k}\Omega \parallel 100\text{k}\Omega) = \frac{20.00 \cdot 100}{20.00+100} = 16.67\text{k}\Omega$. The current is therefore $I = \frac{6\text{V}}{10\text{k}\Omega+16.67\text{k}\Omega}$, which yields:

$$V_{\text{out}} = \frac{6\text{V} \cdot 16.67\text{k}\Omega}{10\text{k}\Omega + 16.67\text{k}\Omega}$$
$$V_{\text{out}} = 3.75\text{V}$$

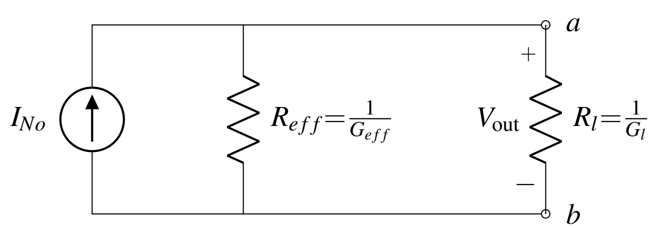
(d) Now we would like to design a divider that would keep the voltage V_{out} regulated for loads for a range of loads R_l. By that, we would like the voltage to remain within a 5% window of 4V. That is, we would like to design the following circuit such that $3.80\text{V} \leq V_{\text{out}} \leq 4.20\text{V}$ for a range of loads R_l. As a first step, what is the Norton equivalent of the circuit on the left? Write I_{No} and G_{eff} in terms of conductance values $G_1 = \frac{1}{R_1}$ and $G_2 = \frac{1}{R_2}$.





Solution:
In open circuit, the current through the resistors is $I = \frac{6\text{V}}{R_1+R_2}$. This means that the open circuit voltage is $V_{Th} = I \cdot R_2 = 6\text{V} \cdot \frac{R_2}{R_1+R_2}$. We now calculate the short circuit current I_{No}. In short circuit, the current is simply $I = \frac{6\text{V}}{R_1}$. Therefore, the effective resistance is $R_{eff} = \frac{V_{Th}}{I_{No}} = \frac{6\text{V} \cdot \frac{R_2}{R_1+R_2}}{\frac{6\text{V}}{R_1}} = \frac{R_1 \cdot R_2}{R_1+R_2} = (R_1 \parallel R_2)$. From this, we can write $I_{No} = 6\text{V} \cdot G_1$ and $G_{eff} = G_1 + G_2$.

(e) For the second step, using the Norton equivalent circuit you found in part (d), what is the range of G_{eff} that achieves $3.80\text{V} \leq V_{\text{out}} \leq 4.20\text{V}$ in terms of I_{No} and G_l?



Solution:
From Ohm's law, $V_{\text{out}} \cdot (G_{eff} + G_l) = I_{No}$. This means that $V_{\text{out}} = \frac{I_{No}}{G_{eff}+G_l}$. From the bound $3.80\text{V} \leq V_{\text{out}} \leq 4.20\text{V}$, we have $3.80\text{V} \leq \frac{I_{No}}{G_{eff}+G_l} \leq 4.20\text{V}$. This yields the following two inequalities:

$$3.80\text{V} \leq \frac{I_{No}}{G_{eff} + G_l}$$
$$\frac{I_{No}}{G_{eff} + G_l} \leq 4.20\text{V},$$

which is equivalent to

$$G_{eff} + G_l \leq \frac{I_{No}}{3.80\text{V}}$$
$$\frac{I_{No}}{4.20\text{V}} \leq G_{eff} + G_l,$$

which is equivalent to

$$G_{eff} \leq \frac{I_{No}}{3.80\text{V}} - G_l$$
$$\frac{I_{No}}{4.20\text{V}} - G_l \leq G_{eff},$$

or concisely

$$\frac{I_{No}}{4.20\text{V}} - G_l \leq G_{eff} \leq \frac{I_{No}}{3.80\text{V}} - G_l.$$

(f) Translate the range of G_{eff} in terms of I_{No} and G_l (that you found in part (e)) into a range on G_2 in terms of G_1 and G_l.

