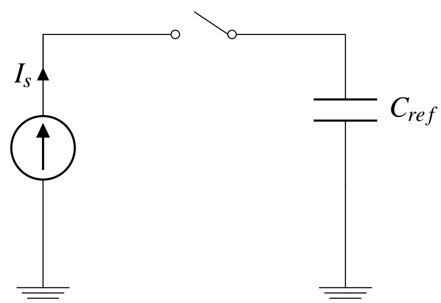
# Design Example Continued

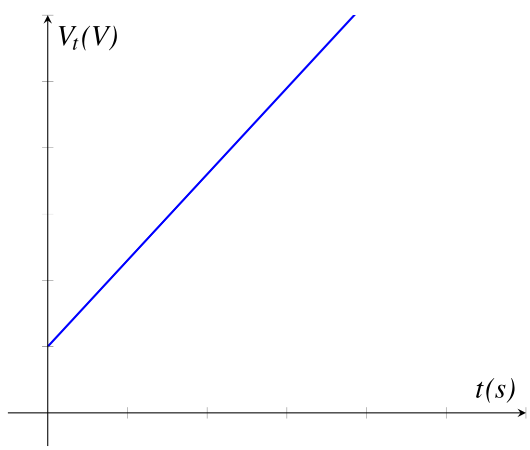
Continuing our analysis for "countdown timer" circuit.



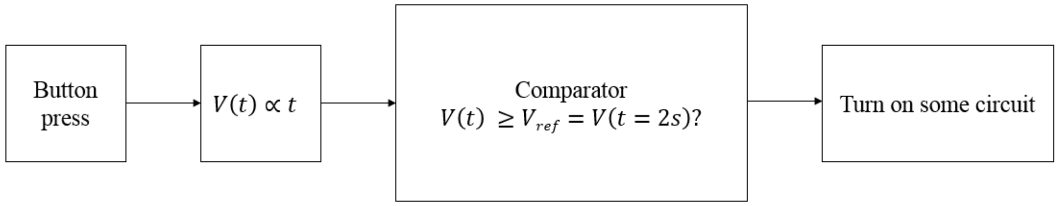

We know for a capacitor $C$:

$$I = C\frac{dV}{dt} \tag{1}$$

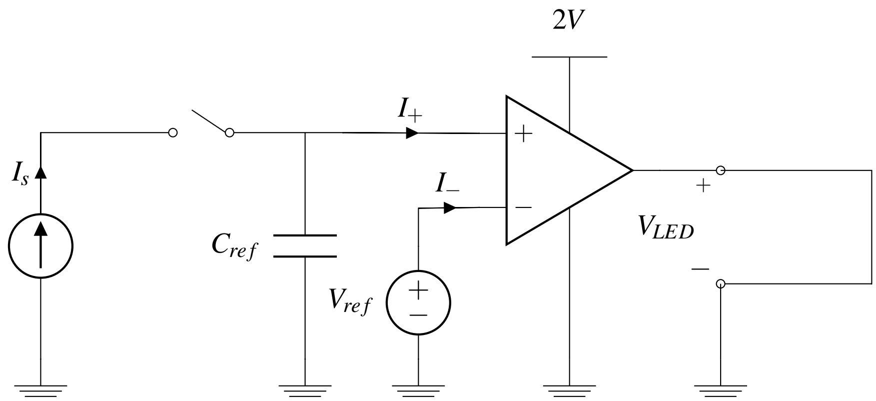
There is a linear relationship between the voltage across capacitor $V(t)$ and charging time $t$.




When a button is pressed, we want to turn on some circuit after 2s. Given that voltage is linearly dependent on charging time, we can use a comparator and a reference voltage $V_{ref}$ to decide if 2s has already passed after the button press to decide whether or not to turn on some circuit.

