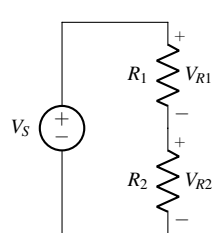
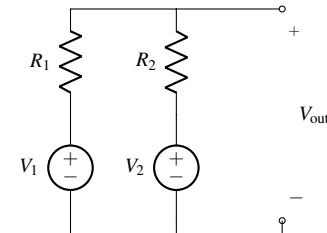
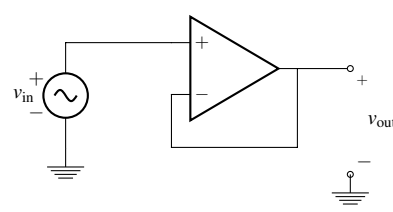
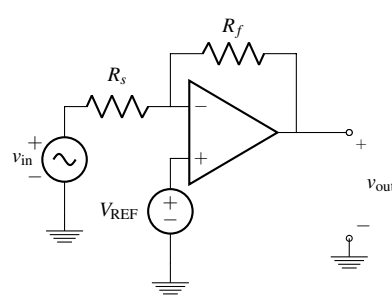
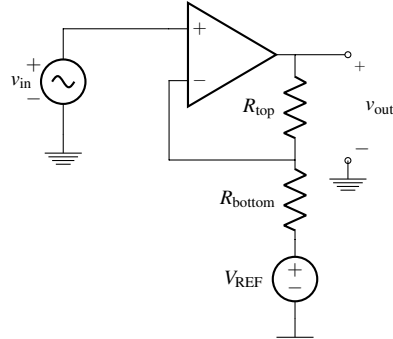
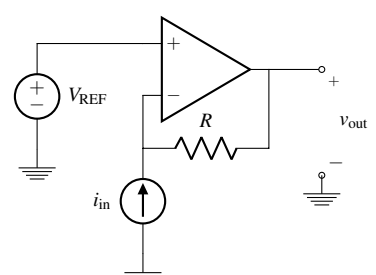


EECS 16A Designing Information Devices and Systems I

Discussion 10B

For Reference: Circuits Cookbook, Abridged

<p style="text-align: center;">Voltage Divider</p>  <p style="text-align: center;">$V_{R2} = V_S \left(\frac{R_2}{R_1 + R_2} \right)$</p>	<p style="text-align: center;">Voltage Summer</p>  <p style="text-align: center;">$V_{out} = V_1 \left(\frac{R_2}{R_1 + R_2} \right) + V_2 \left(\frac{R_1}{R_1 + R_2} \right)$</p>	<p style="text-align: center;">Unity Gain Buffer</p>  <p style="text-align: center;">$\frac{v_{out}}{v_{in}} = 1$</p>
<p style="text-align: center;">Inverting Amplifier</p>  <p style="text-align: center;">$v_{out} = v_{in} \left(-\frac{R_f}{R_s} \right) + V_{REF} \left(\frac{R_f}{R_s} + 1 \right)$</p>	<p style="text-align: center;">Non-inverting Amplifier</p>  <p style="text-align: center;">$v_{out} = v_{in} \left(1 + \frac{R_{top}}{R_{bottom}} \right) - V_{REF} \left(\frac{R_{top}}{R_{bottom}} \right)$</p>	<p style="text-align: center;">Transresistance Amplifier</p>  <p style="text-align: center;">$v_{out} = i_{in} (-R) + V_{REF}$</p>

