Design Example Continued

Continuing our analysis for “countdown timer” circuit.

![Circuit Diagram]

We know for a capacitor $C$:

$$ I = C \frac{dV}{dt} \quad (1) $$

There is a linear relationship between the voltage across capacitor $V(t)$ and charging time $t$.

![Graph]

When a button is pressed, we want to turn on some circuit after 2s. Given that voltage is linearly dependent on charging time, we can use a comparator and a reference voltage $V_{ref}$ to decide if 2s has already passed.
after the button press to decide whether or not to turn on some circuit.

Adding an op-amp comparator and a voltage source $V_{ref}$ to the “countdown timer” circuit:

Obviously, $V_{ref}$ should be set equal to the voltage of $C_{ref}$ after charging for 2s. After 2s since the button is pressed, if the voltage across $C_{ref}$ becomes higher than $V_{ref}$, the comparator outputs $2V$ to the output of the op-amp to turn on the LED.

**Step 4: design verification**

Now, let’s actually analyze the design completely to make sure it works. Before the button is pressed, the circuit on the current source side looks like:

According to KCL,

$$I_s = I_L$$  \hspace{1cm} (2)

$I_s$ is the constant current supplied by the current source, $I_L$ is the current flow into the switch. Before touch, $I_L$ must equal 0 since there is an open circuit. However, the current source guarantees that $I_s$ is nonzero. It is very easy to see that mathematically this is problematic, how do we solve the problem? We can add another switch in the circuit:
Before touch, the switch is on and can be replaced by a wire:

After touch, the switch is off and can be replaced by an open circuit:

In either cases, there is a loop in the circuit for $I_s$ to flow. Now, there is a remaining mystery to be answered, how do we build that current source?

### 20.1 “Almost” current source

In this section, we will use resistors, voltage sources and op amps to build a current source. We know by Ohm’s law,

$$ I = \frac{V}{R} \quad (3) $$

If we have a voltage source $V_s$, we can scale it by a resistance value $R$, then we should get a constant current $I_s$.

Now let’s use our design procedures to build a current source.
**Step 1** restate design goal: we want to build a current source that can output constant current regardless of whatever elements we hook up to it.

**Step 2** Let’s now take a voltage source and connect it to a resistor to output a current:

**Attempt #1**

In the above circuit,

\[ u_1 = V_s \quad (4) \]

\[ I_s = \frac{V_s}{R} \quad (5) \]

However, if we hook up an element between \( u_2 \) and ground, we can tell immediately that the current through \( R \) is no longer the constant \( I_s = \frac{V_s}{R} \).

**Attempt #2** Although our attempt 1 has failed, we have learned an important lesson: if we can somehow set \( u_2 \) to 0V without physically connecting it to ground. The current through \( R \) will always equal \( \frac{V_s}{R} \) (\( I_R = \frac{u_1-u_2}{R} = \frac{V_s-0}{R} \)). According to golden rule #2, we can set both \( V_+ \) and \( V_- \) to 0V if an op amp circuit is in negative feedback. Indeed, we will now use an op amp to build a current source!
According to golden rules:

\[ I_- = 0 \]  \hspace{1cm} (6)

\[ V_- = V_+ = 0V \]  \hspace{1cm} (7)

We also know that the current across \( R \) will always be:

\[ I_s = \frac{u_1 - u_2}{R} \]  \hspace{1cm} (8)

Solving the above equations, we can get the value of \( I_s \):

\[ I_s = \frac{V}{R} \]  \hspace{1cm} (9)

By setting \( u_2 (V_-) \) to 0V by using a negative feedback circuit, we have succesfully built a current source! It is important to keep in mind setting \( u_2 \) to 0V by using a negative feedback circuit is very different from physically connecting the node of \( u_2 \) to ground. If we physically connect \( u_2 \) to ground by adding a wire between \( V_- \) and \( V_+ \):

If we physically connect both \( V_- \) and \( V_+ \) to ground, \( I_L \) will become 0A because it is shorted by the wire between \( V_- \) and \( V_+ \). Instead, now we have \( I_1 = I_s \). So we must not physically connect \( V_- \) to ground. Let’s now hook up a resistor \( R_L \) to the circuit and prove that current flow through \( R_L \) is constant:
According to KCL:

\[ I_L = I_s = \frac{V}{R_s} \]  \hspace{1cm} (10)

From \( I_L \) equation, we can see immediately that \( I_L \) is not affected by changes in \( R_L \). How does the circuit maintain the constant current flow through \( R_L \)?

The op-amp outputs a negative \( V_{out} \) to maintain a constant current flow through \( R_L \). In other words, the voltage drop across \( R_L \) is always given by

\[ V_{R_L} = \frac{V}{R_S} \times R_L \]  \hspace{1cm} (11)

Now let us plug in the current source to the following circuit:

The circuit becomes:
Now \( u_3 \) is connected to ground. By doing this, the voltage across \( C_{ref} \) becomes:

\[
V_{\text{time}} = u_2 - u_3 = 0V - 0V = 0V
\]  

(12)

There is even a worse problem, recall that there is an controlled voltage source inside the op-amp, this op-amp wants to set \( u_3 \) to some nonzero value \( A \cdot V_c \) but \( u_3 \) is also manually connected to 0V. The fix to solve this problem is to simply get rid of the ground connection.

\[
I_s = C_{ref} \frac{dV}{dt}
\]  

(13)

\[
I_s = C_{ref} \frac{d(u_2 - u_3)}{dt}
\]  

(14)

According to golden rule #2, \( u_2 = 0V \).

\[
I_s = C_{ref} \frac{d(0V - u_3)}{dt} = C_{ref} \frac{d(-u_3)}{dt}
\]  

(15)
Solving the above equation:

\[ u_3(t) = -\frac{I_s}{C_{ref}} \times t + u_3(t = 0s) \]  \hspace{1cm} (16)

\[ u_3(t) \] is associated with the initial value \( u_3(t = 0s) \). Before touch, the switch \( S1 \) is on, which sets \( u_3(t = 0s) \) to 0V. Therefore,

\[ u_3(t) = -\frac{I_s}{C_{ref}} \times t = -\frac{V_s}{R_sC_{ref}} \times t \]  \hspace{1cm} (17)

Note there is a term \( R_sC_{ref} \) in the denominator, what is the unit of \( R_sC_{ref} \)?

Unit for \( R_sC_{ref} = \frac{V}{A} \times CV = \frac{C}{A} = \text{second} \)  \hspace{1cm} (18)

It is good to know that this multiplication result of \( RC \) is very useful and common in the timing analysis for circuits, this is an indicator of how fast a circuit is.

There are 2 important points to keep in mind when this current source:

- Do not connect the circuit element that we want to supply the constant current to with ground externally. Doing so may force \( V_{out} \) of the op-amp to 0V and lead to nonidealities.

- The circuit element we hook up to the current source must still keep the op-amp circuit in its negative feedback state. Being in negative feedback allows us to set the node \( u_2 \) to 0V without physically connecting it to ground and hence allows a constant current output.