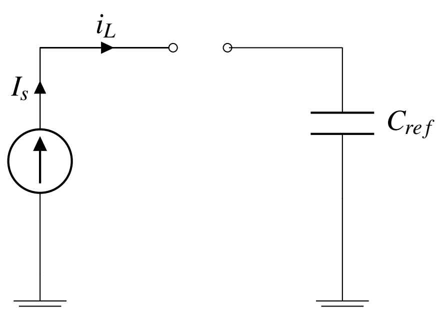



Adding an op-amp comparator and a voltage source $V_{ref}$ to the "countdown timer" circuit:




Obviously, $V_{ref}$ should be set equal to the voltage of $C_{ref}$ after charging for 2s. After 2s since the button is pressed, if the voltage across $C_{ref}$ becomes higher than $V_{ref}$, the comparator outputs $2V$ to the output of the op-amp to turn on the LED.

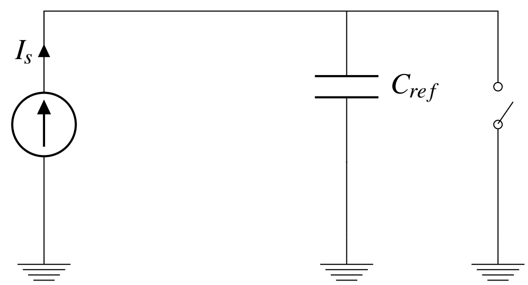
**Step 4: design verfication** Now, let's actually analyze the design completely to make sure it works. Before the button is pressed, the circuit on the current source side looks like:



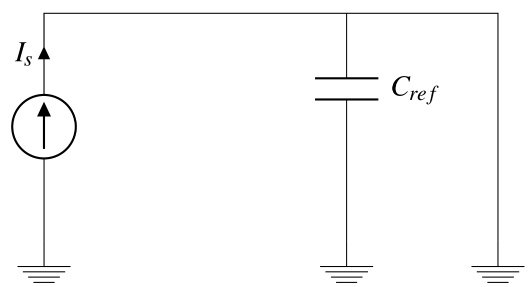

According to KCL,

$$I_s = I_L \tag{2}$$

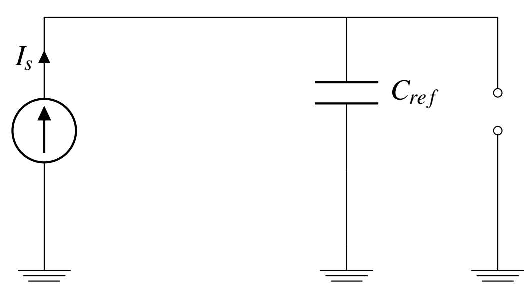
$I_s$ is the constant current supplied by the current source, $I_L$ is the current flow into the switch. Before touch, $I_L$ must equal 0 since there is an open circuit. However, the current source guarantees that $I_s$ is nonzero. It is very easy to see that mathematically this is problematic, how do we solve the problem? We can add another switch in the circuit:




Before touch, the switch is on and can be replaced by a wire:




After touch, the switch is off and can be replaced by an open circuit:




In either cases, there is a loop in the circuit for $I_s$ to flow. Now, there is a remaining mystery to be answered, how do we build that current source?

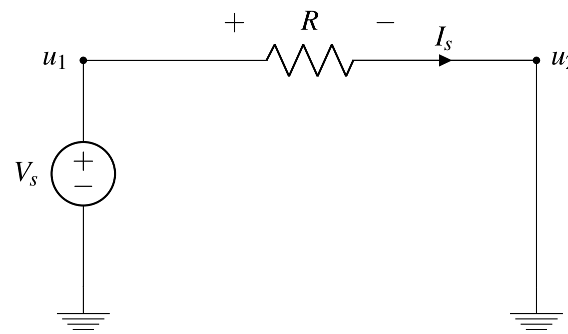
## 20.1 "Almost" current source

In this section, we will use resistors, voltage sources and op amps to build a current source. We know by Ohm's law,

$$I = \frac{V}{R} \tag{3}$$

If we have a voltage source $V_s$, we can scale it by a resistance value $R$, then we should get a constant current $I_s$.
Now let's use our design procedures to build a current source.