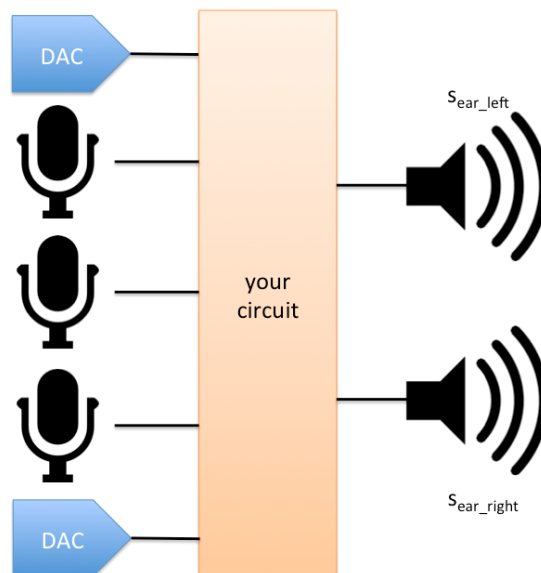
1. Noise Cancelling Headphones Part 2

The basic goal of noise cancelling headphones is for the user to hear only the desired audio signal and not any other sounds from external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources and then feeds a signal in to your speakers that cancels (subtracts out) that externally-generated sound.

Answer:

There are a lot of different solutions for this problem. This solution is aggressive and minimal, so be patient with your understanding. If your solution solves the same problem, you will receive credit.

- (a) In the previous discussion, we had just one speaker and one microphone, but almost all headphones today have two speakers (one for each ear). Adding an extra speaker that can be driven by a separate audio stream typically makes things sound more real to us. For similar reasons, having multiple microphones to pick up ambient sounds from multiple different locations can help us do a better job of cancellation, if we can use that information in the right way.



Let's now assume that our system has 3 microphones and 2 speakers, and that the source of our audio is stereo – i.e., we have two different audio streams s_{left} and s_{right} (produced by two different DACs) that represent the ideal sounds we would like the user to hear in their left and right ear. We have three microphone audio signals s_{mic1} , s_{mic2} , and s_{mic3} , and let's assume that without any active noise cancellation, some fraction of the signal picked up by each microphone would be heard by the user in each of their ears. For example, $a_{1\text{left}}$ would represent the fraction of the signal picked up by microphone 1 that will be heard in the user's left ear, $a_{2\text{right}}$ would represent the fraction of the signal picked up by microphone 2 that will be in the user's right ear, etc.

Let the vector \vec{s}_{noise} represent the noise heard in each ear and \vec{s}_{mic} represent the sound in each mic. Find a matrix \mathbf{A} such that $\vec{s}_{\text{noise}} = \mathbf{A}\vec{s}_{\text{mic}}$.

Answer:

$$\begin{bmatrix} s_{\text{noise_left}} \\ s_{\text{noise_right}} \end{bmatrix} = \begin{bmatrix} a_{1\text{left}} & a_{2\text{left}} & a_{3\text{left}} \\ a_{1\text{right}} & a_{2\text{right}} & a_{3\text{right}} \end{bmatrix} \begin{bmatrix} s_{\text{mic1}} \\ s_{\text{mic2}} \\ s_{\text{mic3}} \end{bmatrix}$$

- (b) Assume no noise canceling, find an equation for \vec{s}_{ear} , the sound heard in each ear in terms of the two audio streams and \vec{s}_{noise} .

Answer:

We can represent this as the matrix multiplication and addition below.

$$\begin{bmatrix} s_{\text{ear_left}} \\ s_{\text{ear_right}} \end{bmatrix} = \begin{bmatrix} a_{1\text{left}} & a_{2\text{left}} & a_{3\text{left}} \\ a_{1\text{right}} & a_{2\text{right}} & a_{3\text{right}} \end{bmatrix} \begin{bmatrix} s_{\text{mic1}} \\ s_{\text{mic2}} \\ s_{\text{mic3}} \end{bmatrix} + \begin{bmatrix} s_{\text{left}} \\ s_{\text{right}} \end{bmatrix}$$

- (c) In order to cancel the noise, we want to create a signal that is the inverse of \vec{s}_{noise} . Let \vec{s}_{cancel} be the vector representing the cancel signal in each headphone. Find a matrix \mathbf{B} in terms of the matrix \mathbf{A} such that $\vec{s}_{\text{cancel}} = \mathbf{B}\vec{s}_{\text{mic}}$.

