EECS 16A Designing Information Devices and Systems I
Spring 2018 Homework 9

**This homework is due April 2, 2018, at 23:59.**
**Self-grades are due April 5, 2018, at 23:59.**

**Submission Format**
Your homework submission should consist of **one** file.

- `hw9.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

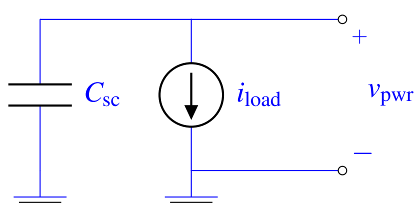
**1. Super-Capacitors**

In order to enable small devices for the "Internet of Things" (IoT), many companies and researchers are currently exploring alternative means of storing and delivering electrical power to the electronics within these devices. One example of these are "super-capacitors" - the devices generally behave just like a "normal" capacitor but have been engineered to have extremely high values of capacitance relative to other devices that fit in to the same physical volume.

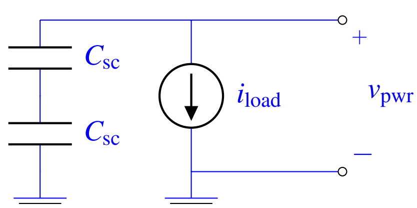
Your startup named **IoT4eva** is designing a new device that will revolutionize the process of making pizza, and you've been put in charge of selecting an energy source for it. You can't find a battery that quite suits your needs, so you decide to try out some super capacitors in various configurations. The super capacitors will be charged up to a certain voltage in the factory and will then act as the power supply (source of voltage) for the electronics in your device.

(a) Assuming that the electronics in your device can be modeled as drawing a constant current with a value of $i_{load}$, draw circuit models for your device using the following configurations of super-capacitors as the power supply for the electronics:

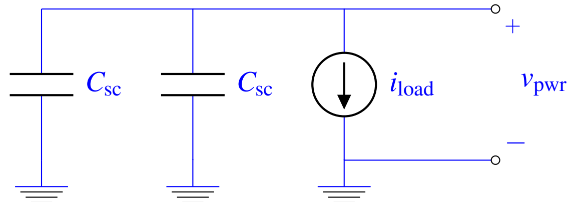
- Config 1: a single super-capacitor
  **Solution:**




- Config 2: two super-capacitors stacked in series
  **Solution:**



- Config 3: two super-capacitors connected in parallel
  **Solution:**




(b) If each super-capacitor is charged to an initial voltage $v_{init}$ and has a capacitance of $C_{sc}$, for each of the three configurations above, write an expression for the voltage supplied to your electronics as a function of time after the device has been activated.

**Solution:**

Let the initial voltage each super capacitor is charged to be $v_{init}$. We'll now consider the three situations:

- Config 1: **Single super capacitor**
  In this case, the initial voltage that the super capacitor provides is $v_{init}$, and the intial charge stored in it is then given by $Q_{init} = v_{init}C_{sc}$. Let the voltage at any time $(t)$ be defined by $v_{pwr}(t)$. The charge drained by the constant current source in time $t$ is given by $i_{load}t$. The effective charge stored in the capacitor after time $t$ is given by $Q(t) = Q_{init} - i_{load}t$. Therefore,

$$\begin{aligned} v_{pwr}(t) &= \frac{Q(t)}{C_{sc}} \\ &= \frac{Q_{init} - i_{load}t}{C_{sc}} \\ &= \frac{v_{init}C_{sc} - i_{load}t}{C_{sc}} \\ &= v_{init} - \frac{i_{load}t}{C_{sc}} \end{aligned}$$

- Config 2: **Two super capacitors in series**
  In this case, the initial voltage that the effective super capacitor provides is $2v_{init}$, and the effective capacitance is $C_{eq} = \frac{C_{sc}}{2}$. Then, the initial effective charge stored in them is then given by $Q_{init} = 2v_{init}C_{eq} = 2v_{init}\frac{C_{sc}}{2} = v_{init}C_{sc}$. Let the voltage at any time $(t)$ be defined by $v_{pwr}(t)$. The charge drained by the constant current source in time $t$ is given by $i_{load}t$. The effective charge stored in the combination after time $t$ is given by $Q(t) = Q_{init} - i_{load}t$. Therefore,

$$\begin{aligned} v_{pwr}(t) &= \frac{Q(t)}{C_{eq}} \\ &= \frac{Q_{init} - i_{load}t}{C_{eq}} \\ &= \frac{v_{init}C_{sc} - i_{load}t}{C_{eq}} \\ &= 2v_{init} - \frac{2i_{load}t}{C_{sc}} \end{aligned}$$

- Config 3: **Two super capacitors in parallel**
  In this case, the initial voltage that the effective super capacitor provides is $v_{init}$, and the effective capacitance is $C_{eq} = 2C_{sc}$. Then, the initial effective charge stored in them is then given by $Q_{init} = v_{init}C_{eq} = 2v_{init}C_{sc}$. Let the voltage at any time $(t)$ be defined by $v_{pwr}(t)$. The charge drained by the

constant current source in time $t$ is given by $i_{\text{load}}t$. The effective charge stored in the combination after time $t$ is given by $Q(t) = Q_{\text{init}} - i_{\text{load}}t$. Therefore,

$$\begin{aligned} v_{\text{pwr}}(t) &= \frac{Q(t)}{C_{\text{eq}}} \\ &= \frac{Q_{\text{init}} - i_{\text{load}}t}{C_{\text{eq}}} \\ &= \frac{2v_{\text{init}}C_{\text{sc}} - i_{\text{load}}t}{C_{\text{eq}}} \\ &= v_{\text{init}} - \frac{i_{\text{load}}t}{2C_{\text{sc}}} \end{aligned}$$

(c) Now let's assume that your electronics require some minimum voltage $v_{\text{min}}$ in order to function properly. For each of the three super-capacitor configurations, write an expression you could use to calculate the lifetime of the device.

