17.1 Capacitive Touchscreen

Viewing the physical structure corresponding to one pixel on the capacitive screen, we want to be able to tell if there is a finger touch on top of the pixel.

The green line represents our finger touching the dielectric, $C_{F-E2}$ is the capacitance between our finger ($F$) and electrode $E2$, and $C_{E1-F}$ is the capacitance between our finger ($F$) and electrode $E1$. Now, we can draw the equivalent circuit model corresponding to the pixel we’re looking at:
From the standpoint of smart phones, a smart phone can only sense something that is between electrodes $E_1$ and $E_2$ as it is impractical to physically connect a measuring device to our finger. So, we redraw our circuit as follows:

We can further simplify the circuit by replacing the series combination of $C_{E_1-F}$ and $C_{F-E_2}$ by some capacitance $C_{\Delta}$. Intuitively, we know $C_{\Delta}$ has some finite capacitance and is greater than zero:

If we consider the parallel combination of $C_o$ and $C_{\Delta}$ as some equivalent capacitance $C_{eq,E_1-E_2}$, we know that $C_{eq,E_1-E_2}$ will have some value greater than $C_o$ because $C_{eq,E_1-E_2} = C_o + C_{\Delta}$ and $C_{\Delta} > 0$. 
When a finger presses on top of a pixel, the equivalent capacitance between $E_1$ and $E_2$ will become greater than the default capacitance without a finger pressing on top. This characteristic allows us to tell if there is a finger pressing on top of each pixel on our 2D capacitive touchscreen.

Given 4 pixels 1, 2, 3, 4 and their corresponding locations, we can measure the capacitance at $(x_1, y_1)$ to know if there is a finger pressing on top of pixel 1. Similarly, to know if there is a finger pressing on top of pixels 2, 3, 4, we can measure the capacitances at $(x_1, y_2)$, $(x_2, y_1)$, $(x_2, y_2)$. If we sense an increase in the capacitance at the pixel location, we know there is a finger pressing on top of this pixel.

### 17.2 Capacitance Measurement

There are various ways you can use to measure capacitance. In this lecture, we will cover one specific method to measure capacitance. First, let’s review some basic physics for a capacitor. As you know:

\[
I = C \times \frac{dV}{dt}, \quad Q = C \times V. \tag{1}
\]

We can build a circuit with a current source $I_s$ and a capacitor $C$. If we charge the capacitor $C$ using the current source $I_s$ for a time interval $t$. 

\[
E_1
\]

\[
C_{eq,E1-E2}
\]

\[
E_2
\]
We can then find $V$ to be:

$$V = \frac{I_s \times t}{C}$$

(2)

$V$ measured depends on the amount of charge $Q = I_s \times t$ and the capacitance value $C$. So, if we charge a capacitor using some current source for a finite time period, we can map the voltage across the capacitor to capacitance. However, building a current source is not that easy, so we have to come up with some workarounds to measure capacitance.

**• Attempt #1** Our first attempt is connecting a capacitor to some voltage source and we measure the voltage across the capacitor directly:

We can immediately see that no matter how $C$ changes, the voltage measured $V_{out}$ is always going to be equal to $V_s$.

**• Attempt #2** In order to measure something different from $V_s$ while still using the voltage source as the power source in our circuit, we must somehow disconnect $V_{out}$ from $V_s$ when we measure $V_{out}$. How? We can add a switch to the circuit. A switch has two states, at its on state, the switch becomes a perfectly conducting wire with zero resistance, and when the switch is turned off, it becomes an open circuit.

For our second attempt, we will add a reference capacitor $C_{ref}$ and a switch to the circuit, and we will measure the voltage across $C_{ref}$.
There are two phases of this circuit.

- **Phase 1**: $s_1$ is off. $C_{eq,E1-E2}$ is connected to $V_s$ and $C_{ref}$ is not connected to anything.

- **Phase 2**: $s_1$ is on, we can replace $s_1$ with a perfect wire.

When $s_1$ is on, $V_{out}$ is always the same as $V_s$ and our second attempt fails.

- **Attempt #3** Let’s now add another switch $s_2$ so that $V_{out}$ no longer always equals $V_s$ when $s_1$ is turned on.
− **Phase 1:** $s_1$ is on and $s_2$ is off.