Answer:

For the setup:

$$\begin{bmatrix} s_{\text{ear_left}} \\ s_{\text{ear_right}} \end{bmatrix} = \mathbf{A} \begin{bmatrix} s_{\text{mic1}} \\ s_{\text{mic2}} \\ s_{\text{mic3}} \end{bmatrix} + \mathbf{B} \begin{bmatrix} s_{\text{mic1}} \\ s_{\text{mic2}} \\ s_{\text{mic3}} \end{bmatrix} + \begin{bmatrix} s_{\text{left}} \\ s_{\text{right}} \end{bmatrix}$$

We want $\mathbf{B} = -\mathbf{A}$.

- (d) Assume that the microphones can be modeled as voltage sources, whose value v_{micn} is proportional to s_{micn} , design and sketch a circuit that would implement the cancellation matrix \mathbf{B} . You should assume that this circuit has three voltage inputs v_{mic1} , v_{mic2} , and v_{mic3} and two voltage outputs $v_{\text{cancel_left}}$ and $v_{\text{cancel_right}}$ (corresponding to the voltages that will be subtracted from the desired audio streams in order to cancel the externally-produced sounds). In order to simplify the problem, you can assume that all of the v_{mic} voltages are already centered at 0V (relative to the DAC ground). Furthermore, assume all entries of the \mathbf{A} matrix are positive. You may use op-amps and resistors to implement your circuit. You do not have to pick specific resistor values, but write expressions for each resistor value.

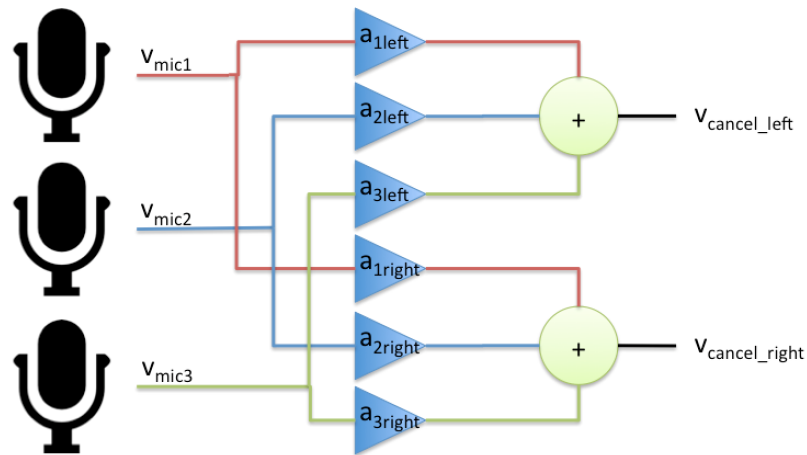
Answer:

Since we want to add $v_{\text{cancel_left}}$ and $v_{\text{cancel_right}}$ with the audio stream output to cancel noise, so we want these values to be

$$v_{\text{cancel_left}} = -(a_{1\text{left}} v_{\text{mic1}} + a_{2\text{left}} v_{\text{mic2}} + a_{3\text{left}} v_{\text{mic3}})$$

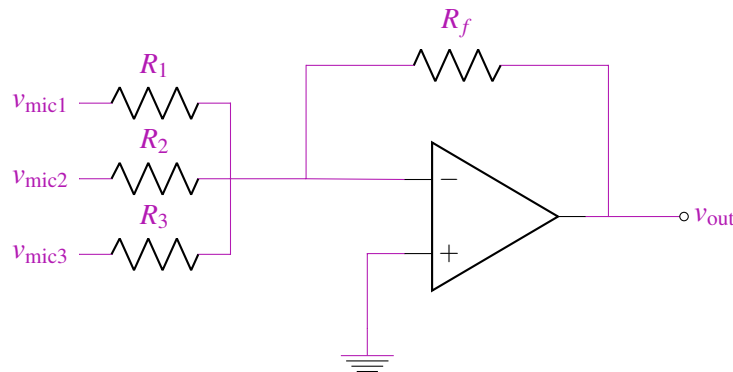
$$v_{\text{cancel_right}} = -(a_{1\text{right}} v_{\text{mic1}} + a_{2\text{right}} v_{\text{mic2}} + a_{3\text{right}} v_{\text{mic3}})$$

Following the design process, we can draw a block diagram for the circuit.