**Solution:**

The lifetime of a device is the time it takes for the $v_{\text{pwr}}(t)$ to hit the threshold $v_{\text{min}}$. For each of the three configurations, let's find out their lifetime (denoted by $t_0$):

- Config 1: **Single super capacitor**
  Let us calculate at what time $t_0$ $v_{\text{pwr}}(t)$ equals $v_{\text{min}}$. We know from the previous part that

  $$v_{\text{pwr}}(t) = v_{\text{init}} - \frac{i_{\text{load}}t}{C_{\text{sc}}}.$$

  Substituting $v_{\text{pwr}}(t) = v_{\text{min}}$, we get

  $$t_0 = \frac{(v_{\text{init}} - v_{\text{min}})C_{\text{sc}}}{i_{\text{load}}}.$$

- Config 2: **Two super capacitors in series**
  Let us calculate at what time $t_0$ $v_{\text{pwr}}(t)$ equals $v_{\text{min}}$. We know from the previous part that

  $$v_{\text{pwr}}(t) = 2v_{\text{init}} - \frac{2i_{\text{load}}t}{C_{\text{sc}}}.$$

  Substituting $v_{\text{pwr}}(t) = v_{\text{min}}$, we get

  $$t_0 = \frac{(2v_{\text{init}} - v_{\text{min}})C_{\text{sc}}}{2i_{\text{load}}}.$$

- Config 3: **Two super capacitors in parallel**
  Let us calculate at what time $t_0$ $v_{\text{pwr}}(t)$ equals $v_{\text{min}}$. We know from the previous part that

  $$v_{\text{pwr}}(t) = v_{\text{init}} - \frac{i_{\text{load}}t}{2C_{\text{sc}}}.$$

  Substituting $v_{\text{pwr}}(t) = v_{\text{min}}$, we get

  $$t_0 = \frac{(v_{\text{init}} - v_{\text{min}})2C_{\text{sc}}}{i_{\text{load}}}.$$

**Note**: We could have also figured it out by finding out how much charge needs to be removed to cause the voltage at the effective capacitance to drop to $v_{\text{min}}$. Thus, we have

$$\Delta Q = (v_{\text{init}} - v_{\text{min}})C_{\text{eq}},$$

which gives us

$$t_0 = \frac{\Delta Q}{i_{\text{load}}}.$$

(d) Assuming that a single super-capacitor doesn't provide you sufficient lifetime and so you have to spend the extra money (and device volume) for another super-capacitor, which configuration would you pick and why would you pick one over the other?

- Config 2: two super-capacitors stacked in series
- Config 3: two super-capacitors connected in parallel

**Solution:**

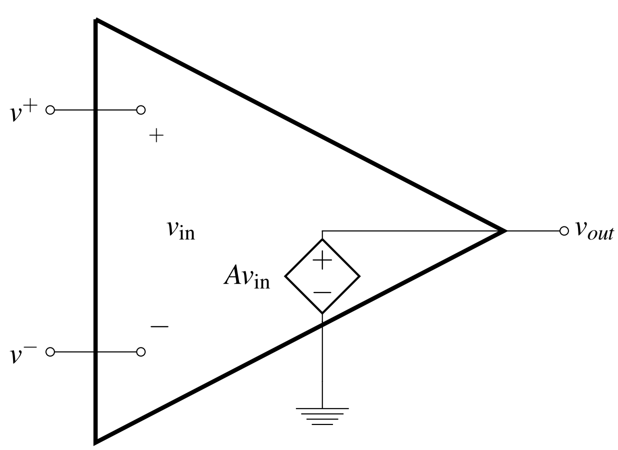
It is not obvious from the previous part whether configuration 2 or 3 will provide a longer lifetime. In fact, it depends on what $v_{\text{init}}$ is with respect to $v_{\text{min}}$. Let us now see what conditions we need on $v_{\text{init}}$, such that the parallel configuration provides a longer lifetime, i.e., $t_{0,\text{ parallel}} > t_{0,\text{ series}}$. From the previous part, we get

$$\begin{aligned}
\frac{(v_{\text{init}} - v_{\text{min}})2C_{\text{sc}}}{i_{\text{load}}} &\geq \frac{(2v_{\text{init}} - v_{\text{min}})C_{\text{sc}}}{2i_{\text{load}}} \\
2(v_{\text{init}} - v_{\text{min}}) &\geq v_{\text{init}} - \frac{v_{\text{min}}}{2} \\
v_{\text{init}} &\geq \frac{3}{2}v_{\text{min}}
\end{aligned}$$

Thus, we see that when $v_{\text{init}} \geq \frac{3}{2}v_{\text{min}}$, the parallel configuration is better; otherwise the series configuration is better.

