EECS 16A Designing Information Devices and Systems I
Spring 2019 Discussion 1A

1. Gaussian Elimination

Use Gaussian elimination to solve the following systems. Does a solution exist? Is it unique?

(a)

$$\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 3 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ -1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

Answer:
The solution is not unique. One solution is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

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(c)

$$\begin{bmatrix} 1 & 4 & 2 \\ 1 & 2 & 8 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

Answer:
No solution. When you do Gaussian elimination, you will get a row that looks like $\left[\begin{array}{ccc|c} 0 & 0 & 0 & a \end{array}\right]$, where $a \neq 0$ in the augmented matrix.

(d)

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$$

Answer:
There are many solutions. One solutions is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

Answer:
No solution.

2. Solving Systems of Equations

(a) Systems of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following system of equations, state whether or not a solution exists. If a solution exists, list all of them.

i. $\begin{cases} 49x+7y=49 \\ 42x+6y=42 \end{cases}$ ii. $\begin{cases} 5x+3y=-21 \\ 2x+y=-9 \end{cases}$ iii. $\begin{cases} 49x+7y=60 \\ 42x+6y=30 \end{cases}$

iv. $\begin{cases} 2x+2y+4z=-1 \\ y+z=-2 \\ x+2y+3z=2 \end{cases}$ v. $\begin{cases} 2x+2y+4z=6 \\ y+z=1 \\ x+2y+3z=4 \end{cases}$ vi. $\begin{cases} x+y+z=4 \\ 3z=6 \\ y+z=3 \end{cases}$

Answer:

i. $\begin{bmatrix} 0 \\ 7 \end{bmatrix} + \begin{bmatrix} a \\ -7a \end{bmatrix}, \forall a \in \mathbb{R}$ ii. $\begin{bmatrix} -6 \\ 3 \end{bmatrix}$ iii. no solution

iv. no solution v. $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} a \\ a \\ -a \end{bmatrix}, \forall a \in \mathbb{R}$ vi. $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

(b) Systems of equations can also be interpreted graphically. We will try to build a graphical intuition for the results you found in the previous part. Follow along as your TA walks through `dis1A.ipynb`.

Answer:

i. The lines lie on top of one another (i.e. they are the same line), so there are an infinite number of solutions. This system is referred to as underdetermined, which means that there are more unknowns than equations. Though it appears we have two equations and two unknowns, dividing the top equation by 7 and the bottom one by 6 quickly reveals that they are both the same equation.
ii. The lines intersect at one point, so the solution is unique.
iii. The lines do not intersect (a little algebraic manipulation on the equations would reveal that the two lines are parallel). There is no solution.
iv. The intersection of the planes is null. There is no solution.
v. The intersection of the planes is a line, so there is an infinite number of solutions. This system is also underdetermined. Subtracting the second equation solution from the third and multiplying the result by two yields the first equation. In other words including the first equation is redundant because any point that satisfies the second and third equation will certainly satisfy the first. In effect, we have two equations and one unknown.
vi. The intersection of the planes is a single point, so there is a unique solution.