We can see here that the two channels are actually independent from each other. The only point where they meet is at the microphone voltage. Thus, we can start by designing one channel.

For each channel, we want to build a circuit that sums its inputs and negates them. We have many options to pick from; the easiest will be an inverting summer.



This topology gives us

$$V_{\text{out}} = - \left(\frac{R_f}{R_1} V_{\text{mic1}} + \frac{R_f}{R_2} V_{\text{mic2}} + \frac{R_f}{R_3} V_{\text{mic3}} \right)$$

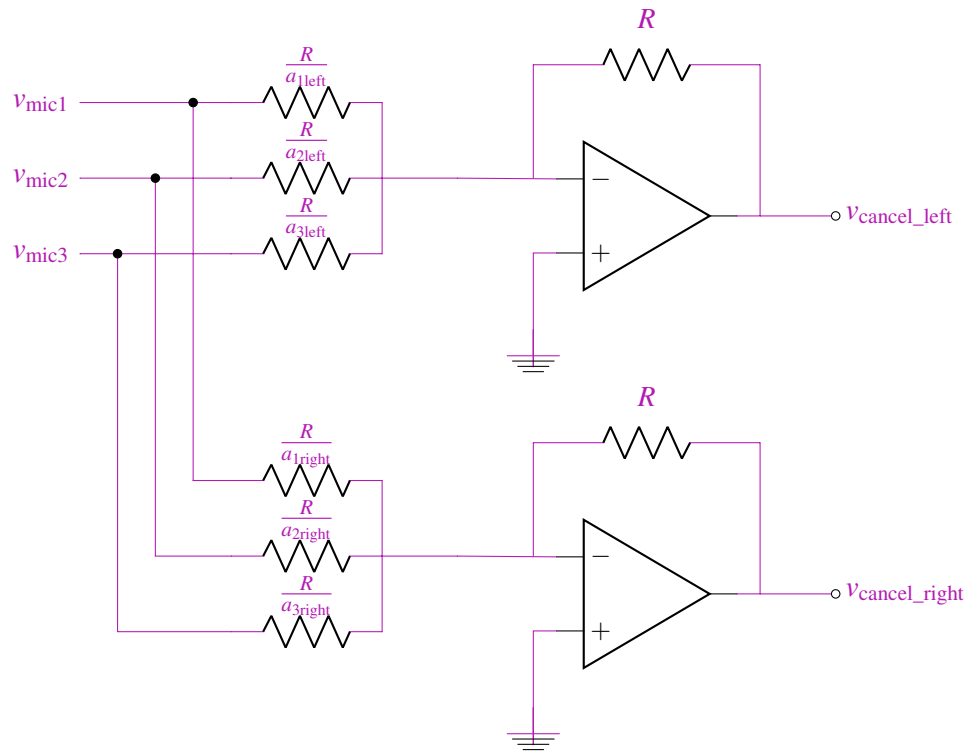
For the correct gains, we want

$$a_1 = \frac{R_f}{R_1}, a_2 = \frac{R_f}{R_2}, a_3 = \frac{R_f}{R_3}$$

We can pick R_f arbitrarily. Then we set

$$R_1 = \frac{R_f}{a_1}, R_2 = \frac{R_f}{a_2}, R_3 = \frac{R_f}{a_3}$$

Now that we have all the building blocks we need, we can construct the two-channel noise cancelling circuit. $v_{\text{mic}i}$ is connected to the output of the microphone buffers. We can choose an arbitrary value for R , for example, 1 k Ω .