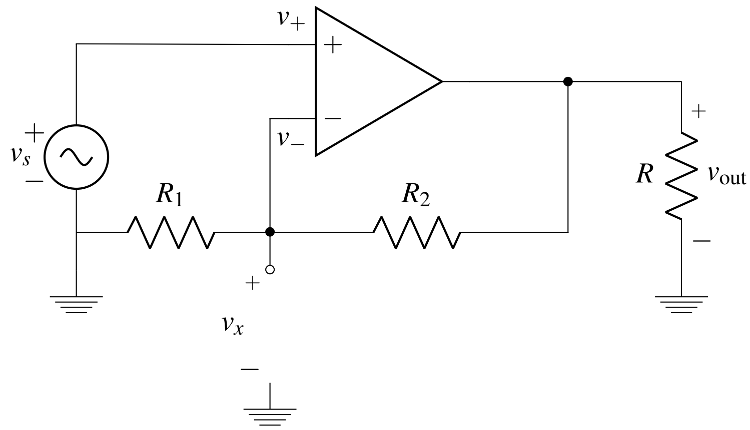
## 2. Op-Amp Golden Rules

In this question, we are going to show that the Golden Rules for op-amps hold by analyzing equivalent circuits and then taking the limit as the open-loop gain approaches infinity. Below is a picture of the equivalent model of an op-amp we are using for this question.



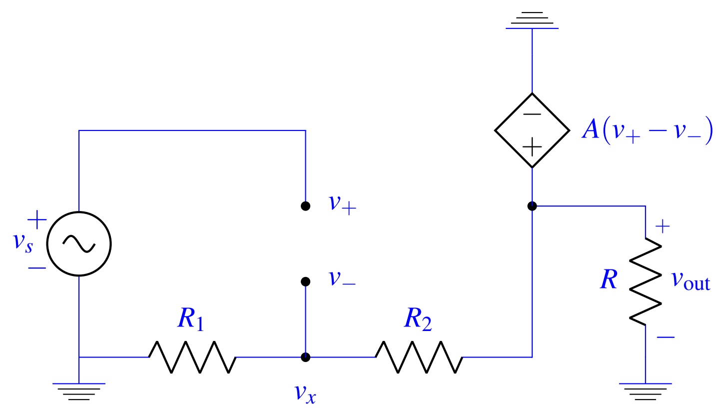
(a) Now consider the circuit below.

Draw an equivalent circuit by replacing the op-amp with the op-amp model shown above and calculate $v_{\text{out}}$ and $v_x$ in terms of $A$, $v_s$, $R_1$, $R_2$ and $R$. Is the magnitude of $v_x$ larger or smaller than the magnitude of $v_s$? Do these values depend on $R$?




**Solution:**

This is the equivalent circuit of the op-amp:




Since $v_{\text{out}}$ is connected to the output of the op-amp, which is a voltage source, we can determine $v_{\text{out}}$:

$$\begin{aligned} v_{\text{out}} &= A(v_+ - v_-) \\ &= A(v_s - v_x) \end{aligned}$$

Since there is no current flowing through the nodes $v_+$ and $v_-$ (because we are assuming that $R_{\text{in}}$ is infinite), $R_1$ and $R_2$ form a voltage divider and $v_x = v_{\text{out}}\left(\frac{R_1}{R_1+R_2}\right)$. Thus, substituting and solving for $v_{\text{out}}$:

$$v_{\text{out}} = A\left(v_s - v_{\text{out}}\frac{R_1}{R_1+R_2}\right)$$

$$v_{\text{out}} = v_s\left(\frac{1}{\frac{R_1}{R_1+R_2}+\frac{1}{A}}\right)$$

Knowing $v_{\text{out}}$, we can find $v_x$:

$$v_x = \frac{v_s}{1+\frac{R_1+R_2}{AR_1}}$$

Notice that $v_x$ is slightly smaller than $v_s$, meaning that in equilibrium in the non-ideal case, $v_+$ and $v_-$ are not equal. $v_{\text{out}}$ and $v_x$ do not depend on $R$, which means that we can treat $v_{\text{out}}$ as a voltage source that supplies a constant voltage independent of the load $R$.

(b) Using your solution to part (a), calculate the limits of $v_{\text{out}}$ and $v_x$ as $A \to \infty$. Do you get the same answers if you apply the Golden Rules ($v_+ = v_-$ when there is negative feedback)?

**Solution:**

As $A \to \infty$, the fraction $\frac{1}{A} \to 0$, so

$$v_{\text{out}} = v_s \left( \frac{1}{\frac{R_1}{R_1+R_2} + \frac{1}{A}} \right)$$

converges to

$$v_s \left( \frac{1}{\frac{R_1}{R_1+R_2} + 0} \right) = v_s \left( \frac{R_1 + R_2}{R_1} \right).$$

Therefore, the limits as $A \to \infty$ are:

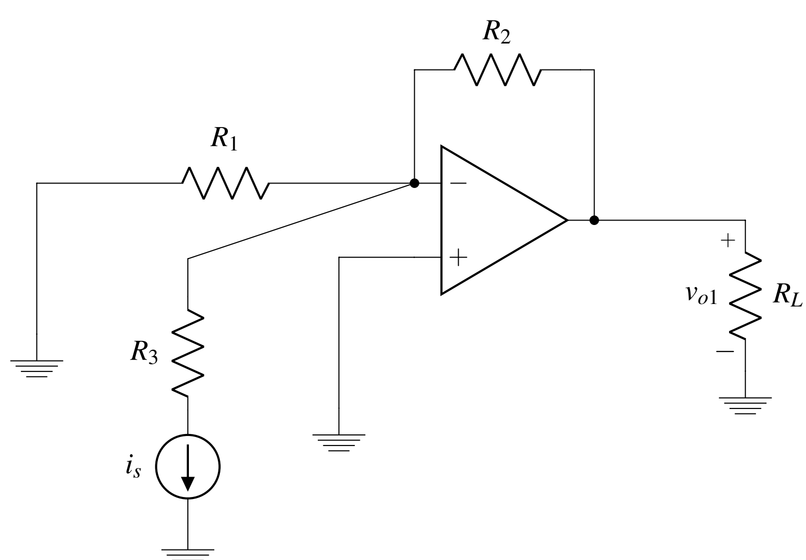
$$v_{\text{out}} \to v_s \left( \frac{R_1 + R_2}{R_1} \right)$$