Solution:

By plugging in the values of G_{eff} and I_{No} into $\frac{I_{No}}{4.20\text{V}} - G_l \leq G_{eff} \leq \frac{I_{No}}{3.80\text{V}} - G_l$, we get

$$\frac{6\text{V}G_1}{4.20\text{V}} - G_l \leq G_1 + G_2 \leq \frac{6\text{V}G_1}{3.80\text{V}} - G_l,$$

or equivalently

$$\frac{6\text{V}G_1}{4.20\text{V}} - G_l - G_1 \leq G_2 \leq \frac{6\text{V}G_1}{3.80\text{V}} - G_l - G_1,$$

which is equivalent to

$$G_1\left(\frac{6\text{V}}{4.20\text{V}} - 1\right) - G_l \leq G_2 \leq G_1\left(\frac{6\text{V}}{3.80\text{V}} - 1\right) - G_l.$$

(g) Say we want to support loads in the range $1\text{k}\Omega \leq R_l \leq 100\text{k}\Omega$ with approximately constant voltage as described above (that is, $3.80\text{V} \leq V_{\text{out}} \leq 4.20\text{V}$). What is the range of G_2 in terms of G_1 now? Translate the range of G_2 in terms of G_1 into a range of R_2 in terms of R_1.

Solution:

Note: The unit of conductance is Siemens and is denoted by S.

We plug $R_l = 1\text{k}\Omega$ (equivalently $G_l = \frac{1}{1}\text{mS}$) into the bound found above and get

$$G_1\left(\frac{6\text{V}}{4.20\text{V}} - 1\right) - \frac{1}{1}\text{mS} \leq G_2 \leq G_1\left(\frac{6\text{V}}{3.80\text{V}} - 1\right) - \frac{1}{1}\text{mS}.$$

Similarly, we plug $R_l = 100\text{k}\Omega$ (equivalently $G_l = \frac{1}{100}\text{mS}$) into the bound found above and get

$$G_1\left(\frac{6\text{V}}{4.20\text{V}} - 1\right) - \frac{1}{100}\text{mS} \leq G_2 \leq G_1\left(\frac{6\text{V}}{3.80\text{V}} - 1\right) - \frac{1}{100}\text{mS}.$$

The intersection of the above two sets of inequalities is

$$G_1\left(\frac{6\text{V}}{4.20\text{V}} - 1\right) - \frac{1}{100}\text{mS} \leq G_2 \leq G_1\left(\frac{6\text{V}}{3.80\text{V}} - 1\right) - \frac{1}{1}\text{mS}$$

In terms of R_2 that range is (by taking the reciprocal of *all* sides and using $G = \frac{1}{R}$)

$$\frac{1}{\frac{1}{R_1}\left(\frac{6\text{V}}{4.20\text{V}} - 1\right) - \frac{1}{100\text{k}\Omega}} \geq R_2 \geq \frac{1}{\frac{1}{R_1}\left(\frac{6\text{V}}{3.80\text{V}} - 1\right) - \frac{1}{1\text{k}\Omega}}.$$

(h) Note that conductance is always non-negative. From the bounds on G_2 you found in the previous part, derive a bound on G_1 that ensures that G_2 is always non-negative and non-empty (that is, the whole range of possible G_2 values is non-negative and is not empty). Translate this range into a range of possible R_1 values.

Hint: In addition to the conductance being non-negative, also make sure that the range for G_2 is non-empty.

Solution:

Since conductance has to be positive, we need the range above to consist of positive values (that is, the lower bound has to be positive and the upper bound is larger than the lower bound) which yields:

$$0 \leq G_1\left(\frac{6\text{V}}{4.20\text{V}} - 1\right) - \frac{1}{100}\text{mS}$$

and

$$G_1\left(\frac{6\text{V}}{4.20\text{V}} - 1\right) - \frac{1}{100}\text{mS} \leq G_1\left(\frac{6\text{V}}{3.80\text{V}} - 1\right) - \frac{1}{1}\text{mS},$$

which is equivalent to

$$\frac{1}{100}\text{mS} \leq G_1\left(\frac{6\text{V}}{4.20\text{V}} - 1\right)$$