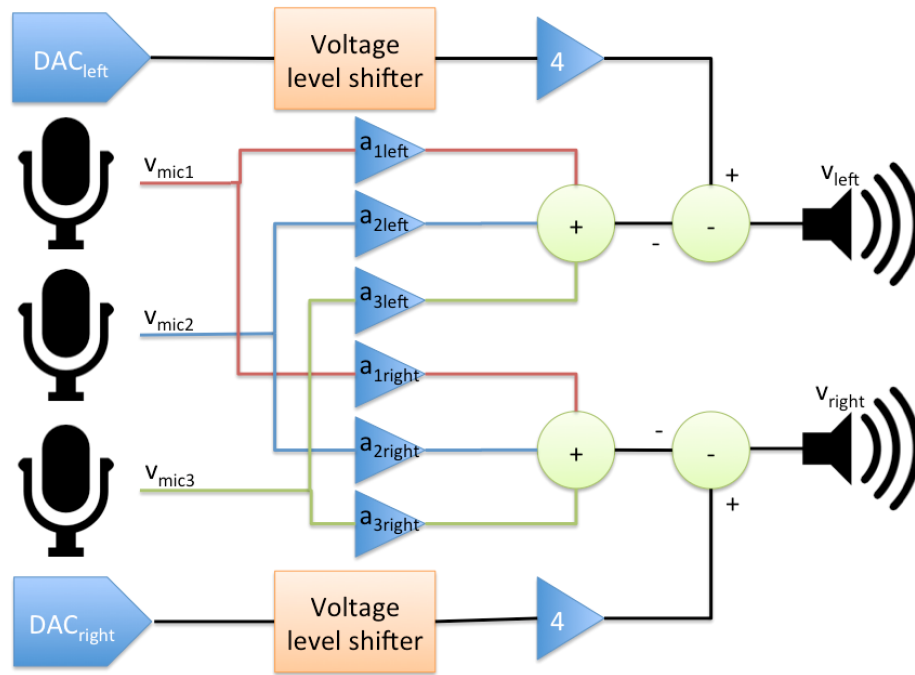


We just pick R to be some value, we then calculate the appropriate resistances in the rest of the circuit, and we are good to go!

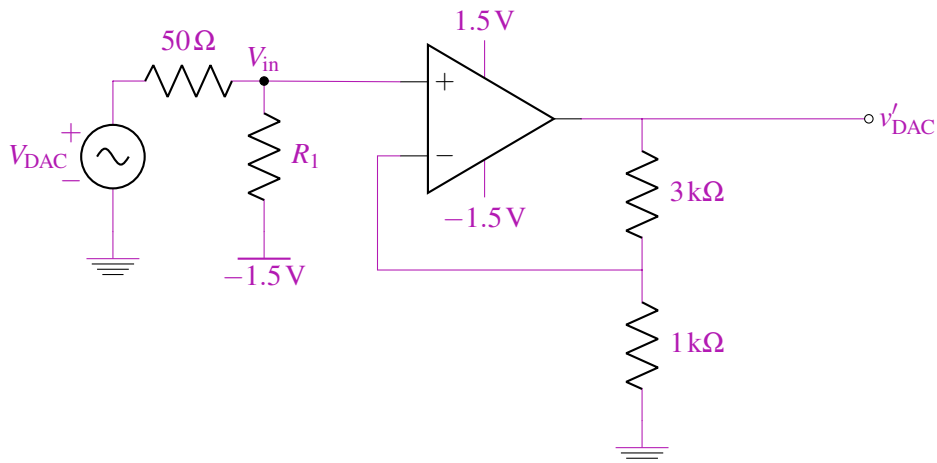
- (e) **PRACTICE:** Building upon your solutions to all previous parts, and otherwise making the same assumptions about the relative voltage ranges of v_{mic1} , v_{mic2} , and v_{mic3} and available supply voltages, sketch the complete circuit you would use to create the stereo audio on the two speakers while cancelling the noise picked up by the three microphones.