$$v_x \to v_s$$

If we apply the Golden Rules, we get $v_x = v_s$. Then the current $i$ flowing through $R_1$ to ground is $\frac{v_s}{R_1}$. By KCL, this same current flows through $R_2$ since no current flows through $v_-$. Thus, the voltage drop between $R_2$, $v_{\text{out}} - v_x$, is $i \cdot R_2 = v_s \left( \frac{R_2}{R_1} \right)$. Therefore, $v_{\text{out}} = v_s + v_s \left( \frac{R_2}{R_1} \right) = v_s \left( \frac{R_1+R_2}{R_1} \right)$. The answers are the same if you take the limit as $A \to \infty$.

**3. Amplifier with Multiple Inputs**

(a) Use the Golden Rules to find $v_{o1}$ for the circuit below.




**Solution:**

Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is 0. The voltage drop across $R_1$

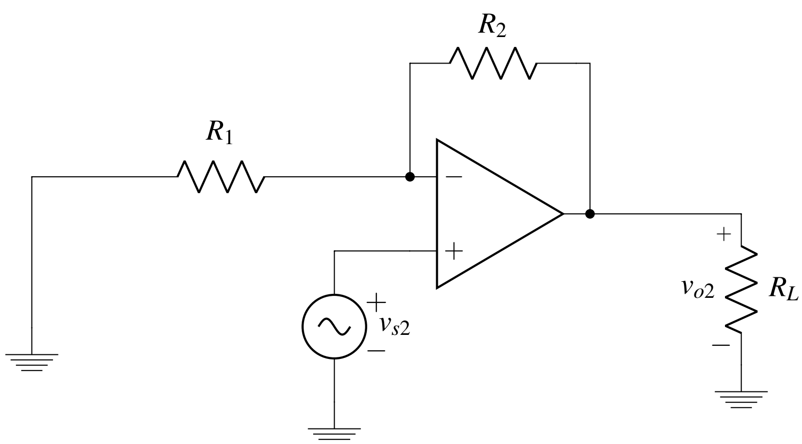
is 0 and no current flows through it. In addition, no current flows into the op-amp from the negative terminal due to its infinite input resistance (the negative terminal is connected to an "open" circuit). By KCL at the negative terminal of the op-amp, this means that the current going through $R_3$ and $R_2$ is $i_s$. Taking the positive terminal of $R_2$ to be on the right, the voltage drop across $R_2$ is $v_{o1}$. By Ohm's law, we conclude:

$$\frac{v_{o1}}{R_2} = i_s$$

Rearranging we get:

$$v_{o1} = i_s \cdot R_2$$

(b) Use the Golden Rules to find $v_{o2}$ for the circuit below.




**Solution:**

Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $V^- = v_{s2}$. In addition, since no current can enter into the negative terminal of the op-amp, $R_1$ and $R_2$ are in series. This means that the voltage at the negative terminal of the op-amp can be expressed in terms of $v_{o2}$ using the voltage divider formula:

$$v^- = v_{o2}\left(\frac{R_1}{R_1 + R_2}\right)$$

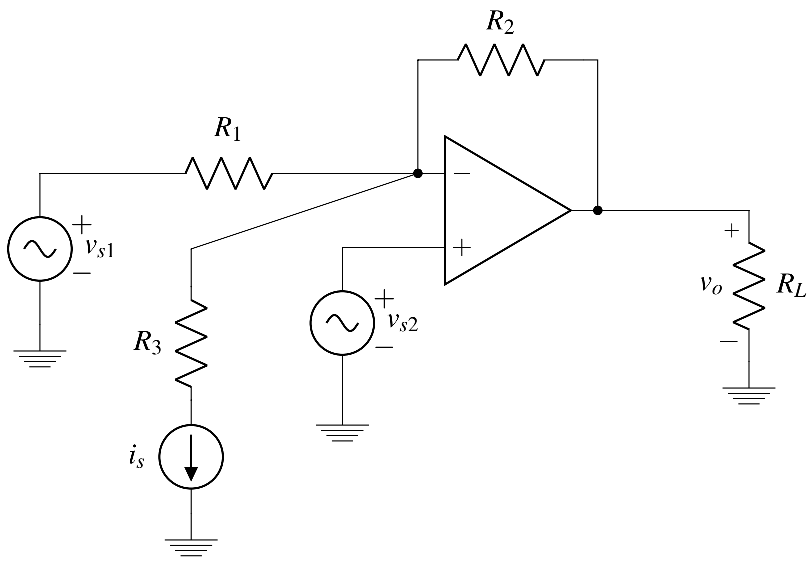
We also know that $v^- = v_{s2}$ and conclude:

$$v_{s2} = v_{o2}\left(\frac{R_1}{R_1 + R_2}\right)$$

After rearranging, we have:

$$v_{o2} = v_{s2}\left(\frac{R_2}{R_1} + 1\right)$$

(c) Use the Golden Rules to find the output voltage $v_o$ for the circuit shown below.