**Step 1** restate design goal: we want to build a current source that can output constant current regardless of whatever elements we hook up to it.
**Step 2** Let's now take a voltage source and connect it to a resistor to output a current:
**Attemp #1**

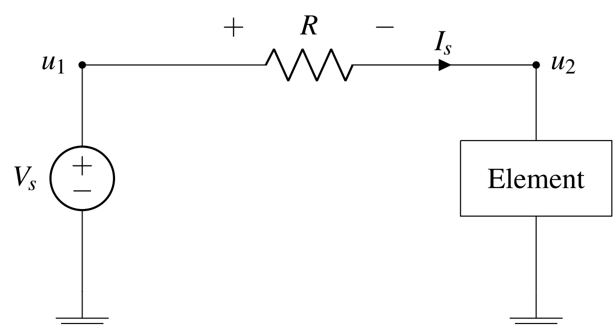



In the above circuit,

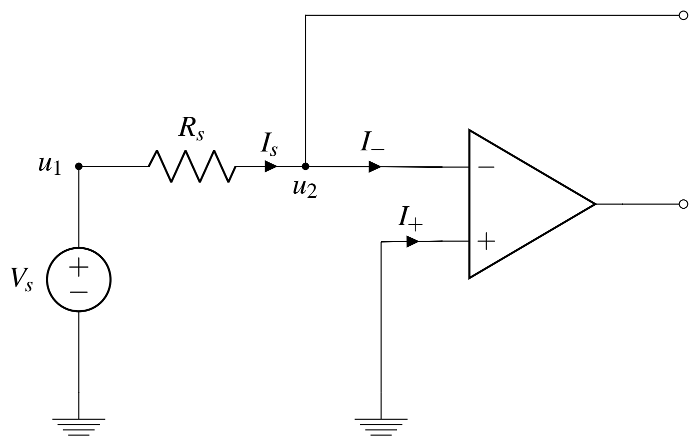
$$u_1 = V_s \tag{4}$$

$$I_s = \frac{V_s}{R} \tag{5}$$

However, if we hook up an element between $u_2$ and ground, we can tell immediately that the current through $R_s$ is no longer the constant $I_s = \frac{V_s}{R}$.




Recall that it is important to guarantee a constant current output from the current source regardless of the element added to the circuit.
**Attempt #2** Although our attempt 1 has failed, we have learned an important lesson: if we can somehow set $u_2$ to $0V$ without physically connecting it to ground. The current through $R$ will always equal $\frac{V_s}{R}$ ($I_R = \frac{u_1 - u_2}{R} = \frac{V_s - 0}{R}$)! According to golden rule #2, we can set both $V_+$ and $V_-$ to $0V$ if an op amp circuit is in negative feedback. Indeed, we will now use an op amp to build a current source!




According to golden rules:

$$I_- = 0 \tag{6}$$

$$V_- = V_+ = 0V \tag{7}$$

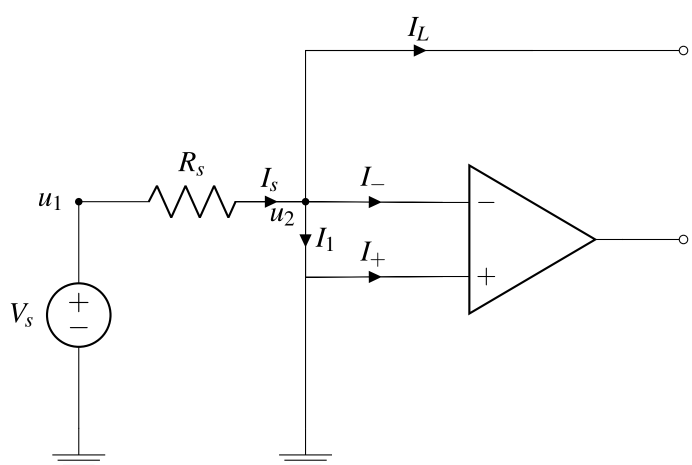
We also know that the current across $R$ will always be:

$$I_s = \frac{u_1 - u_2}{R} \tag{8}$$

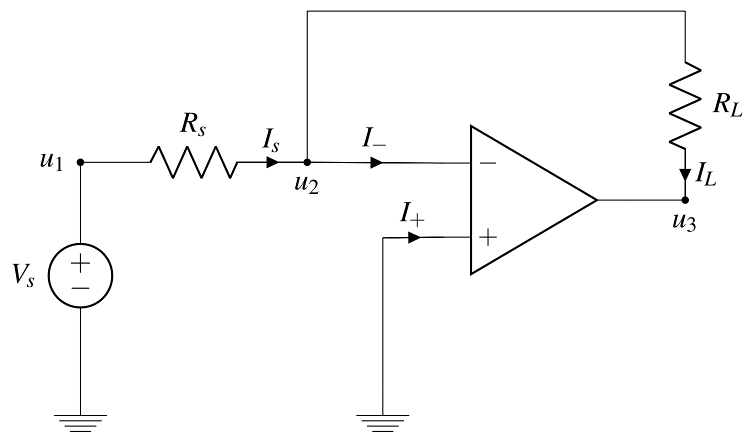
Solving the above equations, we can get the value of $I_s$:

$$I_s = \frac{V}{R} \tag{9}$$

By setting $u_2$ ($V_-$) to $0V$ by using a negative feedback circuit, we have succesfully built a current source!
It is important to keep in mind setting $u_2$ to $0V$ by using a negative feedback circuit is very different from physically connecting the node of $u_2$ to ground. If we physically connect $u_2$ to ground by adding a wire between $V_-$ and $V_+$:




If we physcially connect both $V_-$ and $V_+$ to ground, $I_L$ will become $0A$ because it is shorted by the wire between $V_-$ and $V_+$. Instead, now we have $I_1 = I_s$. So we must not physically connect $V_-$ to ground.
Let's now hook up a resistor $R_L$ to the circuit and prove that current flow through $R_L$ is constant:



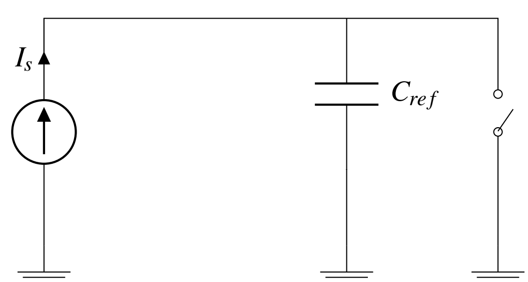

According to KCL:

$$I_L = I_s = \frac{V}{R_s} \tag{10}$$

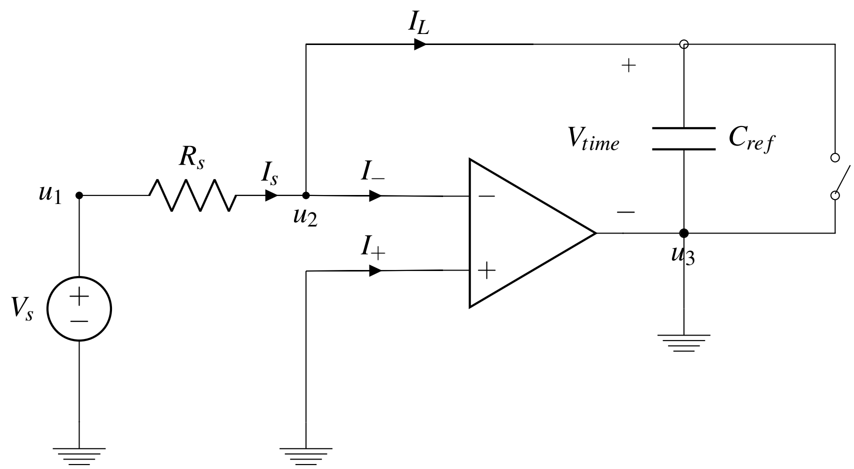
From $I_L$ equation, we can see immediately that $I_L$ is not affected by changes in $R_L$. How does the circuit maintain the constant current flow through $R_L$?
The op-amp outputs a negative $V_{out}$ to maintain a constant current flow through $R_L$. In other words, the voltage drop across $R_L$ is always given by

$$V_{R_L} = \frac{V}{R_S} \times R_L \tag{11}$$

Now let us plug in the current source to the following circuit:



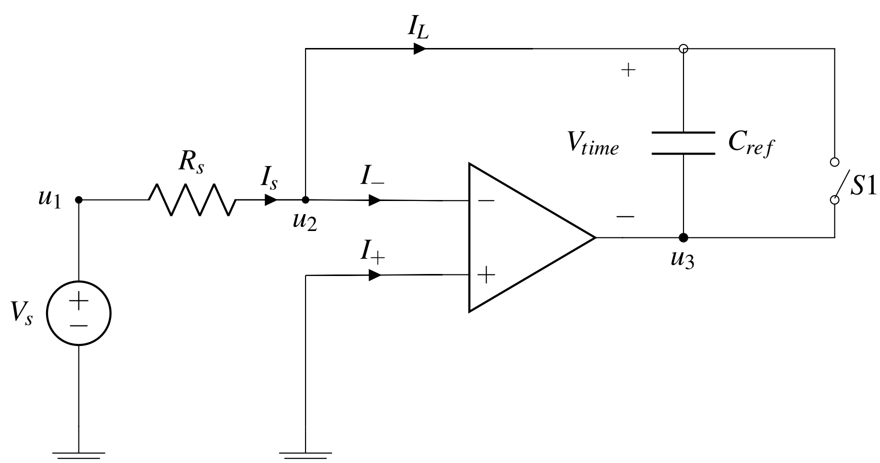

The circuit becomes:




Now $u_3$ is connected to ground. By doing this, the voltage across $C_{ref}$ becomes:

$$V_{time} = u_2 - u_3 = 0V - 0V = 0V \tag{12}$$

There is even a worse problem, recall that there is an controlled voltage source inside the op-amp, this op-amp wants to set $u_3$ to some nonzero value $A \cdot V_c$ but $u_3$ is also manually connected to $0V$. The fix to solve this problem is to simply get rid of the ground connection.




$$I_s = C_{ref}\frac{dV}{dt} \tag{13}$$

$$I_s = C_{ref}\frac{d(u_2 - u_3)}{dt} \tag{14}$$

According to golden rule #2, $u_2 = 0V$.

$$I_s = C_{ref}\frac{d(0V - u_3)}{dt} = C_{ref}\frac{d(-u_3)}{dt} \tag{15}$$

Solving the above equation:

$$u_3(\mathrm{t}) = -\frac{I_s}{C_{ref}} \times t + u_3(\mathrm{t = 0s}) \tag{16}$$

$u_3(\mathrm{t})$ is associated with the initial value $u_3(\mathrm{t = 0s})$. Before touch, the switch $S1$ is on, which sets $u_3(\mathrm{t = 0s})$ to $0V$. Therefore,

$$u_3(\mathrm{t}) = -\frac{I_s}{C_{ref}} \times t = -\frac{V_s}{R_s C_{ref}} \times t \tag{17}$$

Note there is a term $R_s C_{ref}$ in the denominator, what is the unit of $R_s C_{ref}$?

$$\text{Unit for } R_s C_{ref} = \frac{V}{A} \times CV = \frac{C}{A} = \text{second} \tag{18}$$

It is good to know that this multiplication result of $RC$ is very useful and common in the timing analysis for circuits, this is an indicator of how fast a circuit is.
There are 2 important points to keep in mind when this current source:

- Do not connect the circuit element that we want to supply the constant current to with ground externally. Doing so may force $V_{out}$ of the op-amp to $0V$ and lead to nonidealities.
- The circuit element we hook up to the current source must still keep the op-amp circuit in its negative feedback state. Being in negative feedback allows us to set the node $u_2$ to $0V$ without physically connecting it to ground and hence allows a constant current output.