and

$$\frac{1}{1}\text{mS} - \frac{1}{100}\text{mS} \leq G_1\left(\frac{6\text{V}}{3.80\text{V}} - \frac{6\text{V}}{4.20\text{V}}\right),$$

which is equivalent to

$$0.0233\text{mS} = \frac{1}{\frac{6\text{V}}{4.20\text{V}} - 1} \cdot \frac{1}{100}\text{mS} \leq G_1$$

and

$$6.5835\text{mS} = \frac{\frac{1}{1}\text{mS} - \frac{1}{100}\text{mS}}{\frac{6\text{V}}{3.80\text{V}} - \frac{6\text{V}}{4.20\text{V}}} \leq G_1.$$

The intersection of both ranges yields

$$6.5835\text{mS} \leq G_1,$$

which in terms of R_1 is equivalent to

$$R_1 \leq 0.15189\text{k}\Omega$$

(i) Pick the values of R_1 and R_2 that achieve $3.80\text{V} \leq V_{\text{out}} \leq 4.20\text{V}$ for $1\text{k}\Omega \leq R_l \leq 100\text{k}\Omega$ while minimizing the power consumed by the voltage divider circuit in open circuit (when there is no load attached to the output). What are these values R_1 and R_2? How much power is consumed in this case? Calculate and report this power consumption using both the original circuit and the Norton equivalent circuit. Are the power you calculated using the original circuit and the power you calculated using the Norton equivalent circuit equal?

Solution:

The power in the original circuit (no load) can be calculated as $\frac{V^2}{R_1+R_2}$. In order to minimize the power, we should pick the largest possible $R_1 + R_2$. This can be achieved by choosing $R_1 = 0.15189\text{k}\Omega$ (the highest possible) and $R_2 = \frac{1}{\frac{1}{0.15189\text{k}\Omega}\left(\frac{6\text{V}}{4.20\text{V}}-1\right)-\frac{1}{100\text{k}\Omega}} = 0.35567\text{k}\Omega$ (the upper bound). The power in this case is $P = \frac{(6\text{V})^2}{0.15189\text{k}\Omega+0.35567\text{k}\Omega} = 70.9276\text{mW}$. If we use the Norton equivalent circuit, we have $R_{eff} = 0.1064\text{k}\Omega$ which yields the power consumption $P = I_{No}^2 \cdot R_{eff} = 166.0861\text{mW}$. The powers are not equal since Thévenin and Norton equivalents don't preserve power consumption of the circuit (think about the Thévenin equivalent in open circuit: the power consumption is always 0W).

(j) Now using the same values R_1 and R_2 from the previous part, load the circuit with a load of $51\text{k}\Omega$. How much power is consumed by each of the three resistors, R_1, R_2 and R_l (use the original circuit to compute the power)?

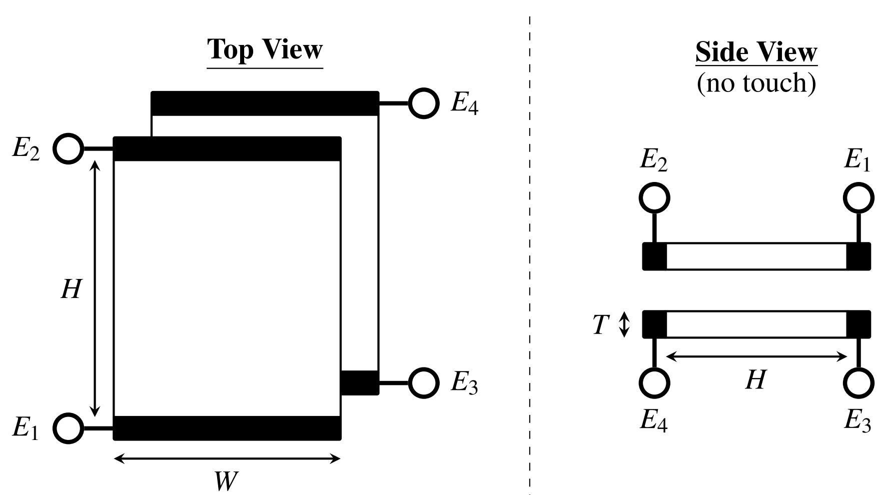
Solution:

The effective resistance in the output is $(R_2 \parallel R_l)$ which is equal to $0.3532\text{k}\Omega$. This means that the current from the source is $I = \frac{6\text{V}}{R_1+(R_2\|R_l)} = 11.8789\text{mA}$. Therefore, the power consumed by R_1 is $P_{R_1} = I^2 \cdot R_1 = 21.4330\text{mW}$. The output voltage is $V_{\text{out}} = I \cdot (R_2 \parallel R_l) = 4.1957\text{V}$, which means the power consumed by R_2 is $P_{R_2} = \frac{V_{\text{out}}^2}{R_2} = 49.4953\text{mW}$ and the power consumed by R_l is $P_{R_l} = \frac{V_{\text{out}}^2}{R_l} = 0.3452\text{mW}$ for a total power of $P = 71.2735\text{mW}$.

Note: Try doing the whole problem using resistance directly without going through conductances. You will see that the math will not be as "pretty" as the one presented here (using conductances). Sometimes using conductance yields easier derivation than the derivation using resistance.

5. Multitouch Resistive Touchscreen

In this problem, we will look at a simplified version of the multitouch resistive touchscreen. In particular, rather than measuring the position of two potential touch points in both dimensions (i.e. a pair of coordinates (x_1, y_1) and (x_2, y_2) corresponding to two touch positions), let's think about a version where we are interested in measuring only the vertical position of the two touch points (i.e. y_1 and y_2). Therefore, unlike the touchscreens we looked at in class, both of the resistive plates (i.e. both the top and the bottom plate) would have conductive strips placed along their top and bottom edges, as shown below.

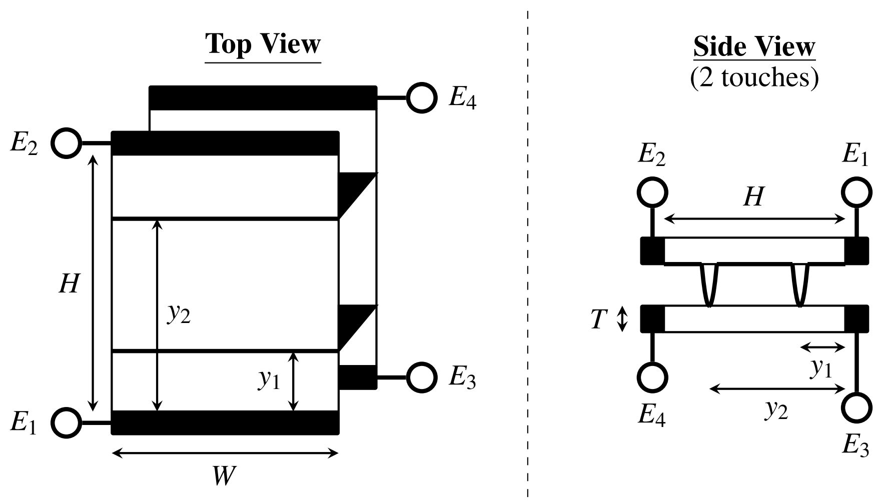


(a) Assuming that both of the plates are made out of a material with $\rho = 1\,\Omega\text{m}$ and that the dimensions of the plates are $W = 3\,\text{cm}$, $H = 12\,\text{cm}$, and $T = 1\,\text{mm}$, with no touches at all, what is the resistance between terminals E_1 and E_2 (which would be the same as the resistance between terminals E_3 and E_4)?