**Solution:**

Applying the Golden Rules we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $v = v_{s2}$. Then we write a KCL equation at the node connected to the minus terminal of the op-amp (recalling that no current flows into or out of the op-amp's terminals). All currents are defined as flowing out of the node:

$$i_{R_1} + i_{R_2} + i_{R_3} = 0$$

Because of the independent current source, we know:

$$i_{R_3} = i_s$$

By Ohm's law, we know:

$$i_{R_1} = \frac{v^- - v_{s1}}{R_1}$$

and

$$i_{R_2} = \frac{v^- - v_o}{R_2}$$

Then, substituting back into the original KCL equation, we have:

$$\frac{v^- - v_{s1}}{R_1} + \frac{v^- - v_o}{R_2} + i_s = 0$$

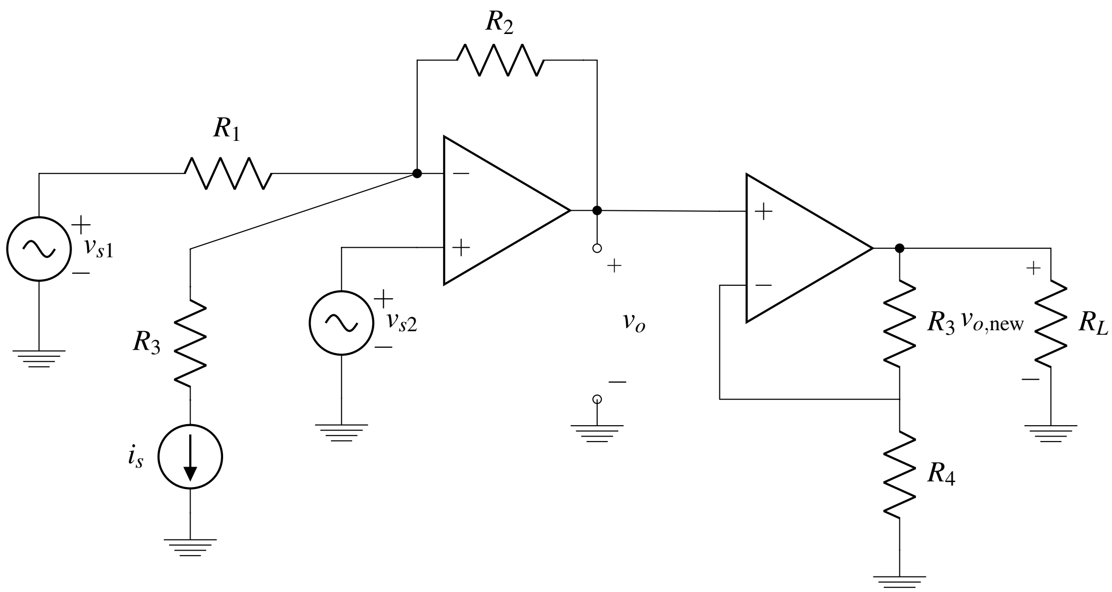
and substituting $v^- = v_{s2}$, we have:

$$\frac{v_{s2} - v_{s1}}{R_1} + \frac{v_{s2} - v_o}{R_2} + i_s = 0$$

which we rearrange to find $v_o$, giving:

$$v_o = v_{s2}\left(1 + \frac{R_2}{R_1}\right) + i_s \cdot R_2 - \left(\frac{R_2}{R_1}\right) v_{s1}$$

(d) Now add a second stage as shown below. What is $v_{o,\text{new}}$? Does $v_o$ change between part (c) and this part? Does the voltage $v_{o,\text{new}}$ depend on $R_L$?




**Solution:**

Adding the second stage does not change the voltages in the first stage. This is because the circuit connected to the positive and negative terminals of the first stage op-amp "sees" an open circuit/infinite input resistance in the op-amp.

Call the output voltage of the first stage $v_{o1}$. Then it remains unchanged from part (c).

$$v_{o1} = -\left(\frac{R_2}{R_1}\right) v_{s1} + i_s \cdot R_2 + v_{s2}\left(\frac{R_2 + R_1}{R_1}\right)$$

By the Golden Rules, the negative terminal of the second op-amp must have the same voltage as the plus terminal, which is $v_{o1}$. No current can flow into the negative terminal, so $R_3$ and $R_4$ are in series and have the same current, so we know:

$$\frac{v_{o1}}{R_4} = \frac{v_o - v_{o1}}{R_3}$$

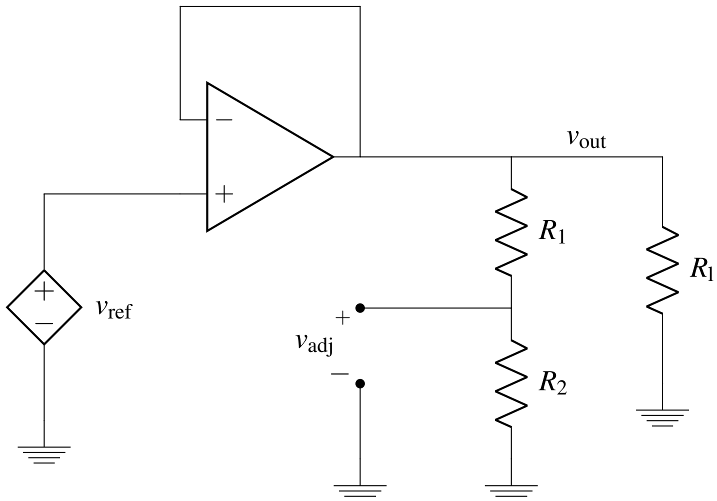
Therefore:

$$v_o = \left(\frac{R_3 + R_4}{R_4}\right) v_{o1} = \frac{R_3 + R_4}{R_4}\left(-\frac{R_2}{R_1} \cdot v_{s1} + i_s \cdot R_2 + v_{s2} \cdot \frac{R_2 + R_1}{R_1}\right)$$

**Solution:**

**4. Op-Amps and State Transition Matrices**

Consider the following circuit that where we are given that $v_{ref} = v_{adj} + 1.25$.