Answer:

We already have a circuit that does subtraction and a circuit that computes the noise cancelling signal. We just have to combine the two circuits such that it implements the matrix **B**.



Recall our circuit from last discussion for voltage shifting and changing the range of a DAC:

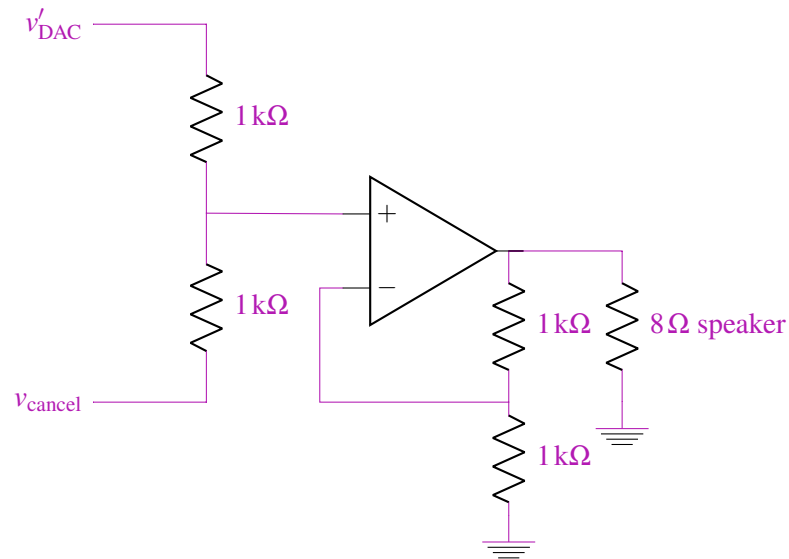


Now we want

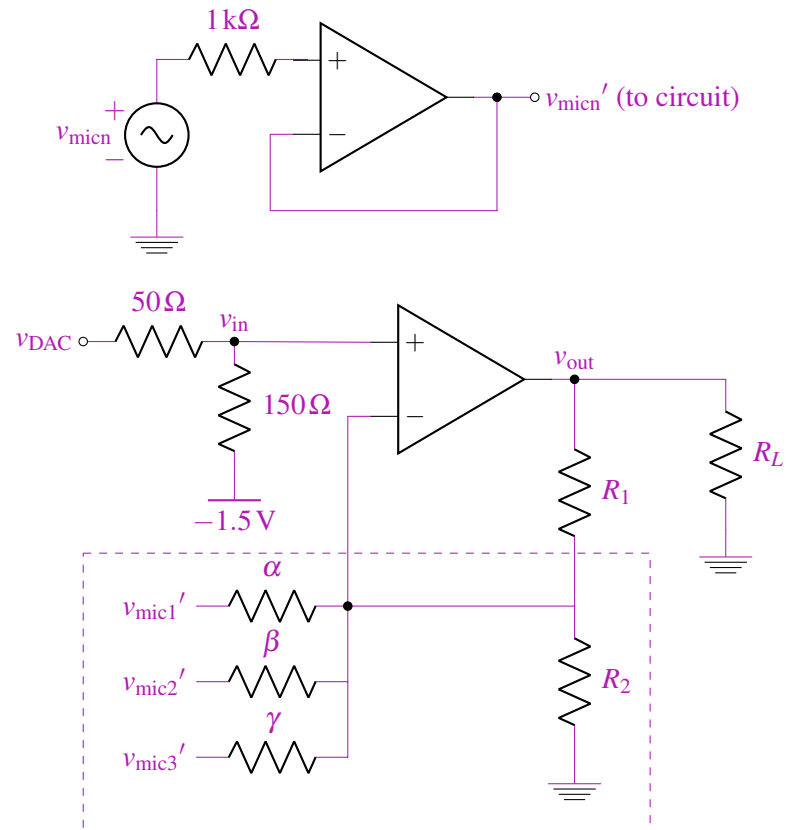
$$v_{\text{left}} = v'_{\text{DAC, left}} + v_{\text{cancel_left}}$$

$$v_{\text{right}} = v'_{\text{DAC, right}} + v_{\text{cancel_right}}$$

Note that v_{cancel} already inverted the microphone signal so we are adding it to v_{DAC} . To sum, we can use a noninverting summer. Again, we consider the channels independently.