Solution:

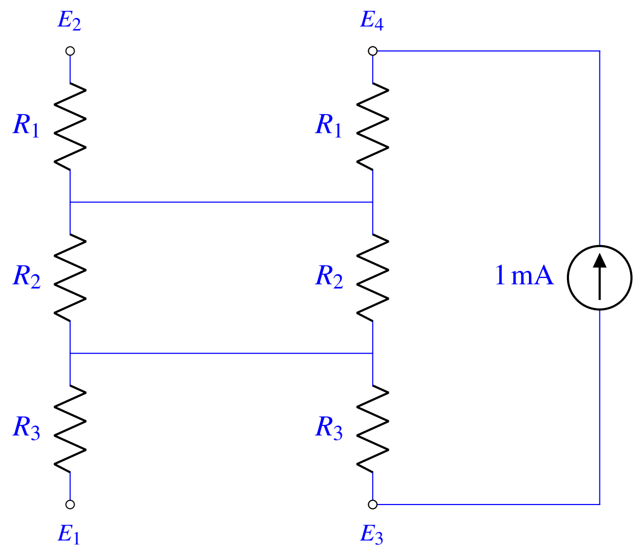
$$R = \rho \cdot \frac{L}{A} \implies R_{E1-E2} = \rho\left(\frac{H}{W \cdot T}\right)$$
$$R_{E1-E2} = 1\,\Omega\,\text{m}\left(\frac{12\times10^{-2}\,\text{m}}{3\times10^{-2}\cdot 1\times10^{-3}\,\text{m}}\right)$$
$$R_{E1-E2} = 4\,\text{k}\Omega$$

(b) Now let's look at what happens when we have two touch points. Let's assume that at wherever height the touch occurs, a perfect contact is made between the top plate and the bottom plate along the entire width of the plates (i.e. you don't have to worry about any lateral resistors), but that otherwise none of the electrical characteristics of the plates change. Defining the bottom of the plate as being $y = 0\,\text{cm}$ (i.e. a touch at E_1 would be at $y = 0\,\text{cm}$), let's assume that the two touches happen at $y_1 = 3\,\text{cm}$ and $y_2 = 7\,\text{cm}$ and that your answer to part (a) was $8\,\text{k}\Omega$ (which may or may not be the right answer). Draw a model with 6 resistors that captures the electrical connections between E_1, E_2, E_3, and E_4 and calculate their resistances. Note that for clarity, the system has been redrawn below to depict this scenario.

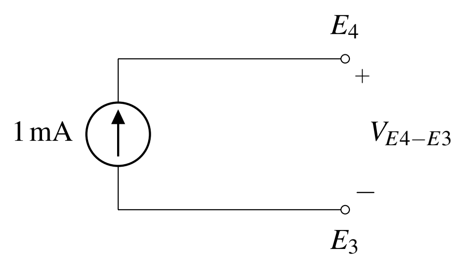


Solution:

$$R_3 = \frac{3\,\text{cm}}{12\,\text{cm}} \cdot R_{E2-E1} = 2\,\text{k}\Omega$$
$$R_2 = \frac{7\,\text{cm} - 3\,\text{cm}}{12\,\text{cm}} \cdot R_{E2-E1} = 2.667\,\text{k}\Omega$$
$$R_1 = \frac{12\,\text{cm} - 7\,\text{cm}}{12\,\text{cm}} \cdot R_{E2-E1} = 3.334\,\text{k}\Omega$$



(c) Using the same assumptions as part (b), if you drove terminals E_3 and E_4 with a 1 mA current source (as shown below) but left terminals E_1 and E_2 open-circuited, what is the voltage you would measure across $E_4 - E_3$ (i.e. V_{E4-E3})?



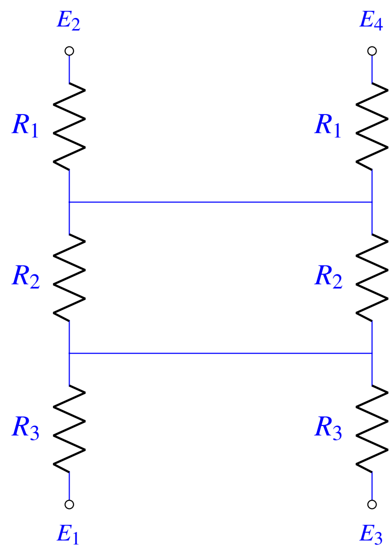
Solution:

The equivalent resistance between $E_4 - E_3$ is

$$R_{E4-E3} = R_1 + R_2 \parallel R_2 + R_3 = R_1 + \frac{R2}{2} + R_3$$
$$= 6.667\,\mathrm{k\Omega}$$
$$V_{E4-E3} = 1\,\mathrm{mA} \cdot R_{E4-E3} \implies V_{E4-E3} = 6.667\,\mathrm{V}$$