(a) Express $v_{out}$ in terms of the other voltages and resistor values. Then express $v_{adj}$ in terms of $v_{out}$.

**Solution:**

$$v_{out} = v_{ref}$$
$$v_{adj} = v_{out}\frac{R_2}{R_1 + R_2}$$

(b) Let us model the nodal voltages as updating once every $dt$ units of time to see how the long term steady state of the system behaves in this circuit. Use the state vector given below and construct a state transition matrix for the circuit. More precisely find the matrix $\vec{s}(t+dt) = A\vec{s}(t)$.

$$\vec{s} = \begin{bmatrix} v_{out} \\ v_{adj} \\ v_{ref} \\ 1.25 \end{bmatrix}$$

**Solution:** Using the equations from part a and the expression for $v_{ref}$ in the question, we get

$$\begin{bmatrix} v_{out}(t+dt) \\ v_{adj}(t+dt) \\ v_{ref}(t+dt) \\ 1.25 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{R_2}{R_1+R_2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{out}(t) \\ v_{adj}(t) \\ v_{ref}(t) \\ 1.25 \end{bmatrix}$$

(c) Now find the eigenvalues of the matrix. You may use an online tool like Wolfram Alpha for this part. What do these eigenvalues say about the existence steady state of the system? If there exists a steady state, write it down.

**Solution:** The eigenvalues of this system are $1, k^{\frac{1}{3}}; k = \frac{R_2}{R_1+R_2}$. Note that the latter root is repeated 3 times. An eigenvalue of 1 is always present, which means that the system has a steady state. Moreover the other eigenvalue is always smaller than 1, which means the system converges to it's steady state. The eigenvector corresponding to eigenvalue 1 is

$$\vec{v} = \begin{bmatrix} \frac{R_1+R_2}{R_1} \\ \frac{R_2}{R_1} \\ \frac{R_1+R_2}{R_1} \\ 1 \end{bmatrix}$$

The eigenvector associated with the steady state must have the value of 1.25 in the last entry. So we multiply by the appropriate constant to get the steady state as

$$\vec{s'} = 1.25 \begin{bmatrix} \frac{R_1+R_2}{R_1} \\ \frac{R_2}{R_1} \\ \frac{R_1+R_2}{R_1} \\ 1 \end{bmatrix}$$

**5. Cool For The Summer**

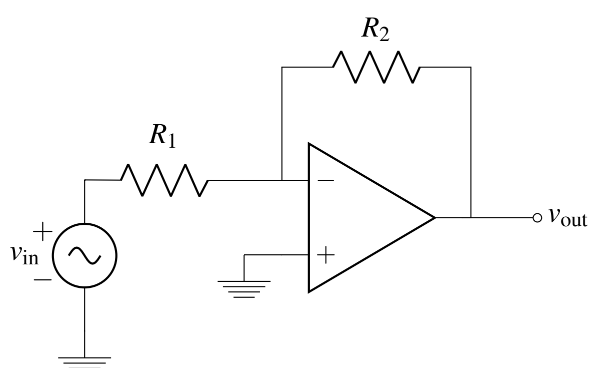
You and a friend want to make a box that helps control an air conditioning unit. You both have dials that display a voltage: 0 means that you want to leave the temperature as it is. Negative voltages mean that you want to reduce the temperature. (It's hot, so we will assume that you never want to increase the temperature – so, we're not talking about a Berkeley summer...)

Your air conditioning unit, however, responds to positive voltages. The higher the magnitude of the voltage, the stronger it runs. At zero, it is off.

Therefore, you need a box that is an inverting summer – it outputs a weighted sum of two voltages where the weights are both negative. The sum is weighted because each of you has your own subjective sense of how much to turn the dial down, so you need to compensate for this.

This problem walks you through this using an op-amp.

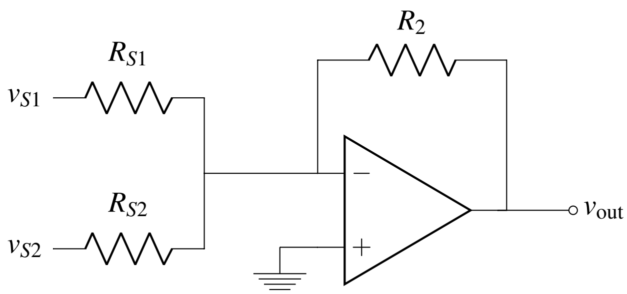
(a) As a first step, find $v_{\text{out}}$ in terms of $R_2$, $R_1$, $v_{\text{in}}$.




**Solution:**

The general inverting amplifier shown above has a voltage gain $v_{\text{out}} = -\frac{R_2}{R_1} v_{\text{in}}$.

(b) Now we will add a second input to this circuit as shown below. Find $v_{\text{out}}$ in terms of $v_{S1}$, $v_{S2}$, $R_{S1}$, $R_{S2}$ and $R_2$.