Below is alternative circuit that does the same with one op-amp. The approach here is to start with by attenuating the mic voltages and summing them with a voltage divider and solving for the correct resistances. Then we feed the the microphone voltage into the negative terminal and the DAC signal into the positive terminal of an op amp to subtract the signals.



Recall that the v_{in} range is -0.375 V to 0.375 V , and it has to be amplified 4 times. We can write the

KCL equation in the inverting input of the op-amp.

$$\frac{v_{\text{mic1}} - v_{\text{in}}}{\alpha} + \frac{v_{\text{mic2}} - v_{\text{in}}}{\beta} + \frac{v_{\text{mic3}} - v_{\text{in}}}{\gamma} + \frac{v_{\text{out}} - v_{\text{in}}}{R_1} + \frac{0 - v_{\text{in}}}{R_2} = 0$$

$$\frac{v_{\text{out}}}{R_1} = v_{\text{in}} \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{v_{\text{mic1}}}{\alpha} - \frac{v_{\text{mic2}}}{\beta} - \frac{v_{\text{mic3}}}{\gamma}$$

$$v_{\text{out}} = v_{\text{in}} \left(\frac{R_1}{\alpha} + \frac{R_1}{\beta} + \frac{R_1}{\gamma} + 1 + \frac{R_1}{R_2} \right) - \frac{R_1}{\alpha} v_{\text{mic1}} - \frac{R_1}{\beta} v_{\text{mic2}} - \frac{R_1}{\gamma} v_{\text{mic3}}$$

Just as before, we can compare this formula to the output we want. In this case, we want $v_{\text{out}} = 4v_{\text{in}} - a_1 v_{\text{mic1}} - a_2 v_{\text{mic2}} - a_3 v_{\text{mic3}}$. Thus,

$$\begin{aligned} \frac{R_1}{\alpha} + \frac{R_1}{\beta} + \frac{R_1}{\gamma} + 1 + \frac{R_1}{R_2} &= 4 & \frac{R_1}{\alpha} &= a_1 & \frac{R_1}{\beta} &= a_2 & \frac{R_1}{\gamma} &= a_3 \\ \alpha &= \frac{R_1}{a_1} & \beta &= \frac{R_1}{a_2} & \gamma &= \frac{R_1}{a_3} \end{aligned}$$