(d) Now let's try to generalize the situation by assuming that the two touches can happen at any two arbitrary points y_1 and y_2, but with y_1 defined to always be less than y_2 (i.e. y_1 is always the bottom touch point). Leaving the setup the same as in part (c) except for the arbitrary y_1 and y_2, by measuring only the voltage between E_4 and E_3, what information can you extract about the two touch positions? Please be sure to provide an equation relating V_{E4-E3} to y_1 and y_2 as a part of your answer, and note that you may want to redraw the model from part (b) to help you with this.

Solution:



For general

$$R_3 = \frac{y_1}{12\,\text{cm}} \cdot 8\,\text{k}\Omega$$
$$R_2 = \frac{y_2 - y_1}{12\,\text{cm}} \cdot 8\,\text{k}\Omega$$
$$R_1 = \frac{12\,\text{cm} - y_2}{12\,\text{cm}} \cdot 8\,\text{k}\Omega$$

$$\begin{aligned} R_{E4-E3} = R_1 + \frac{R_2}{2} + R_3 &= \left(12\,\text{cm} - y_2 + \frac{y_2 - y_1}{2} + y_1\right) \cdot \frac{8\,\text{k}\Omega}{12\,\text{cm}} \\ &= \left(12\,\text{cm} + \frac{y_1}{2} - \frac{y_2}{2}\right) \cdot \frac{8\,\text{k}\Omega}{12\,\text{cm}} \end{aligned}$$

So

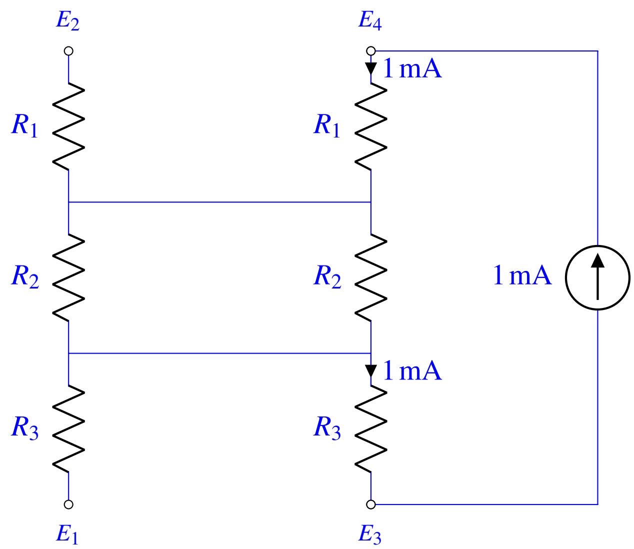
$$V_{E4-E3} = 1\,\text{mA} \cdot \frac{12\,\text{cm} - \frac{y_2 - y_1}{2}}{12\,\text{cm}} \cdot 8\,\text{k}\Omega = \frac{12\,\text{cm} - \frac{y_2 - y_1}{2}}{12\,\text{cm}} \cdot 8\,\text{V}$$

This means that by measuring V_{E4-E3}, we can only measure the distance between the two touch points $(y_2 - y_1)$.

(e) One of your colleagues claims that by measuring the appropriate voltages, not only can they extract what both y_1 and y_2 are in this system, but they can even do so by formulating a system of three independent voltage equations related to y_1 and y_2. As we will see later, this will allow us to gain some robustness to noise in the voltage measurements.

In order to facilitate this, write equations relating V_{E4-E2} and V_{E1-E3} to y_1 and y_2. (The third voltage we'll use is V_{E4-E3}, which you should have already derived an equation for in the previous part of the problem.)

Solution:



$$V_{E4-E2} = I \cdot R_1 = \frac{12\,\text{cm} - y_2}{12\,\text{cm}} \cdot 8\,\text{V}$$
$$V_{E1-E3} = I \cdot R_3 = \frac{y_1}{12\,\text{cm}} \cdot 8\,\text{V}$$

6. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.