**Solution:**

We can find the overall voltage gain of this amplifier using superposition. When $v_{S1}$ is on, we can ignore $R_{S2}$. From the Golden Rules, we know that the voltage at the $-$ terminal of the op-amp must be equal to the voltage at the $+$ terminal. Thus, the voltage across $R_{S2}$ is $0\,\mathrm{V}$. Now apply the equation from part (a) $v_{\text{out}} = -\frac{R_2}{R_{S1} v_{S1}}$. Similarly, when $v_{S2}$ is on, we get $v_{\text{out}} = -\frac{R_2}{R_{S2}} v_{S2}$. Combining the two equations, we get $v_{\text{out}} = -R_2 \left( \frac{v_{S1}}{R_{S1}} + \frac{v_{S2}}{R_{S2}} \right)$.

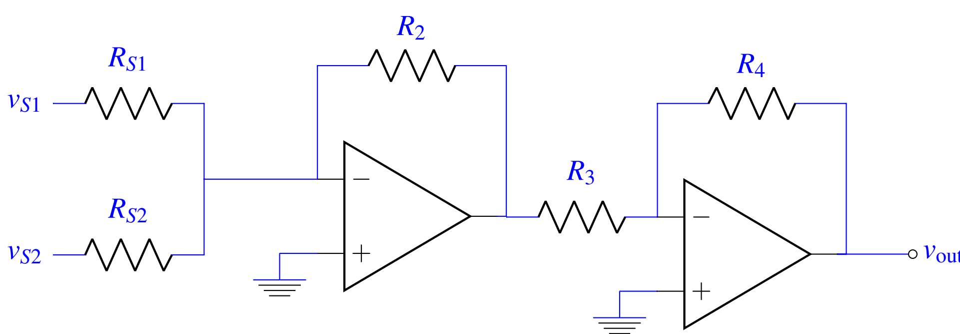
(c) Let's suppose that you want $v_{\text{out}} = -\left(\frac{1}{4} v_{S1} + 2 v_{S2}\right)$ where $v_{S1}$ and $v_{S2}$ represent the input voltages from you and your friend. Select resistor values such that the circuit implements this desired relationship.

**Solution:**

One possible set of values is $R_2 = 2\,\mathrm{k}\Omega$, $R_{S1} = 8\,\mathrm{k}\Omega$, and $R_{S2} = 1\,\mathrm{k}\Omega$. There are many possible answers here.

(d) Now suppose that you have another AC unit that you want to add to the same room. This unit however, functions opposite to the already existing unit; it responds to negative voltages. You want to run both units at the same time. Add another op-amp based circuit to your existing circuit to create an output for the second AC unit.

**Solution:**




Here, we add another inverting op-amp stage with a voltage gain of 1, and we can pick any equal-valued resistors for $R_3$ and $R_4$.

## 6. IoT4eva Revisited

After guiding them to make an intelligent selection for their super-capacitors, IoT4eva was so happy with your performance that you got a promotion! The good news is that you're getting paid more, but the "bad" news is that you have more responsibilities too. In particular, you are now responsible not only for selecting the super-capacitors used to power the device, but also for building the rest of the circuitry associated with the power supply.

In practice, many real circuits (especially sensors that are trying to detect very small signals) don't like to operate with supply voltages that vary substantially over time. Remembering that the voltage on our super capacitors drops linearly as we pull current out of them. This means that if we want to use these super capacitors for our device, we need to build another circuit. This circuit is powered by the super-capacitor and produces a constant voltage at its output, where this voltage will then be used to supply power to rest of the device. These circuits are often referred to as "voltage regulators," and in this problem we'll explore how to build the simplest form of such a voltage regulator.

(a) The first problem we have to solve to realize such a voltage regulator is to figure out how to build a reference that would allow us to set the voltage at the output of our regulator to a known absolute value. Fortunately, someone else in the company has already built one of those and made it available to you – the Thévenin equivalent of this circuit is a voltage source whose value is 0.8 V and a resistance of 1 kΩ. (The internals of this voltage reference circuit aren't important for this problem, but as you should see shortly, this circuit by itself is not appropriate for supplying power to the rest of the device.)

Now that we have a reference, we can focus on the core of the voltage regulator itself. Using this reference circuit, an op-amp, and resistors, design a circuit that is powered by the super-capacitor voltage $V_{sc}$ (which for now you can assume is always high enough for the circuit to work) and that would produce a constant 1.2 V supply voltage for the rest of the device. Note that you can model the load from the rest of the device as a 10 mA current source; please be sure to choose specific values for any resistors you use in your circuit as well.

*Hint:* Remember that the op-amp itself needs to be supplied with power, and the only source of power we have available is the super-capacitor.

**Solution:**

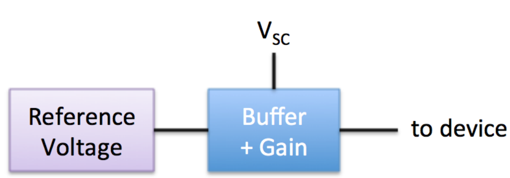
Let's practice the design method.

*Step 1*:

In this problem, the ultimate objective is to output a 1.2 V node that is capable of driving the load modeled as a 10 mA current source. We are also required to power the voltage regulator using the super-capacitor.