In phase 1, $C_{eq,E1-E2}$ is connected to voltage source $V_s$ and $C_{ref}$ is not connected to anything.

− **Phase 2:** $s_1$ is off and $s_2$ is on.

In phase 2, $C_{eq,E1-E2}$ is disconnected from $V_s$ and $C_{ref}$ is connected to $C_{eq,E1-E2}$. However, there is one serious problem with this circuit, since we are not sure what is the initial charge of $C_{ref}$, we cannot perform any circuit analysis. Our third attempt fails because the initial condition of $C_{ref}$ is not clear.

**Attempt #4** We learned from our third attempt that we must somehow know the initial condition of $C_{ref}$. How do we know the initial condition of $C_{ref}$? It turns out that we can acquire this missing piece of information by adding another switch $s_3$!
- **Phase 1**: $s_1$ is on, $s_2$ is off, $s_3$ is on.

In this phase, the voltage across $C_{eq, E1-E2}$ is $V_s$, and the voltage across $C_{ref}$ is zero.

- **Phase 2**: $s_1$ is off, $s_2$ is on, $s_3$ is off.

In this phase, the voltage across $C_{eq, E1-E2}$ is the same as the voltage across $C_{ref}$.

How do we analyze this circuit? We know from last lecture that charge must be preserved for a capacitor, this indicates that:

$$Q_{total, phase 1} = Q_{total, phase 2}. \quad (3)$$

The total charge in each phase can be calculated by summing up $Q_{eq, E1-E2}$ and $Q_{ref}$.

$$Q_{total, phase 1} = Q_{eq, E1 - E2} + Q_{eq, ref} = C_{eq, E1 - E2} \times V_s + C_{ref} \times 0V. \quad (4)$$
$$Q_{\text{total, phase 2}} = Q_{\text{eq, E1 - E2}} + Q_{\text{eq, ref}} = C_{\text{eq, E1 - E2}} \times V_{\text{out}} + C_{\text{ref}} \times V_{\text{out}}. \quad (5)$$

From equations (4) and (5), we have:

$$C_{\text{eq, E1 - E2}} \times V_{s} + C_{\text{ref}} \times 0V = C_{\text{eq, E1 - E2}} \times V_{\text{out}} + C_{\text{ref}} \times V_{\text{out}}. \quad (6)$$

Therefore, we can derive the following equation for $V_{\text{out}}$:

$$V_{\text{out}} = \frac{C_{\text{eq, E1 - E2}}}{C_{\text{eq, E1 - E2}} + C_{\text{ref}}} \times V_{s}. \quad (7)$$

This particular circuit analysis approach we have used is called "charge sharing". "Charge sharing" refers to the sharing of charge among capacitors. Between different steady phases for a circuit, one important ground rule remains true: the total charge stored by all capacitors must always be the same. We can use the "charge sharing" approach to solve many capacitor problems.

Now we can solve for $C_{\text{eq, E1 - E2}}$ given $V_{\text{out}}$. Recall that we only want to figure out whether or not our finger is touching this pixel rather than $C_{\text{eq, E1 - E2}}$, we need some device that can transform the continuous $C_{\text{eq, E1 - E2}}$ value to some binary value: is there a finger pressing on top of this pixel? Or, more specifically, is $C_{\text{eq, E1 - E2}}$ greater than its default value when there is no finger touch?

### 17.3 Comparator & Op-amp Basics

We want to figure out if there is a finger pressing on top of a pixel from $V_{\text{out}}$. In order to do so, we can use a comparator. Before explaining how a comparator works, we will first introduce the concept of op-amp.

Op-amp (operational amplifier), by definition, is an amplifier that can amplify something small into something much bigger. For example, a speaker is an audio amplifier, if you connect your smart phone to a speaker, it can generate sounds much louder than your phone can. The circuit symbol for an op amp is shown below:

![Op-amp Circuit Symbol](image)