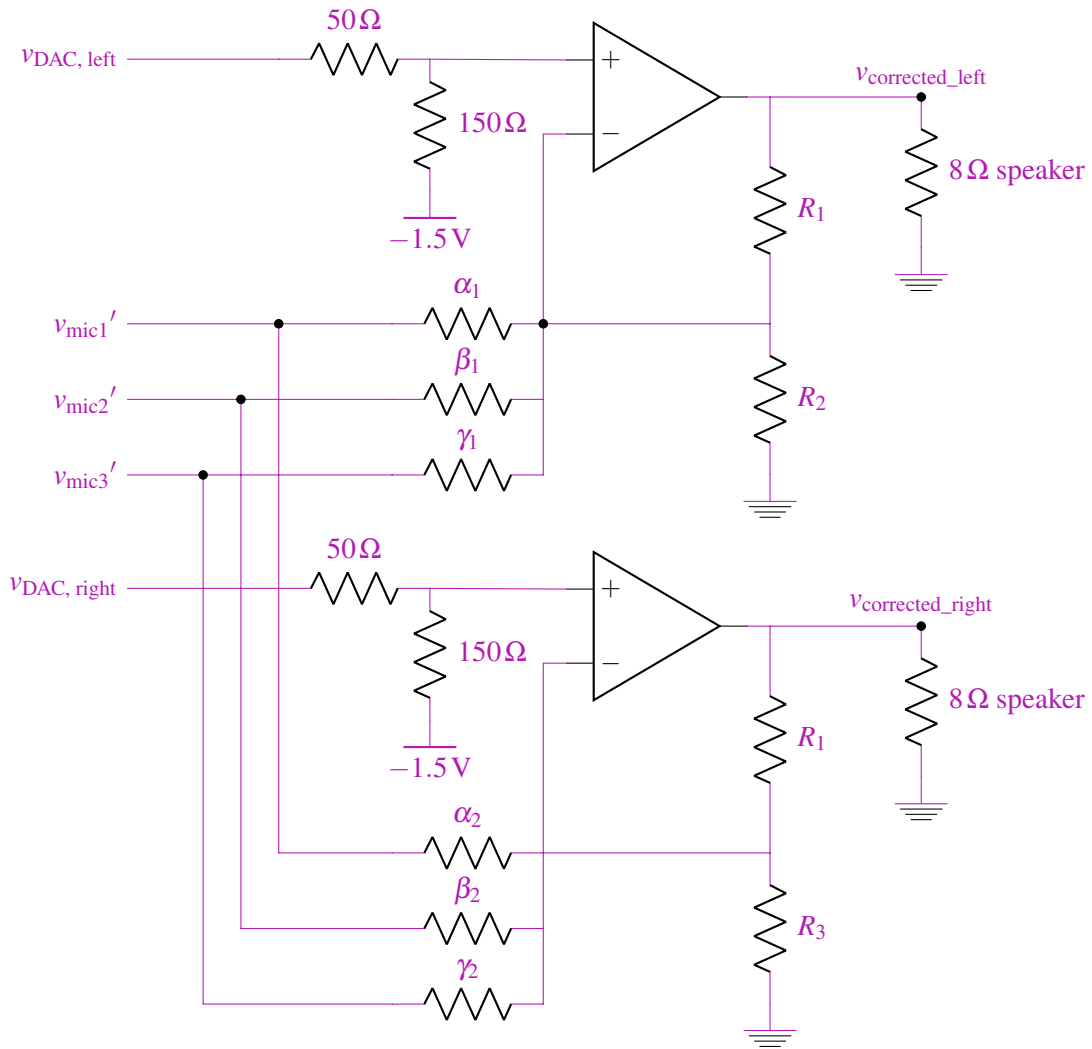
From the first equation,

$$a_1 + a_2 + a_3 + 1 + \frac{R_1}{R_2} = 4$$

$$R_2 = \frac{R_1}{3 - a_1 - a_2 - a_3}$$

Thus, if we pick a value for R_1 , we can use the formulas above to calculate α , β , γ and R_2 .

Now that we have a working circuit for one speaker, we can duplicate this circuit to have two speakers. Notice that in the circuit below we can use the same value for R_1 in the two channels, but we have to keep R_2 as a variable (hence it is replaced with R_3 in the right channel). This is because R_1 is a free variable. If we choose a value for R_1 arbitrarily, we can calculate what the other resistor values have to be with the equations we have derived.



We have seen that if we choose values for R_1 and R_3 arbitrarily, we can find the other resistor values.

$$\alpha_1 = \frac{R_1}{a_{1\text{left}}} \quad \beta_1 = \frac{R_1}{a_{2\text{left}}} \quad \gamma_1 = \frac{R_1}{a_{3\text{left}}} \quad R_2 = \frac{R_1}{3 - a_{1\text{left}} - a_{2\text{left}} - a_{3\text{left}}}$$

$$\alpha_2 = \frac{R_1}{a_{1\text{right}}} \quad \beta_2 = \frac{R_1}{a_{2\text{right}}} \quad \gamma_2 = \frac{R_1}{a_{3\text{right}}} \quad R_3 = \frac{R_1}{3 - a_{1\text{right}} - a_{2\text{right}} - a_{3\text{right}}}$$