*Step 2*:

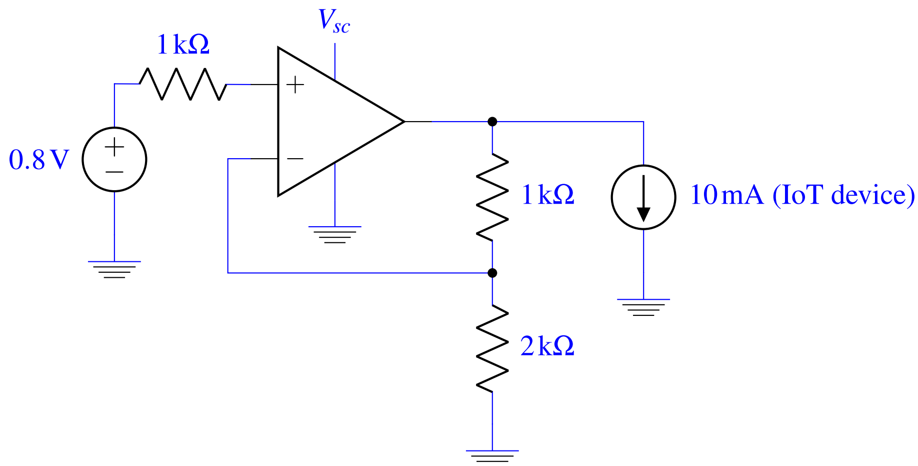
We are given a reference voltage source that we can use to power the IoT device. However, the voltage of the reference is not high enough and it has a high source resistance (in fact, if we run 0.8 mA of current through the source resistance, the voltage drop across the resistor would be the same as the voltage source itself). The only other source of voltage is the super-capacitor, but the problem is that it has a variable voltage. Thus, we need to build a circuit that buffers (provides low output resistance) and amplifies the reference voltage which is powered by the super-capacitor.




*Step 3*:

Now that we have a high-level block diagram of the circuit, we can think about how to implement it. We need some form of buffer, so we will definitely need an op-amp. Moreover, we also know that the output of the op-amp must be directly connected to the device for it to act as a buffer. We also know that the circuit can only be powered by the super-capacitor, so we power the op-amp using $V_{sc}$.

The other functionality we have not dealt with is the gain. We need a gain of $\frac{1.2\,\text{V}}{0.8\,\text{V}} = 1.5$, and we have seen that we can implement this using a non-inverting amplifier. Since the gain of a non-inverting amplifier is $\frac{R_2+R_1}{R_1}$, we can set any value to $R_1$ and $R_2$ such that this ratio is 1.5. For example, we can use $R_1 = 2\,\text{k}\Omega$ and $R_2 = 1\,\text{k}\Omega$. Note that the source resistance of the reference voltage doesn't play a role here since there is no current flowing into the inputs of the op-amp. The circuit is shown below.




(b) Now that we've built the voltage regulator and we know that we want its output voltage to stay fixed at 1.2 V, what is the minimum voltage we need on our super capacitors $V_{sc,min}$ to ensure that the regulator can indeed produce a fixed 1.2 V output?

**Solution:**

The op-amp will not be able to produce 1.2 V at its output if $V_{sc} < 1.2\,\text{V}$, so $V_{sc,min} = 1.2\,\text{V}$.

(c) One of the most important things to evaluate about a voltage regulator is its efficiency – i.e., the power dissipated by the load circuits (in this case, the rest of the IoT4eva device) divided by the total amount of power delivered by the power supply. Continuing to model the rest of the IoT4eva device as a 10 mA current source, how much power is dissipated by the 10 mA current source? Assuming that all of the IoT4eva's 10 mA current flows through the super-capacitor, and that no other current is added by the op-amp itself, how much power is delivered by the super-capacitor? In this case, what is the overal efficiency of our design?

**Solution:**

The 10 mA curent source is supplied with 1.2 V, so

$$P_{\text{device}} = 1.2\,\text{V} \cdot 10\,\text{mA} = 12\,\text{mW}$$

Our reference voltage does not output any current since the current into the input of an op-amp in negative feedback is 0 A, so the power associated with that source is 0 W. It is important to note that the op-amp by itself cannot generate any power. Any current flowing to the 1.2 V output node has to come from the op-amp, and any current that flows out of the op-amp must come from the super-capacitor. The op-amp is not supplying any power, it just dissipates a part of the power it receives and passes on the rest to the output. The total power supplied by the super-capacitor is the product of the output current of the op-amp with the voltage of the super-capacitor. This output current is the sum of the current source and the current flowing through the negative feedback resistors to ground.

$$I_{\text{op-amp}} = 10\,\text{mA} + \frac{1.2\,\text{V}}{3\,\text{k}\Omega} = 10.4\,\text{mA}$$

Power from super-capacitor is thus $V_{sc} \cdot 10.4\,\mathrm{mA}$. So, the efficiency is

$$\frac{1.2\,\mathrm{V} \cdot 10\,\mathrm{mA}}{V_{sc} \cdot 10.4\,\mathrm{mA}} = \frac{12}{10.4 V_{sc}}$$

*Note*: The efficiency might differ depending on your choice of resistor values.

Notice that $V_{sc}$ shows up in the denominator. Thus, if we increase the super-capacitor voltage, our efficiency drops. This might be counterintuitive at first, but the reason this happens is that the op-amp is configured in a way that forces the output to a certain voltage. That means there is some voltage drop that happens inside the op-amp itself, and this voltage drop is wasted since it does not get delivered to the IoT device.

Even though the efficiency of this circuit is quite low, this circuit is actually used in real life quite often. This is because the circuit is small (in PCB area) and has nice properties in terms of isolating different components in a circuit from each other.

**7. Homework Process and Study Group**

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

**Solution:**

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.