We have two input terminals named $V_{+}$ and $V_{-}$, two power supply terminals named $V_{DD}$ and $V_{SS}$, and one output terminal named $V_{\text{out}}$. The op-amp circuit can transform a voltage signal ($V_{+} - V_{-}$) into $V_{\text{out}}$. For the op amp to function correctly, we need to connect the two external voltage sources $V_{DD}$, $V_{SS}$ to the op amp. The symbol actually represents the following equivalent circuit:
In the first circuit, we see a new symbol — a diamond with +/- signs inside. This represents a voltage-controlled voltage source where the voltage across it depends on the voltage(s) in other parts of the circuit. In this case, the voltage across the top voltage-controlled voltage source is $A(V_+ - V_-)$ where $A$ is a constant. For any good op amps, the constant $A$ term is very large – approaching infinity. The bottom voltage-controlled voltage source is dependent on the power supplies of the op-amp. If $V_+$ and $V_-$ are exactly equal, $V_{out}$ is the average of the supply voltages. Note that the dependent voltage sources are referenced to $V_{SS}$, not $V_{DD}$.

Equivalently, we have the following plot that describes the relationship between $V_{out}$ and $V_+ - V_-$. 

For any ideal op amp, when $V_+ < V_-$ ($V_+ - V_- < 0$), we have $V_{out}$ equals $V_{SS}$; when $V_+ - V_- > 0$, we have $V_{out}$ equals $V_{DD}$. If we zoom in around $V_+ - V_- = 0$ by a million times, we will be able to see there is actually some finite slope associated with the transition region. 

Now, we add an op-amp to the circuit model we built to extract $C_{eq, E1-E2}$. 

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If the analog voltage $V_{ana}$ is greater than $V_{ref}$, the output voltage $V_{out}$ will be 3.3V. If $V_{ana} < V_{ref}$, $V_{out}$ will be 0V. Recall for $V_{ana}$, we have

$$V_{ana} = \frac{C_{eq,E1-E2}}{C_{eq,E1-E2} + C_{ref}} \times V_s. \quad (8)$$

When there is touching, $C_{eq,E1-E2} = C_{E1-E2} + \Delta C$ in which $\Delta C > 0$, we have:

$$V_{ana, touch} = \frac{C_{E1-E2} + \Delta C}{C_{E1-E2} + \Delta C + C_{ref}} \times V_s \quad (9)$$

When there is no touching:

$$V_{ana, no-touch} = \frac{C_{E1-E2}}{C_{E1-E2} + C_{ref}} \times V_s \quad (10)$$

For any finite capacitance $\Delta C > 0$, $V_{ana, touch}$ is always greater than $V_{ana, no-touch}$.

$V_{ana}$ is connected to the positive voltage input ($V_+$) of the op-amp, $V_{ref}$ is connected to the negative voltage input ($V_-$) of the op-amp. If we set up $V_{ref}$ such that:

1. $V_{ana, touch} > V_{ref}$, $V_+ - V_- = V_{ana, touch} - V_{ref} > 0$, $V_{out}$ will be equal to 3.3V when there is touching,
2. $V_{ana, no-touch} < V_{ref}$, $V_+ - V_- = V_{ana, no-touch} - V_{ref} < 0$, $V_{out}$ will be equal to 0V when there is no touching.

Now we have built a comparator circuit that can compare $V_{ana}$ against $V_{ref}$ using an op-amp!

**How should we set up $V_{ref}$?** Intuitively, we want to set it to be half between $V_{ana, no-touch}$ and $V_{ana, touch}$. This actually allows our comparator circuit to be robust against non-idealities, we will have the same amount of margin for any $V_{ana}$ that deviates from its ideal values, either from $V_{ana, touch}$ or from $V_{ana, no-touch}$.

A second important question is that **how should we set up $C_{ref}$?** Recall that $V_{ana} = \frac{C_{eq,E1-E2}}{C_{eq,E1-E2} + C_{ref}} \times V_s$. We realize that $V_{ana}$ is associated with $C_{ref}$. As long as we do not set $C_{ref}$ to some value much greater (for example, 1000 times greater) than $C_{eq,E1-E2}$ so that $V_{ana}$ becomes too small (for our op-amp to pick up the signal), we can get some reasonable $V_{ana}$ which then allows us to set up $V_{ref}$ according to $V_{ana, touch}$ and $V_{ana, no-touch}$. Practically, we will set up one of $C_{ref}$, $V_{ref}$ first, and then set up the other value according to $V_{ref} = \frac{1}{2}(V_{ana, touch} + V_{ana, no-touch})$. 